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Institute

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# JOURNAL

OF THE

# INSTITUTE OF ACTUARIES

AND

# ASSURANCE MAGAZINE.

"I hold every man a debtor to his profession, from the which as men of course do seek to receive countenance and profit, so ought they of duty to endeavour themselves by way of amends to be a help and ornament thereunto."—BACON.

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JOURNAL

OF THE

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AND

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## *On the Origin and Nature of some of those Limited and Contingent Interests in Property which are commonly submitted to Actuaries for Valuation.* By C. J. Bunyon, M.A., *Vice-President of the Institute of Actuaries.*

[Read before the Institute, 28 April 1873.]

THE present paper is essentially elementary, and must therefore in a great measure be considered as addressed to the junior members of the Institute, although, perhaps, even to those more experienced persons to whom in the course of years these things have become familiar, a recapitulation, such as I now propose, may not be wholly uninstructive.*

These interests, then, appear to divide themselves into two great classes:—(1) interests arising under settlements of either real or personal estate, and (2) interests or liabilities which are the fruits of tenure, as in copyholds, or which require valuation on account of the imperfection or limitation of the subject or property to be valued, as in the case of renewable leaseholds, or terms of years in gross, or estates *pour autre vie*. On the present occasion I propose to confine myself to the consideration of the first class of interests only.

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* The statements of law in this paper are principally taken from "A Profitable Book upon Domestic Law," published by Messrs. Longman & Co., 1873—a volume of legal essays giving in a readable form as much technical information on these subjects as is likely to be useful to persons not actually in legal practice.

Under the general term of settlements I include all dispositions, either by deed or will, under which successive interests are given in any property, either real or personal. There is no difference in the estates or interests which may be conferred, save only that as a will operates from the death of a testator, and a deed from its execution, a testator can, in effect, tie up his property for a longer period than a settlor or donor by deed, as he can give an estate to the child of an unborn child, provided only that the latter be born in his own lifetime, and be sufficiently designated. In other respects, the same rules against perpetuities apply which limit the power of a settlor, and forbid the attempt to create any estate which shall not vest during the life or lives of existing persons and 21 years afterwards, or to confer an estate in any case upon the child of an unborn child as purchaser.

In examining a settlement by deed, and especially a marriage settlement, it will be found that the object of the conveyancer is, in most cases, the exact opposite of that of the actuary or those who consult him. The object of the conveyancer is to tie up the property and keep it in the family, as a provision, as long as the law will permit. On the other hand, the object of the actuary is to resolve into ready money the limited interests parcelled out, and to reduce into possession the promise of the future. Let us, then, consider the usual forms of the limitations in the settlement of personal property.

In such a case, when the money is derived from the lady's family, a first life interest is generally given to her in the income for her separate use, with a restriction against anticipation, thus providing for her a certain and inalienable provision, with remainder to her husband for his life, with remainder, as to the capital, to their children or more remote descendants, in such shares as they may by deed, or the survivor of them may by deed or will, appoint, with remainder in default of appointment to the children equally, to become vested interests in such as may be boys on attaining the age of 21, and in such as may be girls at 21 or previous marriage. In case there shall be no children, the ultimate trust is sometimes for the survivor of the husband and wife, and sometimes, if the husband is the survivor, excluding him for the benefit of the next-of-kin of the wife, being such persons as would have been entitled in case she had died intestate and unmarried, thus returning the money to her own family; or, when the lady's father is the settlor, the ultimate trust, in default of issue of the marriage, may be for him and his personal representatives. When the husband is in trade, it is not unusual to restrict his life interest in remainder, giving it to him only until alienation or bankruptcy, with a gift over on any such event to the children or other persons next in succession, or with a discretionary trust enabling the trustees to apply the income during his life for the benefit of any of these objects, thus indirectly making a provision for him. Sometimes, when it is desired to put the husband on his good behaviour, he takes no immediate interest; but a power is given to his wife to give him by her will a life interest in priority of any children, and in default of children a general power of appointment is vested in her.

When the money is derived from the husband, he is clearly entitled to the first life interest; but as it is one of the principal objects of a marriage settlement to make a provision for the wife and children in the event of the husband's bankruptcy, if he is in trade, it is very usual to give it to the lady for her separate use, without power of anticipation. It would undoubtedly be very convenient here, as in the last case, to give it to him until alienation or bankruptcy; but it has been thought that, except to the extent of any fortune received by him with his wife, it is against the policy of the bankrupt laws to allow a man to settle his own property upon himself to the exclusion of his creditors.

Let us then suppose that the marriage has been solemnized, and that in the course of years a thriving progeny has been born. We have then a number of limited interests; but if the most restrictive form of settlement has been adopted, it may well happen that in no case is any one of them tangible, so as to be worth valuation by the actuary. The life interest of the lady is for her separate use, but without power of anticipation, consequently she cannot deal with it. The interest of the husband may either terminate on alienation, or may depend upon the will of his wife, which is in its nature revocable, or, as the lawyers say, ambulatory, and which will would scarcely be made for the benefit of a purchaser. Over the interests of children dominates the power of appointment under which the share of any particular child, although vested in default of appointment, may be at any time divested by the parents, who, again, are not likely to give much consideration to the claims of a purchaser from an improvident child. Nevertheless, even under the most stringent and restrictive settlement, as time rolls on, the limited interests may become saleable. The husband may die, and thereupon the interest of the wife becomes one with which she may deal with absolute power; for it must be noted that the restraint upon anticipation is the creature of equity, operative only during coverture, and wholly ineffective when the disability of marriage determines. The parents, again, may exercise their power of appointment irrevocably by deed over the whole or part of the fund, without intending to exercise the power of advancement, and affect their own life interests, whereupon the appointed share becomes absolute, and may be dealt with as an absolute reversion.

Should there be but one child, and no prospect of any further family, that child will be entitled to the whole of the settled property, since the power of appointment is one of selection only, and there can be no competitor with him. In like manner, if the children can agree to divide the inheritance, they can do so, and neutralize the power. The interest of the father is not susceptible of valuation, even as a matter of curiosity, for the chance of a forfeiture by him cannot be calculated; but by an equitable bargain with his children, when of age and capable of contracting, he may obtain a release of this liability. We say equitable, because a bargain of this nature, to be upheld, must be free from the obligations arising from the exercise of undue influence, or what is termed a fraud on the power, as where the right of selection is given for the benefit of the children, and it is exercised to obtain a benefit to the party exercising it.

In dealing with reversionary property of the kinds described, there will also be a latent danger still to be inquired after. It is usual in such settlements to provide for a future marriage, by vesting in the person from whom the property proceeds a power of appointment, on that event, over part of the funds in favour of a second husband or wife and family. Should such a power exist, and it is said that it should never be omitted, since it is unreasonable that fortunes should be provided for the first family and the children of a second marriage left destitute, it may wholly prohibit, or, at least, greatly interfere with any dealing with the fund, except through the exercise of an absolute appointment.

It may, however, happen that the life interest of the husband is not terminable by a gift over on alienation, and a mere prohibition or condition subsequent is generally of no effect. In that event he may, of course, sell or charge it, whether it is in possession or reversion, and the life interest of the wife may be for her separate use without any restraint upon anticipation, in which case she may dispose of it as if a single woman. We have then, in the several cases supposed, the following limited interests which may be submitted for valuation to an actuary :—(1) Life interests in possession. (2) Life interests in remainder. (3) Simple reversions subject to such life interests. (4) We may have this much more complicated case, not by any means uncommon, in which the power of appointment does not exist, or has for any reason become inoperative—as, for example, would be the case where it was a joint power, and the death or lunacy of one parent has happened, and the actuary is asked to value the interest of a child who has attained his majority and therewith a vested interest in the fund, subject to be increased by the deaths of his brothers and sisters under age or unmarried, according to the terms of the trust. If there is a large family, the actuary will find in this case ample scope for his ingenuity; but unless the number of very young children is large, will scarcely fail to disappoint his client, with the small value at which he will put his rights of accruer, and possibly be little satisfied himself at the inadequate result of his own labours. (5) We have the interest of the settlor of the fund, probably the father of the lady, if the ultimate reversion is reserved to him and his representatives on the deaths of the husband and wife, and the failure of any issue of the marriage living to attain a vested interest in the fund. (6) Lastly, where the ultimate trust is for the next of kin of the lady, exclusive of the husband when he survives her, we may have the possible interests of a variable class to be ascertained only at her death. Such interests are not likely to be submitted to an actuary for valuation, except in the form of simple reversions, when the reversioner has been ascertained by the death of the first tenant for life. On the hypothesis that that event is still in the future, the student may consult a table deduced from the Statute of distributions of the personal estates of intestates, and from it propose to himself problems of infinite complexity, and sufficient to satisfy the strongest appetite for such mental pabulum. Here, however, he will encounter a contingency which is not susceptible of valuation, namely, that the law, or the construction of the law, shall be the same at the time when he works his solution, and at some future time when he may hope to apply it. As a proof that this is not mere mockery, I may observe that, within the last two years, the Court of Chancery (*in re* Ross Trusts, L. R., 13 Eq., 293) pointed out the true construction of the statute of Charles the Second in so simple a case as that of determining the shares of grandchildren and great grandchildren in the estate of an intestate, in flat contradiction to the dicta of the text books and the probable practice of nearly two centuries.

Under settlements by deed contingent reversions in personalty rarely arise, but under wills they are exceedingly common, for a testator very frequently selects as the objects of his bounty such members of a class as may survive the tenants for life. In them we have here (7) a further interest for valuation.

In all these instances we have been treating of the devolution of personalty, which, subject to prior life interests or on the happening of the particular contingency, when it vests, vests absolutely in the remainderman; but when we deal with landed property, we meet with estates tail which require further consideration. An estate tail is an estate in a man and his heirs of a particular description; if an estate in tail general, in a man and his lineal descendants of either sex being his heirs; if an estate in tail male, descending to his sons and son's sons alone; if an estate in tail female, descending to daughters only and their descendants of the same sex, and any such estate may be granted either to a man or woman. If an estate in fee tail is created, it will clearly not exhaust the fee simple, but a reversion or remainder in fee simple will remain undisposed of, expectant upon the failure of the estate tail, for want of the particular class of descendants entitled to take under it. In like manner an estate may be granted for life or for years, and all these estates may be given in succession until the fee simple is exhausted. Thus the limitations may run to A B for fifty years, if he shall so long live, remainder to C D for life, remainder to E F in tail male, remainder to G H in tail general, remainder to J K in fee simple. The next estate is always called the remainder or reversion, and is entitled to any rents and services payable by the owner of the preceding estate, but, until that estate determines, takes nothing else.

The student of real property will remember that, in the time of Edward I., estates in fee tail had become to be construed as estates in fee simple conditional, under which the tenant in tail, after the birth of heirs, became entitled to alien the land; but that by the statute de Donis, 13 Ed. I., A.D. 1286, it was provided that the form of the gift should be strictly observed, and the land should become inalienable; that, nevertheless, by fine or recovery, alienation became permissible; and that the law is on this subject now governed by the Fines and Recoveries Act, 3 and 4 Wm. IV., c. 74. By this Act, passed in 1833, the law was consolidated and amended by the abolition of fines and recoveries, and the substitution of more simple modes of assurance. Under it the tenant in tail, if in possession (and if in remainder, with the consent of the protector of the settlement, usually the tenant for life), can convert the estate tail into a fee simple barring all subsequent estates; and, without the consent of the protector, the tenant in tail in remainder can convert his estate tail into a base fee or fee simple, conditional upon the continuance of issue inheritable under the entail. In this latter case the remainders over are not affected by the disposition of the tenant in tail, unless, after the estate tail falls into possession, he executes a further deed in conformity with the Act; but if possession under the assurance which creates the base fee continues for 20 years after the termination of the prior estate, or, more accurately, for twenty years after the commencement of the time at which such assurance, if it had been then executed by the tenant in tail, or the person who would have been entitled to his estate tail if the base fee had not been created would have been an absolute bar, the Statute of Limitations, 3 and 4 Wm. IV., c. 27, s. 23, expands this base fee into a fee simple.

When land is settled so as to create life estates and estates tail in succession, it is commonly said to be placed in "strict settlement." Let us then consider shortly the form of such a conveyance. It usually runs as follows:—To trustees for a term of years, commencing with the solemnisation of the marriage, upon trust to raise an annual sum for the pin-money of the lady, with remainder to the husband for life, with remainder to the use that the intended wife, if surviving her husband, shall receive an annual rent-charge for her jointure in bar of dower, with powers of distress and entry upon the land for securing it, with remainder to another set of trustees for a term of years as a further security for the jointure, with remainder to a third set of trustees for a term of years in trust to raise by way of mortgage or sale, or out of the rents and profits, a sum of money for the portions of younger children, the amount generally depending upon their number, with remainder to the unborn children in tail male, first to the sons and next to the daughters successively, in order of seniority of birth, with remainder to the sons successively in tail general, with remainder to the daughters also successively in tail general, and so that the elder shall always be preferred before the younger. In default of such issue, life estates may be given to other members of the family, such as the brothers of the settlor, with remainders to their children in tail, but in all cases the ultimate limitation is to the settlor in fee simple.

The foregoing relates to estates of freehold tenure, but copyhold and leasehold estates are also included in settlements of this nature. This is effected in the case of copyholds by a covenant to surrender, followed by an actual surrender of the land to the uses of the settlement, if authorised by the custom of the manor; or, which is more common, to the use of trustees, according to the custom of the manor; upon trusts as nearly as possible corresponding with the uses declared of the freeholds as the rules of law will permit. Leaseholds are, in like manner, dealt with according to the last described device; but as an estate tail cannot exist in leaseholds, or any personal property, an attempt to create one is simply equivalent to an absolute gift to the first tenant in tail, which would be transmissible to his personal representatives, instead of the next inheritor of the settled property; in order to keep the freeholds and leaseholds together, it is usual to add a condition, that the leasehold shall not vest absolutely for the purpose of transmission in any such first tenant in tail taking by purchase who does not attain the age of 21 years, but shall, on his death under age, go over to the next person entitled to the freeholds under the settlement. Personal property of any other description which it is desirable to keep with the estate, such as jewels, or pictures, or the furniture of the principal mansion, are treated in the same manner, and are then, in popular language, sometimes described as heirlooms, although heirlooms, strictly so called, are only such chattels as by custom belong to the freehold—muniments of title, for example, and the chests in which they are contained. When property has been thus settled, it is said to be in strict settlement, or, in common language, entailed.

The leading idea in every such settlement is to give life estates alone to existing persons, and estates in remainder in tail to those who are not in being. The rules against perpetuities are thus observed, for successive remainders in tail are permitted in any number, on the ground that any tenant in tail in possession may acquire the fee simple, although it is possible that the property may continue in one family under an estate tail for generations.

Under such a settlement as we have described it is obvious that there are a variety of interests which may become the subject of the calculations of an actuary. There may be successive life estates, which may be valued in the same way as if they were life estates in the income of a personal fund, and there are successive estates tail, which may be treated as valuable, although a vested estate tail in remainder following a prior estate tail, whether in remainder or not, can scarcely be a security, unless circumstances of a special character, such as idiotcy or incurable lunacy, have destroyed the power of the first tenant in tail to deal with his estate. There are, however, two cases commonly arising in practice upon these settlements which have been already much discussed in the pages of the *Journal*. They are, first, the interest of a tenant in tail in remainder expectant upon the lapse of a prior life estate, usually that of his father, and, secondly, the life interest expectant not only upon the determination of the previous life estate, but also upon the failure of any issue of the tenant for life to take as purchaser, being the case usually of a nephew, heir presumptive under the settlement of his uncle. The difference between the two is compensated by an insurance against issue of the tenant for life.

Insurances against the birth of issue, which have sprung into common use within the memory of many of us, appear to be rather a surprise to those who are outside of the profession, although, as may be learnt from the amusing pages of Francis on *Life Assurance*, sporting risks of every description of the nature of wagers, and to cover every possible contingency before the Gambling Act, were commonly undertaken as assurances. In the second edition of the *Law of Life Assurance*, I observed that the difficulty of estimating the premium for such an insurance might have been thought almost insuperable, but that the skill and industry of actuaries had overcome it. On this, however, a very leading authority by whom the book was reviewed, in terms of which the author had certainly no right or desire to complain, made the following comment:—"We can hardly suppose that an "actuary of Mr. Bunyon's experience can be serious in such an "assertion. He must surely be poking fun at the charlatanery "which would pretend that the chances of a bachelor over "middle age falling into the toils of matrimony, and if then "having a family, is in any honest sense a subject for actuarial "calculations."

These insurances are not solely in use when a security is proposed upon an estate tail in remainder. In one case which I can remember the limitations were to a father for life, remainder to his eldest son for life, remainder to the issue of the eldest son as purchasers in tail, remainder to the second son for life, remainder to his issue as purchasers in tail, and when the father and eldest son, who were of middle age and married, had exhausted in a great measure their powers of borrowing upon the life estates, the life estate of the younger brother is brought into use, and by the help of an insurance against issue of the elder, fresh loans were raised upon the insurance of a young and fresh life only.

In dealing with interests such as we have described, it is very important to know accurately the advantages which they possess, and hence we should note that in all cases the income is apportionable between the representatives of the particular tenant and the next remainderman. In former times, rents received, annuities, and like periodical payments, were not apportionable, but by a series of Acts of Parliament this evil has been sought to be remedied, the first of such Acts having been passed in the reign of Geo. II., the last, by which the law is now governed, being the "Apportionment Act, 1870" (34 & 35 Vict., c. 75). This latter Act is couched in the most sweeping language.

For our present purpose, however, apportionment is principally noteworthy as showing that the interests with which we have to deal are represented by complete annuities, while the formulæ with which we usually seek to measure them are expressed in the ordinary amounts payable only once a year, and not apportionable.

The subject of the valuation of reversionary interests, namely, the simple reversion, the contingent reversion, and reversionary life interest, has been already repeatedly discussed before the Institute, and has been treated of in papers written by some of our most eminent members. I do not, therefore, propose to retrace familiar and already well-trodden ground, but bearing in mind the last observations which I have made, I pause to point out that from one well known formula, slightly modified, all the problems usually proposed for official calculations can be solved with great facility. The formula to which I allude is that of Mr. Jellicoe, already so familiar to all present,

$$1-(P+d)(1+a_{xy}),$$

the value of a contingent reversion.

In this case the unit of measurement, less the value of the life interest and the costs or allowance for insurance, gives the value of the reversion, and the only question can be that of the accuracy of the expression of the idea. It is, however, obvious that the symbols do not fully express it. We want a complete annuity for the life interest, and are driven to an annuity paid in advance, while in either of the cases in which an annual premium is involved such premium requires adjustment, since it is not calculated to fit the transaction, but varies with the rules of the office. If, however, we vary the formula by writing thus,

$$1-i\overset{\circ(2)}{a}_{xy}-\mathrm{P}'(1+a_{xy}),$$

marking P with an accent, to show that it requires adjustment, and consider the expression that of the value of 1 charged upon the reversion, and protected by insurance when necessary, our problem works out with wonderful facility.

$i$ is the interest which we desire to realize, while the charge continues reversionary; $\overset{\circ(2)}{a}_{xy}$ is a complete annuity, or rather an annuity payable half-yearly and apportionable to death. The values of complete annuities have been discussed with great learning by our Vice-President, Mr. Sprague, but for our present purpose it seems sufficient to consider such an annuity $=a_{xy}+\frac{1+\mathrm{A}_{xy}}{4}$, and if in an office calculation we remember that the extreme limit of the fraction is ·5, and take the turn in favour of the office, no great injustice is done, and we need not wait long to find our complete annuity.

P′ next requires adjustment. If the reversion is absolute, it has no value, and the term in which it is involved vanishes; if, on the other hand, it is the premium for a contingent insurance, it must be adjusted to insure not only 1 but the interest of 1 for the term of delay which will occur between the death and the payment of the claim. If, again, we are dealing with a life interest only, and the premium may require to be paid, after the death of the tenant for life, before a sufficient fund for its payment has accumulated, we must also insure for a year's premium. Assuming that a month is required to prove the death, and that the claim is paid at the end of 3 months, P′ in the two cases assured becomes $\mathrm{P}\left(1+\frac{i}{3}\right)$ or $\mathrm{P}\left(1+\frac{i}{3}+\mathrm{P}\right)$. Working in this way, we rapidly ascertain the present value of our reversion, or the sum which can be given for one pound secured, and if an issue premium is payable, or a whole world single premium, they are also deducted, and scarcely increase the complexity of the calculation.

If a reversionary annuity is desired instead of a reversionary charge, we have an annuity of $i+\mathrm{P}'$ in the place of the latter, and we may take $i$ at any rate which we think most just, charging, as proposed by Mr. Sprague, one rate (6 per cent.) while the charge is in reversion, and a reduced rate (5 per cent.) when it falls into possession. The purchaser of the annuity may have to advance the premium, but he will certainly receive it again, and he will have the policy for an amount which will probably give a small profit, and he will not be liable to account. The real difference between the annuity and the charge is seen by considering the former a charge which cannot be paid off, but for the principal of which the borrower is not responsible.

The great advantage of dealing with the problem in this form appears to be that the calculator has its component parts clearly before him. If he uses 3½ per cent. interest for the annuity, and requires 5 per cent. interest on his advances, he can ask himself the question whether, if he can make 4¼ per cent. upon an ordinary security, it is worth his while to grant an immense and exceptional annuity, on the Carlisle or any other table, with a loading of rather more than 17½ per cent. on the value, or 15 per cent. on the result, the difference between 4¼ and 5 per cent. the interest reserved. Again, he may deal with his insurances with the greatest certainty; he may even think it worth while to share his risk with the other companies with whom he divides his insurances, by paying a single premium for the joint lives, a course which has actually occurred in practice, and may perhaps at once realise a profit on that part of the transaction.

To determine the nature of the insurance to be effected appears one of the most important parts of the problem. If the tenant in tail in remainder has children, since the base fee continues during the existence of issue inheritable under the estate tail, it will be sufficient to insure for a sum payable upon the death of the last survivor of the whole; and as the object will be to reduce the rate of premium to a minimum, the purchaser would perhaps be sufficiently secured by an insurance payable on the death of such survivor during 21 years after the death of the tenant for life. As these reversionary charges are usually created by very young men, it generally happens that they have no descendants at the time, and no life beyond their own can be looked to. It seems, however, that the following course is occasionally adopted:—The borrower, having executed a deed of disposition under the Act, and acquired a base fee, executes a power of attorney authorising the lender and his solicitor, and perhaps half-a-dozen of his clerks, or any one of them, as soon as the tenant for life dies, to execute and enrol the further disentailing deed, or deed necessary to convert the base fee into a fee simple, and bar the remainder. This deed is then prepared ready for execution, and as soon as intelligence is received of the death of the tenant for life, one of the attorneys executes the deed, which is then enrolled without any further consent of the borrower. Several names are inserted in the power to meet the risk of the attorney dying prematurely. The risk of the death of the borrower before the life tenant is met by insurances on his life against that of the latter, and the very remote risk of his surviving him, and dying before the execution of the deed, is met by making the risk under the contingent policies extend for, say, three months after the death of the tenant for life.

Upon this the important question arises, is it perfectly secure? I must admit that I am not wholly satisfied that it is so. The plan has never received any judicial confirmation, and it is certainly driving a coach and six through the Fines and Recoveries Act, which provides (s. 40) that no disposition by a tenant in tail resting only *on contract*, express or implied, or otherwise, and whether supported by a valuable consideration or not, shall be of any force at law or in equity under the Act, and which requires the enrolment in the Court of Chancery of the disposition of the tenant in tail within 6 months after execution (s. 41). If, moreover, the vendor filed a bill to set aside the security, and expressly revoked the power of attorney, which bill was pending, at the time of the death of the tenant for life, is it clear that the deed would be enrolled, or would be the deed of the donor of the power, and satisfy the statute? A power of attorney, it may be said, when given for valuable considerations, is not revocable in equity, and a Court of Equity will enforce a contract to bar an estate tail, but it is always optional for a court of equity to interfere, and if the dispute is protracted, and the tenant in tail dies before it is decided, without having issue, what is to prevent the next remainderman from entering upon the property?

The student will not fail to observe that the whole of these limited interests in succession, when valued, are the equivalent of the absolute or entire interest; as all the successive estates, which, according to the rules of law, can possibly be created together, represent the fee simple. Inquiry may then be made respecting the long terms of years which we have mentioned, and whether they are susceptible of or require valuation. To this the answer is in the negative. So long as they remain vested in the trustees, or their representatives, they are mere securities for the due payment of the rent-charges, or the due raising of the portions, and in modern conveyancing rarely, if ever, are called into operation or dissociated from the freehold—becoming, in legal language, terms in gross. They are rather the machinery by which the family charges are fixed upon the freeholder, who is compelled to provide for their extinguishment. As the object for which the terms are created are fulfilled, the terms themselves become extinguished, either by virtue of clauses expressly providing for that event—technically, provisoes for the cesser of the terms—or by becoming satisfied terms, or terms the object of whose creation is fulfilled, when, by "the satisfied terms Act," they absolutely cease. Another somewhat curious mode of dissolution for long terms of years is by merger. By the doctrine of merger, whenever two estates, one of which is immediately reversionary to the other, meet in the same person and in the same right, the prior being the lesser estate, unless it is an estate in tail, will merge or become extinguished in the latter, the union of the two having the effect in law of the surrender of the former. A term of years will merge in a life estate or estate of freehold, and a life estate in a fee. And one term will merge in another if immediately reversionary to it, and that although the previous unextinguished term should be of the longer duration. If there were a dozen terms, of which eleven were for periods of one thousand years immediately reversionary to one another, and the last term the reversion to the whole were for two years only, and all became vested in one person in his own right, instead of his becoming owner of the property for the accumulated period of the successive terms, the little term of two years, like Moses' rod, would swallow up and extinguish the rest, and there would be no law or equity to restore them.

To prevent such a merger contrary to the wishes of the owner, is one of the arts of the conveyancer, and is readily effected by assigning the term not intended to merge, or in such a case as that of the twelve consecutive terms every alternate term, to a trustee for him.

Although the object of the settlement is permanence, it will be found, like all other human institutions, to have in itself the germ of its decay in the necessity of raising portions for the younger children of the family. The jointure of the widow is onerous, but terminates with her life, the portions subtract a part of the capital of the estate itself. The eldest son cannot always marry a heiress of sufficient fortune to disencumber the property; and should an hereditary susceptibility always prefer personal charms to the money bags, in two or three generations, where the tenant for life only lives to the majority of the eldest son, leaving behind him a jointress and a large family of younger children, the incumbrances may so accumulate as to lead, first, to serious embarrassment, and ultimately to the sale of the estate. As a remedy for this, an eminent conveyancer prescribes life insurance, proposing that each successive tenant for life should in this manner provide for his own children. To this excellent plan there is only one objection, and that is the premium for the insurance. It is indeed suggested that not only should the tenant for life insure, but be so virtuous as to pay up his premiums in fifteen years, and thus get rid of the burthen when the children become expensive. But the natural tendency is to devise some plan to reduce the expense at the outset. This may be done in more ways than one. One mode, which we have seen in practice, is to insure for a sum payable on the death of the survivor of the tenant for life and his wife. Another plan would be to insure for a sum not payable in any event for twenty years from the date of the policy, but with the premium payable only during the life of the tenant for life, and which premium might then admit of a considerable reduction.

To these suggestions no possible objection can be made by the Institute of Actuaries, and it may be worth while considering whether a table might be formed according to the last proposal. There would be no difficulty in its calculation, as the formula would obviously be

$$\frac{v^{20}\mathrm{D}_x - \mathrm{D}_{x+20} + \mathrm{M}_{x+20}}{\mathrm{N}_{x-1}}.$$

It would, however, require one practical warning, namely, that the measure of the probability of death in the first twenty years should be taken as sufficiently onerous. If the Northampton table were used, a good result would undoubtedly be obtained; but if either the New Experience, or that of the 17 Offices', were adopted, the office would run the risk of losing the value of the benefit of selection, or of the light mortality of the early years of insurance, which notably forms the principal source of profit of a young company.

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The following account of the discussion which followed the reading of the paper is abridged from the *Insurance Record:*—

Mr. SPRAGUE remarked that several of the points touched upon in the paper were perhaps familiar to but few of them—some, in fact, to none except those who had had some amount of practice as conveyancing barristers, in which respect probably Mr. Bunyon, as an actuary, stood almost alone. That being the case, he would venture to suggest to Mr. Bunyon that it might be advisable to give a few explanations of some of the technical phrases he had used. There was one in particular that it was desirable to have explained. Mr. Bunyon spoke about the modes of assurance—not meaning life or fire insurance, but the method of assuring an estate to the purchaser, and he (Mr. Sprague) thought it would be as well to explain that. Mr. Bunyon's paper is very interesting, showing as it does how the limited interests which are proposed to insurance companies as securities, take their origin. They very rarely come before the companies soon after the settlement which creates them. It may sometimes happen that a young man comes to borrow money upon his life interest almost immediately after his marriage, but that is a rare case. These interests usually come to them after there has been a lapse of time sufficient to produce changes in the state of affairs, and it is curious to trace the great changes that do sometimes take place. In practice, what an actuary would call the most improbable combination of deaths sometimes takes place—a state of things results that they would have thought almost inconceivably improbable; and it is interesting to trace out what would be the legal questions when these improbable contingencies arose. Several of these points are suggested by the paper. One has regard to the restriction against anticipation by a married woman of her life interest. Mr. Bunyon says that any clause of that sort against anticipation is only good during coverture; and that when the lady becomes a widow, she may, notwithstanding any such clause, charge or encumber, mortgage or sell, her interest. Then, there is another point mentioned, that it has been thought contrary to the policy of the law that a man should be able to settle his own property on himself to the detriment of his creditors. He (Mr. Sprague) did not quite gather whether that is a legal decision or merely the dictum of some text-book writer, and perhaps Mr. Bunyon would give them some information on this point. The only other matter he had to mention, was with reference to the description of the process of the tenant in tail giving a charge upon his estate tail. That was a subject which he (Mr. Sprague) had gone into at some length in one of his papers, and in listening to Mr. Bunyon it almost seemed that one or two of his sentences were quoted from his paper.

Mr. Bunyon remarked that they certainly were, because Mr. Sprague had put the matter as clearly as it was possible to express it.

Mr. Sprague said that the most valuable part of the paper, perhaps, was the new formula for the valuation of reversionary life interests. He had thought this subject was exhausted, that they had got every combination of formula which was likely to be serviceable; but it appeared that Mr. Bunyon had suggested something quite new.

Mr. Walford said it seemed important that the students of the Institute should, as far as possible, make themselves familiar with those law terms which had perhaps been newly introduced to their notice that evening; so that when considering securities in connection with the increased combinations and powers of conveyancing, they might have a definite and distinct idea of their meaning. With regard to the term "assurance", Mr. Sprague had properly pointed out that it required to be defined. Up to the time when Mr. Babbage used the word "assurance" to signify life as distinguished from fire "insurance", it had an express legal meaning. It was a mistake to use the word assurance in the sense of insurance, which was the proper term to represent the profession to which they belonged.

Mr. **Bailey** was not aware that the word assurance was first introduced by Mr. Babbage, but he did recollect that Professor de Morgan, in one of his papers, pointed out as a distinction, between life assurance and fire insurance, which the courts of law had not found out—that the one is a contract of indemnity and the other is nothing of the sort. If any gentleman would do him the honour to call upon him at the London Assurance Corporation, he would show him a life policy dated 1721, by which the Governors of that corporation did "assure" that a particular gentleman should continue in this life for the term of one year, and they added the words, "by permission of Almighty God". In the case of Godsell *v.* Boldero, the term "assurance" was applied to life, and "insurance" to goods, ships, and so forth. The particular term might not be the happiest possible, and might sometimes be misleading; but, still, he did not know any other to suggest in its place.

Mr. R. P. **Hardy** said he thought that Mr. Bunyon's paper was very seasonable at the present time. Most of them were aware that the syllabus for the Institute examinations has recently been revized, and among the subjects added is the law of real property, and special reference is made to a book containing a series of questions, amounting in fact to a digest of Joshua Williams's book. It was thought necessary to draw attention particularly to that subject at the present time, when dealings with reversionary property are assuming such large proportions; and there is little doubt that it is almost impossible to get on without some knowledge of these legal questions. Not very long ago he had before him a lengthy statement which set out almost in the manner of an abstract of title the various estates that had been carved out of a fee simple, with remainder in fee to a certain person; and the problem was to find a value for this ultimate reversion. It might have been possible to take out at length the successive values of the several intermediate life interests, but when they remembered that it was possible for any tenant in tail to bar the remainder over by making a base fee, or getting the consent of the protector of the settlement, it was quite evident that the reversion was liable to be defeated at any time, and consequently he stated that that reversion was valueless. Another case came under his notice where it was proposed to raise money by way of reversionary charge. The ages of the parties were such as would not make the transaction practically possible; but having just a smattering of the knowledge of conveyancing, he was enabled to suggest a means of doing it by barring the entail. The father objected on principle; but it turned out, however, that he would have no objection to the course suggested by him (Mr. Hardy), of a resettlement of the property, reserving to the tenant in tail a power of appointment for as much money as was required to make up the reversionary charge. By adopting this method they not only saved the insurance premiums, but the transaction became possible. He mentioned these cases to show that when one gets a smattering of legal knowledge on these subjects, it is more easy to deal with them financially. He should like to ask Mr. Bunyon to add to his paper some few remarks upon some practical difficulties of dealing with reversions which beset those who are newly engaged in dealing with them. One is what is termed a fraud upon the power, where the transaction becomes tainted by a corrupt understanding between the appointer and appointee. The power of appointment is a trust which may be exercised at unfettered discretion, but must be exercised *bonâ fide*. In a case which had come under his notice, the father for a consideration appointed a large portion of the fund to his eldest son, who immediately raised money upon it, and they shared the proceeds. That transaction ran the usual time, as these things always do, until the money had to be repaid, when the son turned round and impeached the transaction, and the court set the whole security aside. Those who deal with reversions should look carefully to see why an unequal distribution of the fund has been made, and if not, they have to bear the burden of proof that the transaction is entirely free from taint. He was pleased to hear from Mr. Bunyon that by the operation of time a base fee could be enlarged into an estate in fee simple. He had thought it must always remain subject to the limitations declared.

Mr. **Bunyon** said that the point was governed by the 24th section of the Statute of Limitations.

Mr. W. B. **Hodge** said that, from Mr. Bunyon's reputation, he quite expected that they would have had an interesting paper, but certainly many of the facts brought before them had raised points which were new and of a most important character. With reference to the remarks of Mr. Hardy about a fraud on the power, there is no doubt great danger in dealing with reversions derived under power of appointment, and great care is required to see that no fraud is attempted. He remembered in the case of a large estate, which was some years ago the subject of continued litigation, and the life tenant was in a state of great destitution, there was a provision for £40,000 to be charged upon the fee simple for the younger children of the life tenant. There were three children—the eldest son, the second son, and a daughter. The second son fell into very bad health, and if he died during the lifetime of his father without the appointment being made, the whole of the funds would have gone to the daughter; but when the death of the second son appeared certain, the father made an appointment of the whole sum to him, and then, when the son died, made out a claim of £40,000 against his first son as legal representative of his deceased son. A bill was filed in the Court of Chancery, and the whole transaction was set aside, the daughter inheriting the £40,000. One thing was remarkable with regard to Mr. Bunyon's paper, which was greatly at variance with popular prejudice. There is a notion that the law favours the aristocracy in entailing estates in perpetuity. The law does nothing of the kind, but has been engaged in attempting to break down these entailed estates, and in giving facilities for effecting that object. It is not the law that favours the entailing of estates, but the habits of the people. It is the habit of a large class of landowners who are anxious to entail property upon the eldest sons, and the great object of every family is to prevent any one getting at the fee simple of the estate. And that is done in various ways. When a man comes of age and is going to marry, or wants money, a resettlement of the estate is proposed, and the estate is shut up for another generation, and in this way becomes a largely-entailed estate. If it were the opinion of the world generally that a man should not give the whole of the property to the eldest son, that would be done away with; but in the present state of society nothing but the law would be able to prevent them.

The PRESIDENT said this subject was very interesting, as the cases with which it dealt were frequently occurring, and he had himself had several within a short time. He was struck with one mode of settlement that Mr. Bunyon had mentioned, and that was where a husband had been put on good behaviour, and the power of settlement was given to the wife. That was sometimes very fortunate for the family. He recollected some years ago a painful case of that nature, where a family of considerable wealth had been reduced to bankruptcy, the husband taking the reversionary life interest under his wife's settlement, and the reversionary income, of course, becoming the property of the creditor. If the husband had survived his wife, he would have had nothing to live upon. It therefore became important for the family to buy his reversionary life interest. It so happened that he was nine or ten years older than his wife, and when the life interest was submitted to him (the President) he could not make it worth anything like half a year's purchase. It was then submitted to another actuary, who made it worth a year and a half's purchase. It was eventually put up to auction, and knocked down for £10 over the half year's purchase, the price the assignee had put upon it. Mr. Bunyon had probably noticed a case reported in the papers a short time ago, where a person was very anxious to keep the estate in his family, but unfortunately he had excluded one daughter who had married contrary to his wishes, and the whole legitimate issue completely failed, except as regarded this one daughter, who had a numerous progeny. The contest was between the illegitimate members of one of the owners and this unfortunate daughter; and he thought the Court decided in favour of the illegitimate branch of the family, but he did not know whether that would stand or not.

Mr. BUNYON, in reply, said the subject of his paper might have appeared trite and familiar to many of those who listened to it; but it occurred to him that it might be useful to the junior members of the Institute, as they could refer to it to see how the interests which they were asked to value arose. One speaker had made an observation as to the settlement of a man's own property in case of bankruptcy. It has been held in the Court of Chancery that altho' a man on his bankruptcy may settle property which comes to him from his wife, he cannot settle his own property so that it shall pass away on his bankruptcy, on the assumed ground that that would be something like fraud on his creditors. He could not help thinking that this might not be the ultimate decision, because as a man can give his wife a life interest in any property he likes, and afterwards leave it to his children, and thus denude himself of it entirely, it seems rather hard that he should not be able to keep some interest when he might have given away the whole. However, that is the law at present, and must be so accepted. As to the question of frauds on the power, he did allude to that in his paper; but it is exceedingly difficult to ascertain when there is a fraud on the power in bargains between father and son. It was to be noted that the bargain may be bad between the parties to it, and, nevertheless, it may be good as regards the mortgagor or purchaser under it, because he may have made every enquiry and there may be no ground for fixing him with fraud. Still, whenever an appointment is set aside, there may be a considerable amount of risk run, and, therefore, purchasers should see that their hands are perfectly clear in dealing with them. With regard to the formula, he did not pretend to say that it is at all new, but it is rather a short cut—a simplification of the more elaborate forms of calculation, and is also useful for checking, as no actuary ought to be content with making his calculations in one way merely.

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## *On the Arithmometer of M. Thomas (de Colmar), and its application to the Construction of Life Contingency Tables. By* Peter Gray, F.R.A.S., F.R.M.S., *Honorary Member of the Institute of Actuaries.*

(Continued from Vol. xvii page 266.)

Problem V.—To construct a complete table of the Values of Temporary Annuities; that is, a table which shall comprise all ages and all durations.

This benefit, being a function of the ages at commencement and cessation, say $x$ and $w$ (to the former of which ages also its value is referred), it will be here for convenience denoted, in accordance with the usual functional notation, by $\phi(x, w)$; and its duration, consequently, will be $w-x=n$.

Its value is

$$\phi(x, w)=\frac{N_x-N_w}{D_x}=D_x^{-1}.(N_x-N_w) \quad . \quad . \quad . \quad (6)$$

The work of construction will be arranged as in the example following, in columns, having at the top the successive values of $x$, and at the side those of $w$. Hence $w$ will vary in the columns, and $x$ in the rows.

The formation in columns being the more convenient, let, first, $w$ vary. Then we have from (6),

$$\Delta_w\phi(x, w)=-D_x^{-1}.\Delta N_w=D_x^{-1}.D_{w+1};$$

whence, $\phi(x, w+1)=\phi(x, w)+D_x^{-1}.D_{w+1}.$

This is of the form $P+QR$, $Q(=D_x^{-1})$ being constant. By the aid of this expression the columns will be successively formed.

| | | 134392 | 139105 | 143852 | 148661 | 153569 | 158614 |
|---|---|---|---|---|---|---|---|
| $w$ | $D_w$ | 10 | 11 | 12 | 13 | 14 | 15 |
| 10 | 744094 | ·0000 | | | | | |
| 11 | 718881 | ·9661 | ·0000 | | | | |
| 12 | 695159 | 1·9004 | ·9670 | ·0000 | | | |
| 13 | 672671 | 2·8044 | 1·9027 | ·9677 | ·0000 | | |
| 14 | 651175 | 3·6795 | 2·8085 | 1·9044 | ·9680 | ·0000 | |
| 15 | 630462 | 4·5268 | 3·6855 | 2·8113 | 1·9053 | ·9682 | ·0000 |
| 16 | 610342 | 5·3470 | 4·5346 | 3·6893 | 2·8126 | 1·9055 | ·9681 |
| 17 | 590641 | 6·1408 | 5·3562 | 4·5389 | 3·6907 | 2·8125 | 1·9049 |
| 18 | 571212 | 6·9085 | 6·1508 | 5·3606 | 4·5399 | 3·6897 | 2·8109 |
| 19 | 551917 | 7·6502 | 6·9185 | 6·1546 | 5·3603 | 4·5373 | 3·6864 |
| 20 | 532763 | 8·3662 | 7·6596 | 6·9210 | 6·1524 | 5·3555 | 4·5314 |
| * | * | * | * | * | * | * | * |

The process is as follows:—The successive values of $D_x^{-1}$ $(=1\div D_x)$ are written in order, each at the top of its proper column, and a card containing a transcript of Column D is placed at the side, as shown. Then, observing that, as appears from (6), when $x=w$, $\phi(x, w)=0$, this may be at once inserted as the initial term in all the columns. Now, set $D_{10}^{-1}=134392$ on F, and multiply successively by $D_{11}$, $D_{12}$, &c.; the results will be the terms in Column 10 opposite the ages 11, 12, &c. And so Column 10 will be filled up.

Moving the card forward into connexion with Column 11, setting the reciprocal at the top of the column upon F, and multiplying successively by $D_{12}$, &c., this column will be filled up in like manner. And so the columns may be filled up in succession, till the table is completed.

The method of construction here may be looked at in a somewhat different manner. The terms occupying, say, the column headed $x$, are the values of annuities on $(x)$ to last one year, two years, three years, &c.; and they are respectively equal to

$$\text{One year, } \frac{D_{x+1}}{D_x},$$

$$\text{Two years, } \frac{D_{x+1}}{D_x}+\frac{D_{x+2}}{D_x},$$

$$\text{Three years, } \frac{D_{x+1}}{D_x}+\frac{D_{x+2}}{D_x}+\frac{D_{x+3}}{D_x},$$

and so on, the annuity for $n$ years being formed by adding another term, $\frac{D_{x+n}}{D_x}$, to that for $n-1$ years. And the process described is nothing else than the orderly formation of these terms, and the adding of each, as it is formed, to the preceding result.*

The intelligent computer, also, will not fail to observe that, as the additions made in passing from each value to the next consist at the outset of no more than four places, which number becomes reduced also, as we descend the columns, to three, two, and one, it would be a waste of power to continue to employ six places in the factors by the multiplication of which they are produced. It must be left to his own judgment to determine the points at which, and the extent to which, curtailment of the factors can with safety be made. The verifications will serve as a check on excessive curtailment.

Verification will be obtained, as in other cases, by the separate formation of a few rows at suitable intervals.

When $x$ varies, we have, from (6),

$$\Delta_x\phi(x, w) = \Delta.(D_x^{-1}N_x) - N_w.\Delta D_x^{-1},$$
$$= \Delta a_x - N_w.\Delta D_x^{-1};$$

whence, $\phi(x+1, w) = \phi(x, w) + \Delta a_x - N_w.\Delta D_x^{-1}$.

This expression is unsuited for our purpose; it does not give a continuous process fitted for the application of the Arithmometer. We must, therefore, arrange for the independent formation of each term.

Resuming (6), the successive values of the numerator, when $x$ varies, are easily formed, as follows:—

$$\Delta_x(N_x - N_w) = -D_{x+1};$$

whence, $(N_{x+1} - N_w) = (N_x - N_w) - D_{x+1}$.

So that, forming this function for $x=10$ and any constant value of $w$, say 20, the succeeding values will be formed by the continuous subtraction of the terms of $D_x$ in order, commencing with $D_{11}$. Then, writing opposite the terms thus formed the reciprocals $D_{10}^{-1}$, &c., also in order, multiplication of the corresponding terms, by the machine, gives the required results.

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* See General Hannyngton's paper, already referred to, p. 252.

The following is the whole of the work for this row:—

| | $N_x - N_{20}$ | $D_x^{-1}$ | $\phi(x, 20)$ |
|---|---|---|---|
| $N_{10}$ | 1796867 | | |
| $N_{20}$ | 1174345 | | |
| 10 | 622522<br>71888 | 134392 | 8·3662 |
| 11 | 550634<br>69516 | 139105 | 7·6596 |
| 12 | 481118<br>67267 | 143852 | 6·9210 |
| 13 | 413851<br>65118 | 148661 | 6·1524 |
| 14 | 348733<br>63046 | 153569 | 5·3555 |
| 15 | 285687<br>61034 | 158614 | 4·5314 |
| 16 | 224653<br>59064 | 163843 | 3·6808 |
| 17 | 165589<br>57121 | 169308 | 2·8036 |
| 18 | 108468<br>55192 | 175066 | 1·8989 |
| 19 | 53276<br>53276 | 181187 | 0·9653 |
| 20 | 00000 | 187701 | 0·0000 |

The formation of three or four rows in this manner will be sufficient, the values formed being inserted in their places before commencing the construction in the columns.

Problem VI.—To construct a complete table of the Values of Deferred Annuities.

This benefit, like the last, being a function of two ages, namely, the present age, $x$, and that at which the annuity is to be entered on, $w$, it will be, for our present purpose, denoted by $\psi(x, w)$.

We have, then,

$$\psi(x, w) = \frac{N_w}{D_x} = D_x^{-1}.N_w \quad . \quad . \quad . \quad . \quad . \quad (7)$$

First let $w$ vary, and we have,

$$\Delta_w \psi(x, w) = D_x^{-1}.\Delta N_w = -D_x^{-1}.D_{w+1}.$$

Whence, $\psi(x, w+1) = \psi(x, w) - D_x^{-1}.D_{w+1}$. . . (8)

By this formula the required table may be constructed, the values in which $x$, the present age, is constant, taking their places in the columns, and those in which $w$, the age at entry, is constant, taking their places in the rows.

The following is a specimen of the commencement of the table.

| | | 134392 | 139105 | 143852 | 148661 | 153569 | 158614 |
|---|---|---|---|---|---|---|---|
| $w$ | $D_w$ | 10 | 11 | 12 | 13 | 14 | 15 |
| 10 | 74409 | 24·1485 | | | | | |
| 11 | 71888 | 23·1823 | 23·9953 | | | | |
| 12 | 69516 | 22·2481 | 23·0283 | 23·8142 | | | |
| 13 | 67267 | 21·3441 | 22·0926 | 22·8465 | 23·6103 | | |
| 14 | 65118 | 20·4689 | 21·1868 | 21·9098 | 22·6422 | 23·3898 | |
| 15 | 63046 | 19·6217 | 20·3098 | 21·0029 | 21·7050 | 22·4216 | 23·1581 |
| 16 | 61034 | 18·8014 | 19·4608 | 20·1249 | 20·7976 | 21·4843 | 22·1901 |
| 17 | 59064 | 18·0076 | 18·6392 | 19·2752 | 19·9196 | 20·5772 | 21·2532 |
| 18 | 57121 | 17·2400 | 17·8446 | 18·4535 | 19·0704 | 19·7000 | 20·3472 |
| 19 | 55192 | 16·4982 | 17·0768 | 17·6596 | 18·2499 | 18·8525 | 19·4718 |
| 20 | 53276 | 15·7823 | 16·3357 | 16·8932 | 17·4579 | 18·0343 | 18·6267 |
| * | * | * | * | * | * | * | * |

The numbers at the tops of the columns are, as in the last problem, the successive values of $D_x^{-1}$, corresponding to the ages over which they are placed, respectively; and the column on the left, represents a card containing the successive values of $D_w$.

In the initial term of each column $x = w$; so that these terms are, in fact, the whole-life annuities at the ages under and opposite which they are found.

For Column 10 we have for the initial term,

$$\psi(10, 10) = D_{10}^{-1}N_{10}.$$

Hence, taking $D_{10}^{-1}(=134392)$ for the in-factor, and, with the regulator at addition, multiplying by $N_{10}(=1796867)$, we get for the first value in this column 24·1485. The regulator is now reversed, in accordance with (8), and multiplying by $D_{11}$, $D_{12}$, &c., the remaining values in this column are successively produced.

For Column 11 the initial term is $D_{11}^{-1}N_{11}, = 139105 \times 1724979$, and the successive multipliers are $D_{12}$, $D_{13}$, &c.; and so on, throughout the remaining columns, the process in all being similar.

The card is moved forward as the work proceeds, being always placed against the column in course of formation.

Values for verification are obtained as follows. Let, in (7), $x$ vary, and we have,

$$\Delta_x\psi(x, w) = N_w . \Delta D_x^{-1};$$

whence, $$\psi(x+1, w) = \psi(x, w) + N_w . \Delta D_x^{-1}.$$

By this formula we should be able to form in succession the values occupying any of the rows; for it is in these that $x$ varies while $w$ is constant.

The following shows the formation of the row corresponding to $w=20$.

| $x$ | $\Delta D_x^{-1}$ | $\psi(x, 20)$ |
|---|---|---|
| 10 | | 15·7823 |
| 11 | 4713 | 16·3357 |
| 12 | 4747 | 16·8932 |
| 13 | 4809 | 17·4579 |
| 14 | 4908 | 18·0343 |
| 15 | 5045 | 18·6268 |
| 16 | 5229 | 19·2408 |
| 17 | 5465 | 19·8826 |
| 18 | 5758 | 20·5588 |
| 19 | 6121 | 21·2776 |
| 20 | 6514 | 22·0426 |
| * | * | * |

The initial term here is $\psi(10, 20) = N_{20}D_{10}^{-1}$; and $N_{20}$ being constant it is taken for the in-factor, and set upon F. Multiplying now by $D_{10}^{-1}(=134392)$ and the differences of the series of which it is the first term (here represented as written on a card), and the regulator being at addition, the required values are produced in succession, as shown. Verification of this series is had in the visible correspondence of its last term, $\psi(20, 20)$, with $a_{20}$.

The formation of three or four of these horizontal series will be sufficient, the terms of which should be inserted in their places before commencing the construction in the columns.

It will be observed that only five places have been used in the terms of $D_w$ employed in the process. The terms of $D_x^{-1}$, the other factor used in the formation of the addends, has six significant figures; and the product of these two can certainly be depended on to five places, while four is the utmost extent of the addends in the columns. An effect of the restriction of the out-factors, when it can be done with safety, is, of course, a saving of time and labour.

Since $\phi(x, w)+\psi(x, w)=a_x$, it follows that, in consequence of the arrangement adopted in the formations of Problems V and VI, the sum of corresponding values in the two formations is equal to the value of the whole-life annuity at the age under which the two values are found. Thus, in Column 13, and opposite 17, we have, for example, $3·6907+19·9196=23·6103$, $=a_{13}$. And in this manner any of the values formed may be checked.

The results of the two problems admit of arrangement in one table, as follows:—

| | 10 | 11 | 12 | 13 | 14 | 15 | |
|---|---|---|---|---|---|---|---|
| 10 | 24·1485 | 23·1823 | 22·2481 | 21·3441 | 20·4689 | 19·6217 | 10 |
| 1 | ·9661 | 23·9953 | 23·0283 | 22·0926 | 21·1868 | 20·3098 | 1 |
| 2 | 1·9004 | ·9670 | 23·8142 | 22·8465 | 21·9098 | 21·0029 | 2 |
| 3 | 2·8044 | 1·9027 | ·9677 | 23·6103 | 22·6422 | 21·7050 | 3 |
| 4 | 3·6795 | 2·8085 | 1·9044 | ·9680 | 23·3898 | 22·4216 | 4 |
| 15 | 4·5268 | 3·6855 | 2·8113 | 1·9053 | ·9682 | 23·1581 | 15 |
| 6 | 5·3470 | 4·5346 | 3·6893 | 2·8126 | 1·9055 | ·9681 | 6 |
| 7 | 6·1408 | 5·3562 | 4·5389 | 3·6907 | 2·8125 | 1·9049 | 7 |
| 8 | 6·9085 | 6·1508 | 5·3606 | 4·5399 | 3·6897 | 2·8109 | 8 |
| 9 | 7·6502 | 6·9185 | 6·1546 | 5·3603 | 4·5373 | 3·6864 | 9 |
| 20 | 8·3662 | 7·6596 | 6·9210 | 6·1524 | 5·3555 | 4·5314 | 20 |
| * | * | * | * | * | * | * | * |

The deferred annuities occupy the upper part of the table, the values of $x$, the present age, being at the side, and those of $w$, the ages at which the annuities are respectively to be entered on, at the top. The temporary annuities occupy the portion of the table below the diagonal, and as regards them the positions of the values of $x$ and $w$ are reversed, those of the former being at the top, and those of the latter, which are the ages at which the annuities are respectively to cease, at the side. The diagonal is legitimately occupied by the whole-life annuities, the indications connected with which are to be read in conformity with those of the deferred annuities. Thus, under 14 and opposite 14 we have $23·3898=a_{14}$, the annuity on (14), commencing at 14.*

**Problem** VII.—To construct Columns $D_{x.y}$ and $N_{x.y}$.

To effect these constructions we may commence with either of the two functions specified, and thence deduce the other by a simple operation.

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* See the *Institute Tables*, Introduction, p. xxxv.

First. To commence with $D_{x.y}$. If $x$ is equal to or greater than $y$, we have,

$$D_{x.y}=l_x l_y v^x = D_x l_y \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Let $y$ vary; then,

$$\Delta_y D_{x.y} = D_x . \Delta l_y = -D_x d_y,$$

since $d_y$ is negative. Hence,

$$D_{x.y+1} = D_{x.y} - D_x d_y.$$

It is most advantageous to form the series in reverse order. Therefore, transposing,

$$D_{x.y} = D_{x.y+1} + D_x d_y.$$

Or, changing $y$ into $y-1$,

$$D_{x.y-1} = D_{x.y} + D_x d_{y-1} \quad . \quad . \quad . \quad . \quad (10)$$

which is of the form P + QR.

The work will be first arranged in columns, in which $x$ is constant, and $y$ consequently variable. And to facilitate subsequent operations it is desirable that values of the function in which $x-y$ is constant should occupy the same row. We have therefore, from (9), for the initial terms in the successive columns, $D_{97}l_{97}$, $D_{96}l_{96}$, $D_{95}l_{95}$, &c.; and (10) is the working formula for the completion of the several columns.

The following is a specimen of the work, showing the commencement of the first four columns.

| | | 5117 | 28696 | 81432 | 170236 |
|---|---|---|---|---|---|
| $x-y$ | $d_y$ | 97 | 96 | 95 | 94 |
| 0 | 9 | ·046053 | 1·406104 | 10·993320 | 46·644664 |
| 1 | 40 | ·250733 | 3·873960 | 22·312368 | 79·840684 |
| 2 | 86 | ·690795 | 7·862704 | 38·191608 | 123·080628 |
| 3 | 139 | 1·402058 | 13·458424 | 58·875336 | 179·088272 |
| 4 | 195 | 2·399873 | 20·747208 | 85·666464 | 248·544560 |
| 5 | 254 | 3·699591 | 30·188192 | 118·890720 | 332·811380 |
| 6 | 329 | 5·383084 | 41·896160 | 159·199560 | 437·506520 |
| 7 | 408 | 7·470820 | 56·100680 | 209·280240 | 569·098948 |
| 8 | 495 | 10·003735 | 73·748720 | 272·227176 | 729·291024 |
| 9 | 615 | 13·150690 | 95·930728 | 348·854688 | 923·019592 |
| 10 | 773 | 17·106131 | 122·933664 | 441·524304 | 1152·157248 |
| * | 941 | * | * | * | * |
| | 1138 | | | | |
| | 1346 | | | | |
| | * | | | | |

The values of $x$ are, as stated, at the top, each in connexion with the column in which it is constant, and over these are placed the values of $D_x$. The values of $x-y$ are at the side, each being constant in the row opposite which it is placed. The values of $d_y$, in reverse order, are on a card represented as applied to Column 97. The first step is, setting $D_{97}=5117$ upon F, to multiply by $9=l_{97}$, which gives $46053=D_{97.97}$. And the column is completed by using continuously as multipliers the successive terms of $d_y$, commencing with 40, the second term in this order.

The card is now moved forward to the next column, and at the same time upwards one line; and by a similar manner of proceeding this column, too, will be filled up. And so on with the other columns in succession.

I have pointed the values formed in the specimen in accordance with a radix of 10,000. In consequence, the necessity for the exhibition of many useless figures, in the values corresponding to the younger ages, will be avoided. Thus we should now have $D_{10.10}=744094000$, instead of 74409400000.

Verification is obtained as follows:—Let, in (9), $x$ vary, and we we have,

$$\Delta_x D_{x.y}=l_y.\Delta D_x;$$

whence,

$$D_{x+1.y}=D_{x.y}+l_y.\Delta D_x.$$

Transposing, and writing $x-1$ for $x$,

$$D_{x-1.y}=D_{x.y}-l_y.\Delta D_{x-1}.$$

The series here indicated, in which $y$ is constant and $x$ decreases by a unit in passing from term to term, ascend diagonally in the preceding formation, as a little consideration will show. And the construction of three or four of them, at intervals, by the aid of the formula just deduced, will suffice for the verification of the whole work.

The following is the formation of a portion of the series commencing with $D_{97.87}=(x-y=10)$. It must be observed that $\Delta D_x$ being negative, the regulator will be set for addition.

| $x-y$ | | 87 |
|---|---|---|
| 10 | $\Delta D_x$ | 17106131 |
| 9 | 23579 | 95930728 |
| 8 | 52736 | 272227176 |
| 7 | 88804 | 569098948 |
| 6 | 129895 | * |
| * | * | |

Here $l_y = l_{87} = 3343$, being constant, it is taken for the *in* factor, and set upon F. Successive and continuous multiplication by $D_{97} = 5117$, and the terms of $\Delta D_x$ in reverse order, will then complete the series, which will consist of eleven terms, terminating with $D_{87.87}$.

The following is the commencement of the formation of the first four columns of $N_{x.y}$.

| $x$ | Diff. 0. | Diff. 1. | Diff. 2. | Diff. 3. |
|---|---|---|---|---|
| 96 | ·046053 | ·250733 | ·690795 | 1·402058 |
| | 1·406104 | 3·873960 | 7·862704 | 13·458424 |
| 95 | 1·452157 | 4·124693 | 8·553499 | 14·860482 |
| | 10·993320 | 22·312368 | 38·191608 | 58·875336 |
| 94 | 12·445477 | 26·437061 | 46·745107 | 73·735818 |
| | 46·644664 | 79·840684 | 123·080628 | 179·088272 |
| 93 | 59·090141 | 106·277745 | 169·825735 | 252·824090 |
| * | * | * | * | * |

The terms of $D_{x.y}$ occupying the rows in the last formation are transcribed in columns, as shown; and by continuous addition the terms of $N_{x.y}$ are formed. The values of $x$ only are inserted. These are constant in the rows; and so the value of $y$ in any particular term is determined by subtracting the Diff. at the top of the column from the value of $x$ pertaining to the row in which the term in question is found.

There is another method of arranging the results in the formation of $D_{x.y}$, by the employment of which the labour of transcribing them for the formation of $N_{x.y}$ is saved. The arrangement is that adopted by General Hannyngton in the example in his paper of the construction of $D_{x.y}$. In it the successive results, which in the preceding formation are written in columns, are written in rows, commencing in all cases in the first column. In consequence, the terms of $D_{x.y}$ in which $x-y$ is constant occupy the same column; and if the rows are written, not on succeeding but on alternate lines, we are prepared immediately, on the completion of $D_{x.y}$, for the formation of $N_{x.y}$.

The following is an example.

| $x$ | $D_x$ | Diff. 0. | Diff. 1. | Diff. 2. | Diff. 3. |
|---|---|---|---|---|---|
| 97 | 5117 | ·046053 | ·250733 | ·690795 | 1·402058 |
| 96 | 28696 | 1·406104 | 3·873960 | 7·862704 | 13·458424 |
| | | 1·452157 | 4·124693 | 8·553499 | 14·860482 |
| 95 | 81432 | 10·993320 | 22·312368 | 38·191608 | 58·875336 |
| | | 12·445477 | 26·437061 | 46·745107 | 73·735818 |
| 94 | 170236 | 46·644664 | 79·840684 | 123·080628 | 179·088272 |
| | | 59·090141 | 106·277745 | 169·825735 | 252·824090 |
| 93 | 300131 | 140·761439 | 216·994713 | 315·737812 | 438·191260 |
| | | 199·851580 | 323·272458 | 485·563547 | 691·015350 |
| 92 | 476556 | 344·549988 | 501·336912 | 695·771760 | 931·666980 |
| | | 544·401568 | 824·609370 | 1181·335307 | 1622·682330 |
| 91 | 714214 | 751·353128 | 1042·752440 | 1396·288370 | 1835·529980 |
| | | 1295·754696 | 1867·361810 | 2577·623677 | 3458·212310 |
| 90 | 102095 | 1490·58700 | 1995·95725 | 2623·84150 | 3413·03585 |
| | | 2786·34170 | 3863·31906 | 5201·46518 | 6871·24816 |
| 89 | 140810 | 2752·83550 | 3618·81700 | 4707·27830 | 6032·30040 |
| | | 5539·17720 | 7482·13606 | 9908·74348 | 12903·54856 |
| 88 | 190659 | 4899·93630 | 6373·73037 | 8167·83156 | 10337·53098 |
| | | 10439·11350 | 13855·86643 | 18076·57504 | 23241·07954 |
| 87 | 255445 | 8539·52635 | 10943·26380 | 13850·22790 | 17288·51760 |
| | | 18978·63985 | 24799·13023 | 31926·80294 | 40529·59714 |
| * | * | * | * | * | * |

The values of $x$ at the side are those that belong to the terms of $D_{x.y}$ in the rows opposite which they are placed; and the values of the same symbol belonging to the terms of $N_{x.y}$, if inserted opposite the rows in which these values are constant, would be each a unit less than that occupying the line above it, observing, however, that $D_{97.97}=N_{96.96}$.

But, secondly, we may commence the construction with $N_{x.y}$, whence it is easy to pass to $D_{x.y}$.

Since, $N_{x.y}=D_{x+1.y+1}+D_{x+2.y+2}+\ \ldots\ .$

therefore, if $x$ and $y$ both vary,

$$\Delta_{x.y}N_{x.y}=-D_{x+1.y+1}=-D_{x+1}l_{y+1}.$$

Whence, $N_{x+1.y+1}=N_{x.y}-D_{x+1}l_{y+1}.$

Transposing, as the series must be formed in reverse order,

$$N_{x.y}=N_{x+1.y+1}+D_{x+1}l_{y+1};$$

or $$N_{x-1.y-1}=N_{x.y}+D_xl_y.$$

And by this formula, which corresponds to $P+QR$, where both Q and R vary in passing from each term to the next, the table may be constructed.

The following is a specimen, showing the commencement of the first four columns.

| $x$ | $D_x$ | $l_y$ | Diff. 0. | Diff. 1. | Diff. 2. | Diff. 3. |
|---|---|---|---|---|---|---|
| 96 | 5117 | 9 | ·046053 | ·250733 | ·690795 | 1·402058 |
| 95 | 28696 | 49 | 1·452157 | 4·124693 | 8·553499 | 14·860482 |
| 94 | 81432 | 135 | 12·445477 | 26·437061 | 46·745107 | 73·735818 |
| 93 | 170236 | 274 | 59·090141 | 106·277745 | 169·825735 | 252·824090 |
| 92 | 300131 | 469 | 199·851580 | 323·272458 | 485·563547 | 691·015350 |
| 91 | 476556 | 723 | 544·401568 | 824·609370 | 1181·335307 | 1622·682330 |
| 90 | 714214 | 1052 | 1295·754696 | 1867·361810 | 2577·623677 | 3458·212310 |
| 89 | 102095 | 1460 | 2786·34170 | 3863·31906 | 5201·46518 | 6871·24816 |
| 88 | 140810 | 1955 | 5539·17720 | 7482·13606 | 9908·74348 | 12903·54856 |
| 87 | 190659 | 2570 | 10439·11350 | 13855·86643 | 18076·57504 | 23241·07954 |
| 86 | 255445 | 3343 | 18978·63985 | 24799·13023 | 31926·80294 | 40529·59714 |
| * | * | 4284 | * | * | * | * |
| | | 5422 | | | | |
| | | 6768 | | | | |
| | | * | | | | |

It will be at once perceived that the process here, as regards the individual columns, is analogous to that of the formation of $N_x$ in Problem III. The card containing $v^x$ there, is here replaced by one containing $D_x$; and the successive operations are, in both, the multiplying together of the two factors in line with each other. On the completion of each column the cards are moved forward into connexion with the next, the one containing $l_x$ being at the same time raised one line, so as to preserve the proper relation as to difference of ages between the factors that come together.

In practice, the alternate columns will be left blank, to receive the terms of $D_{x.y}$, which will be formed by differencing, as in the example of Problem III. This need not be exemplified here.

I am not prepared to say absolutely to which of the two methods that have been explained the preference ought to be awarded. I incline at present to prefer the first, that in which we commence with $D_{x.y}$, arranging the results as suggested by General Hannyngton. This method is certainly attended with less labour and risk of error than the other, inasmuch as in it the setting on F has to be changed only once for each column, while when the second method is used it has to be changed for each value formed. In the first method, also, the terms of $D_{x.y}$ admit of exact verification, as I have shown, and it is easy to secure the correctness of the terms of $N_{x.y}$ by preliminary addition of the terms of $D_{x.y}$ in groups; while on the other hand, when the second method is employed, there is no other way of verifying the work than by repetition of the process.

A drawback in the use of the first method is that, unless an assistant is available, it is less convenient to record the successive results in rows than in columns. This drawback probably will not, in the circumstances, be deemed of much importance.

A more extended experience than has yet been acquired will be necessary to determine the question authoritatively.

In the examples under the present problem the values formed are given to their full extent. But it must be understood that those of $D_{x.y}$ have no claim to be true to more than six places, and those of $N_{x.y}$ to more than seven, perhaps eight. If this extent shall be judged insufficient, it will be no difficult matter, with the means now available, to form a new Column $D_x$ to seven places, for special use in the formation now under consideration.

I may, perhaps, be allowed here to say, that the first of the foregoing methods of construction, apart from General Hannyngton's manner of arranging the results, is substantially one that I developed and practised upwards of twenty years ago.* It is perfected by the employment of the Arithmometer in the formation of the addends.

(*To be continued.*)

## *On Valuation of Policies for Proof in Liquidation.*

THE manner in which current policies should be valued for proof upon the insolvency of a Life Office has been much discussed in our pages, and under cover of the conflicting decisions in Bell's and Lancaster's cases has given rise to much diversity of opinion. The interest of the question can scarcely be said to be exhausted. We are, therefore, glad to be able to present to our readers an elaborate judgment on the subject delivered by Lord Romilly, the late Master of the Rolls, which is not only interesting from the eminence of its author, but important as carrying the weight of a judicial decision. It will be best apprehended by perusal, and we will therefore only state that it unhesitatingly decides in accordance with the rule laid down by Lord Justice James in Bell's case, and against that of Lord Cairns in Lancaster's case. In order, however, to bring the whole matter down to the present time, we must remind our readers that it has been dealt with in the Life Assurance Companies Amendment Act, 1872, and again by the determination of Lord Westbury in the European Arbitration.

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* See *Tables and Formulæ for the Computation of Life Contingencies,* (1849,) pp. 149 to 181; and *Assurance and Annuity Tables,* (1851,) Introduction, p. x.

In the Act of 1872* the rules in Lancaster's case for valuing both annuities and policies are laid down for adoption in winding up a life assurance company, and this might be considered a legislative sanction to them, but such sanction is greatly weakened by the 6th section of the Act, which provides that these rules shall have the same effect as if Rules of Court, made in pursuance of sections 170, 171, 173 of the Companies Act, 1862, and may be altered in manner provided by those sections.† There is no doubt of the convenience of these rules as a rough and ready mode of valuing liabilities, and their practical application in the Albert Liquidation was an important fact; and it would seem thus that the Legislature adopted them on the urgent necessity of the case, but recognized the difference of opinion in giving them only a temporary sanction. And here, as always, we find the difference between a judicial decision and a legislative enactment. The former is sought to be governed by strict logical truth applied to existing laws, while in the latter the spirit of compromize and general convenience prevails. Of this we find notable instances in this very Act of 1872 and the European Society's Arbitration Act, which passed through the House of Lords on consecutive nights, altho' the general was undoubtedly intended to supersede the particular measure, and was the offspring of remonstrances which appeared in the public journals against it. Under these circumstances it is not surprising that Lord Westbury, under the wide discretion reposed in him as Arbitrator, elected to follow the rules laid down in the recent Act; but here again the decision was arrived at without argument, and as a part of the internal administration of the Arbitration,—at least this may be assumed from the judgment in Wallberg's case,‡ in which the direction is given without comment, while every point in dispute is forcibly argued.

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* 35 and 36 Vict., c. 41, s. 5.

† By sec. 170 (25 and 26 Vict., c. 89) the Lord Chancellor, with the advice and consent of the Master of the Rolls and a Vice-Chancellor, or of any two Vice-Chancellors, may make such rules regarding the mode of proceeding to be had for winding up a company in the Court as may from time to time seem necessary. Secs. 171 and 173 refer to Scotland and Ireland.

‡ Minutes of Proceedings, p. 197, 9 Nov. 1872.

## *In re* ENGLISH ASSURANCE COMPANY.

### HOLDICH'S CASE, L. R. 14, Eq. Ca. 72.

### JUDGMENT.

Lord *Romilly*, M.R.:—

This is a summons adjourned from Chambers, and is one of great importance, not merely from its being a representative case which must affect a great number of other cases, but one which will constantly recur whenever an assurance company has to be wound up in a Court of Equity. It is also a case which has given rise to great contrariety of opinion amongst actuaries and all persons connected with assurance companies. It has been decided in one way by the present Lord Justice *James*, when Vice-Chancellor, in *Bell's Case*,* and that decision has been carefully examined and elaborately commented upon by Lord *Cairns*, who has come to an opposite conclusion in *Lancaster's Case*, while acting as parliamentary arbitrator in the *Albert Assurance Company*.

The question therefore is one, not only of considerable difficulty, but one which will probably have ultimately to be determined by the highest Court of judicature in this country.

The question may be stated in the shortest possible terms. It is this: For what amount is the holder of a life policy in the company under liquidation entitled to prove against the assets of the company?

The Lord Justice *James*, when Vice-Chancellor, decided that this was the rule to be followed: Select another company of perfectly solvent reputation, with premiums the same as those of the company in course of liquidation: find for what premiums that company would now insure the life of the person assured in the company in course of liquidation: the person whose life is assured is to be at liberty to show that his then state of health is such that the new company would require a higher premium than that which would have been required merely from the lapse of years, and then, having ascertained the increased premium which the person seeking to prove would have to pay to the new company at the present time, take the difference between the two premiums and capitalize that difference, having regard to the expectancy of the life of the person in question; and that will give you the amount which ought to be paid to him to put him into as good a position as he would have been in if the company in course of liquidation had not been wound up.

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* Law Rep. 9 Eq. Ca. 706.

Lord *Cairns* states his apprehension of the rule as laid down by the Lord Justice *James* very much to the same effect.

Lord *Cairns*, after examining that judgment, comes to the following conclusion: "There must be what is termed and well known as a pure premium valuation, as at the date of the winding-up order, the rate of interest assumed being £4 per-cent, the table being the Seventeen Offices Experience Table. The principle will be adopted in this way: There must be determined on the one hand the present value of the reversion in the sum assured at the decease of the life, and on the other the present value of the future annual premiums. The difference between the two sums represents the value of the policy. But the annual premium payable is divisible into two parts: first, the part which it is calculated will provide for the risk, called the 'pure premium'; and secondly, the addition for office expenses and other charges, which is sometimes called the 'loading'. The pure premium only is to be taken into account. The value of the policy therefore will be the difference between the value of the reversion in the sum assured and the value of a life annuity of an amount equal to the pure premium."

Lord *Cairns* proceeds entirely upon the 158th section of the *Companies Act*, 1862, and the 25th rule of the General Order of 11 November 1862, issued under the authority of that statute. The section is in these words: "In the event of any company being wound up under this Act, all debts payable on a contingency, and all claims against the company, present or future, certain or contingent, ascertained or sounding only in damages, shall be admissible to proof against the company, a just estimate being made as far as is possible of the value of all such debts or claims as may be subject to any contingency or sound only in damages, or for some other reason do not bear a certain value."

The 25th rule of the General Order is in these words: "The value of such debts and claims as are admissible to proof by the 158th section of the said Act shall so far as is possible be estimated according to the value thereof at the date of the order to wind up the company."

The real question is this: what is a fair sum to force a man to take at the date of the winding-up in lieu of a fixed sum contracted to be paid at the time of his death?

It appears to me the first thing to be ascertained in considering this question is, what is the contract which has been entered into between the assured and the company; remembering always that this is not the case of a mutual assurance company, where the policyholders are partners, but strangers who are contracting with each other. The contract I take to be this: *A* insures his life with the company for payment of a given sum at *A's* death, without profits, in consideration of *A* paying an annual premium to the office of a given amount. *A* on one side, to entitle himself to the sum assured, has to pay as long as he lives the annual premium to the company; the company, on the other hand, contracts that when *A* dies they will pay the amount assured, and that no accident or disease which may curtail the chance of living of *A* shall entitle the office to increase the premium.

The conclusion at which Lord *Cairns* has arrived appears to me to overlook this point in the real contract which is made between a person who insures his life and the assurance company. If there were no such things as diseases or accidents, but all men lived to an age when the body decayed by age, more or less rapidly, according to the strength and nature of the constitution, a pure premium valuation would appear to be an easy mode of solving the difficulty; but it is obvious, and indeed notorious, that most if not all persons who insure their lives, do so expressly to guard against the contingency that an unexpected disease or an unforeseen accident may shorten their life, and prevent it from running what, without the occurrence of such contingency, would be the probable term of its duration. It is obvious also that such diseases and such accidents are taken into account by the assurance office upon an average of the lives assured, and form an item in the scale of mortality on which the company calculates the amount of the premium to be paid. You deprive the assured of the benefit of his contract, if you do not give him compensation for an accident which has made his life uninsurable, and consequently his policy more valuable, at the time when the company is wound up.

Suppose such a case as this, and see how it would work under Lord *Cairns's* plan: Two persons of the same age insure their lives, each for the same sum in the same office at the same premium. The company is wound up. One dies a month before liquidation begins; the other, whose health has become seriously impaired, dies shortly after the liquidation has begun. One is entitled, according to every rule, to prove for the whole amount of the sum insured; the other must submit to a reduction which, according to Lord *Cairns's* plan, will deprive him of the greater part of the sum assured, while according to Lord Justice *James*, he is entitled to such a sum as would be sufficient to induce a new office to give the assured a policy for the same amount at the former premium.

Lord *Cairns* seems to be of opinion that the 158th section and the 25th rule make it imperative that the proof shall, when made, be for a sum to be ascertained at the date of the order to wind up the company. But the rule itself notices that in many cases this must be impossible, and accordingly inserts the words "so far as is possible". Now, I am of opinion that this is one of the cases where such ascertainment of the value of the proof is not possible. No doubt the day of the winding-up order is the time at which the value of the policy or annuity is to be calculated; but subsequent facts may be given in evidence for the purpose of showing what the real value was at that time, such as the sudden death of the annuitant or of the assured, in the one case a benefit to the company, in the other to the estate of the assured. If in every case the day of the death of the assured could be ascertained, the problem would be a very easy one, and ought to be calculated on the principle of the value of the reversion less the value of the annuity, both calculated to the day of the death of the assured. This is, from the nature of things, impossible, but an approximation to it is attained, not merely from the age of the assured, but also from his state of health. If the state of health of the assured be varied, and such variation has been provided for and forms a part of the contract, then the principle adopted by Lord *Cairns* appears to me to be no longer applicable. If, as I believe, it be clear to demonstration, that the chance of the premature termination of the life of the assured from causes other than gradual efflux of age forms one of the principal motives for assurance, and one of the contingencies the possible occurrence of which is contracted for, why should not this form part of the proof, if, by the occurrence of it, the assured has sustained special damage? Any other matter that forms part of the contract must follow the usual course in such cases. For instance, if it formed part of the contract that the policies should be valued according to any particular principle or by any specified table of mortality, then this would supersede all other questions. If a particular rate of interest were specified in calculating the value, this would be followed; if no scale of interest were specified the Chancery scale of £4 per-cent might then be adopted.

Consider the case of a contract made for sharing in the profits to be made by the office. This being the case of a company under liquidation, no profits have been made, and therefore the assured gets none; but all persons agree that the assured cannot, for that reason, require the extra amount of premium he has paid in the hope of profits, to be repaid to him, because he has contracted for a matter necessarily uncertain, the occurrence of which was doubtful, and which has not occurred. Why, in all these cases, should the contract be followed, and not in the present case, where, if I am right, the contract expressly undertakes to provide for the special damages which fever or accident may bring upon the assured, and not to require any addition to the premium if any such event should occur which abridges the chances of life? Suppose an office to contract with the assured that if he should be attacked with a pulmonary complaint he will pay an additional premium. He would be compelled to perform his part of the contract. Why is not the company to perform their part of the contract if his insurance, by reason of the same circumstances, has become of double its former value?

The second objection urged by Lord *Cairns* to the principle of the decision in *Bell's Case**, does not appear to me to be well founded. That case, as it appears to me, very properly lays down that, unless the contrary is shown, all lives will be assumed to be in a normal state, and that no change has taken place other than that which arises from the advance of age; but it appears to me that if, from accident or illness, a higher rate of insurance is required, that being a circumstance of special damage contracted against, it may be taken into consideration in ascertaining the amount of the proof, and the burden of proving it lies on the person who seeks to prove. The suggestion of the instance given by Lord *Cairns*, where it requires £200 to produce the life insured, would apply in like manner to every case where the proof of a debt involves considerable expense; and the question of who is to bear the cost of re-examination is a separate matter.

In the case supposed, you must give evidence to satisfy the Court, which evidence is to be laid before another office, not for the purpose of asking that office to insure the life unless the person applying desired to effect such insurance, but for the purpose of asking that office what, in that case, would be the additional premium they would require to insure the life in question. The amount of the evidence given of the accident or illness, or the state of health of the proposed life, would in every case be subject to the control of the Court, and might be objected to by the official liquidater on behalf of the company, as in the case of the proof of any other debt, and only what was proper allowed. Assume that the proof ought to be given of the state of the life at the date of the order to wind up, the subsequent state of health of the assured up to the time of taking in the claim would properly be given in evidence as affording evidence of the value of the life at that time. Neither do I understand the force of the last objection urged by Lord *Cairns*—that it introduces not only "opinion evidence" as to the state of health, but opinion evidence of the worst possible kind, because it is the opinion evidence of another company, who are not going to insure the life. Now it is true that opinion evidence is always uncertain; but it is, in the nature of things, evidence on which, in a vast variety of cases, a Court of Equity is compelled to act. The great objection to it usually is that it never can be tested by any indictment for perjury, and is too often given with a bias in favour of one side, even by perfectly honourable and fair men, and often without their being conscious of the bias; but if there be one species of evidence of opinion more trustworthy than another, it is, as it appears to me, the opinion of an assurance office on the life of an applicant, not only because they can have no bias either for or against the claimant, and because their opinion may immediately be tested by the application for a fresh life policy by that claimant, or by the application for an annuity for life, either of which tests the office, it is obvious, cannot refuse to submit to, if they should be required so to do, as long as they carry on business.

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* Law Rep. 9 Eq. 706.

The case in reality seems to resolve itself into this: the office has contracted to give the assured the pecuniary advantages which may arise to him from accident or disease; is the office, because it has been compelled to undergo a course of liquidation, to escape from the compliance with this part of their contract? And if so, for what reason? Can it be because it is difficult to ascertain? or because it is liable to error?—all of which they knew and undertook, when they accepted the life. If so, a new principle would be introduced in ascertaining the damages sustained by one party to a contract from the breach of it by the other—a principle, I venture to say, of great danger, and extremely difficult of application, if the Court is to determine what species of damage is to admit of no compensation.

Two healthy young men of the same age insure in the same office for the same premium; they meet with the same accident in a railway carriage; one is killed on the spot a few days before the order for winding-up. His estate proves in full against the office;

the other survives a fortnight, and, according to Lord *Cairns's* principle, his estate proves for little or nothing. Surely the deterioration of his life, which, by the express contract of the company, was not to retard or diminish their liability to pay the full sum assured whenever he might die, ought to be taken into account. Observe also the case of annuities; the same rule must apply both to policies and annuities. The deterioration of the health of the annuitant is a benefit to the company. Suppose a young annuitant die suddenly after the winding up, but before any fresh payment of an annuity on his life has occurred; is the company to pay to his representatives a sum calculated on the hypothesis that he may live twenty-five years? If it be said that in such a case the company has performed their part of the contract, and are not further liable, does not a like principle apply if at the date of the winding-up order he had a mortal complaint but might linger for a year? In either case I think the rule must be, What would buy a similar policy or a similar annuity from a perfectly safe office? In one case, the burden of proof of the deterioration of the life, falls on the assured or his representatives; in the case of the annuitant, it falls on the company; but in either case the investigation may be waived by the party interested. I am also unable to understand how, according to Lord *Cairns's* plan, the pure premiums, and that portion which he calls "loading", are to be ascertained. I do not believe that there is the same system in all offices, even in those of the best regulated description. Who is to be judge what it was in the office under liquidation? and how is it to be determined? Part of it is composed of the profit premiums of the persons who contracted for a share of profits. Is this to be deducted?

It appears to me to be the introduction of a merely arbitrary rule, assigning to an office a calculation, which may or may not have made the basis of their tables, but which assuredly was not the contract entered into by the parties to it, but which is a new contract which the Court of Chancery, for its own convenience, thinks fit to substitute for the real contract. I am also unable to understand the principle which makes such a distinction between the case of non-profit policies and that of profit policies, and between them and annuities.

In the case of profit policies the whole of the premiums must be calculated without any deduction; in the case of annuities the whole value of the annuity for the probable duration of life is to be calculated without any deduction. Why is the premium in the case of a non-profit policy to be treated differently, and an attempt made to separate in an arbitrary manner, by a sort of rule-of-thumb principle, what is loading and what is not?—a method which, in nine cases out of ten, was not adopted by the company in question in calculating the tables which formed the basis of the contract. The assured has contracted to pay a certain premium without reference to any question of loading or not loading; that is his contract; why is he not to pay it, or the value of the policy, to be calculated as if he had only to pay a portion of it? The whole plan suggested by Lord *Cairns's* decision appears also to me to involve a false principle, which makes the circumstance of insolvency alter the proportionate part of the assets of a company to which a class of the creditors is to be entitled, so that, while before the liquidation they were entitled to one proportionate part, after the liquidation they would be entitled to another proportionate part of the assets.

I have given the subject the best consideration I have been able; and in my opinion the proper rule to be followed is that which was laid down by Lord Justice *James* in *Bell's Case.* I shall follow it on the present occasion, and direct the costs of all parties to be paid out of the estate.

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Lord *Cairns's* Judgment in Lancaster's Case (Minutes of Proceedings in *Albert Arbitration,* p. 677,) was as follows:—

I have now to dispose of the case which was brought before me for the purpose of determining the amount of proof upon policies and annuities, *Lancaster's Case.*

This is a question of considerable difficulty. I should have been very glad if I had found that any principle had been adopted by the Court of Chancery in the course of the winding-up which would have been applicable, upon satisfactory grounds, both to policies and to annuities, and which, therefore, I could have followed without any further examination. I have been unable, however, to satisfy myself that any principle has been adopted by the Court of Chancery, which will apply upon sound and clear grounds to the case of policies of insurance and of annuities, and which would in my opinion be consistent with the enactments of the Act of 1862.

In considering the question, I have to deal with the case, and only with the case, of those life policies and annuities which were actually current at the date of the winding-up. Where instalments of an annuity had actually become payable, or where a life policy had actually become payable before the date of the winding-up, there no question as to valuation arises. The claim would be admitted for the actual sum that ought to have been paid before the winding-up. I have to deal with those cases where the policy or the annuity contract was current at the date of the winding-up. There are in some or all of the companies other contracts of a more special kind—for instance, endowments, and contracts of surviver-ship—but the general observations will apply, with an alteration of detail only, to those other cases.

[After considering the case of an annuity contract, His Lordship proceeded as follows:—]

I now come to the case of policies of insurance. I take at present whole-life policies. I may repeat shortly what I have often had occasion to say here. What is the nature of the contract of insurance? It is a contract by which the insured is under no obligation to pay the premiums, it is a contract by which the company insuring is under no obligation to continue its business, but the company undertakes that if the person insured will pay the fixed premiums from year to year, the funds of the company for the time being, including the unpaid capital, shall be answerable in proper order to pay the sum insured on the falling in of the life, and on proper proof of that event being given to the company.

That being the nature of the contract, it is to be observed in the first place, that at the date of the winding-up, taking it distinctly as at that moment of time, there is no breach of the contract (I have said that where the life has dropped before the winding-up a policy of that kind stands in a different position; it is an actual claim for the whole amount). It is also to be observed that we have here to deal, not with the case of a going company which is being sued for a breach of contract. Indeed, it is very difficult to see, in a case of a policy of insurance, how there can be any breach of contract short of a refusal to pay upon the falling in of the life. I know it has been suggested that a refusal to receive the premiums from year to year would be a breach of the contract, and that thereupon an action might be brought. I do not desire to express any opinion upon that point, but it is somewhat difficult to see how such an action could be maintained, and certainly, as far as I know, no such action has ever been brought or maintained. But all that may be set aside here, because, whether an action could or could not be maintained for a refusal to receive the premium, I repeat that at the date of the winding-up, even in that sense, there was no refusal to receive the premium, and therefore no breach of contract. If, however, a going company were sued for a real breach of the contract, that is to say, for the non-payment of the sum insured, there, again, it is difficult to imagine how there could be any damages other than the amount which was covered by the policy, which ought to have been paid. In the case with which I have to deal, it is only, as it seems to me, under the operation of the 158th section and the 25th rule that policies current at the date of the winding-up can be provided for, and claims in respect of them entertained under the winding-up at all. I should have thought that, but for an authority in the Court of Chancery, which I shall presently refer to in detail, namely, *Bell's Case* (Law Rep. 9 Eq. 706), the course pointed out by this section and this rule was clear. It is well known what putting an estimate on the value of an insurance means. Persons may differ as to the table by which the valuation is to be made, or as to the rate of interest to be taken, but the way in which the value of an insurance is to be estimated is a thing perfectly well known, and the thing is attempted and succeeded in every day, and I should have thought, but for the decision to which I have referred, the course under the 158th section and the 25th rule was a clear one, and as simple as any somewhat difficult problem of arithmetic could be made.

The effect of the decision in *Bell's Case*, however, as I understand it, is this. The Court there held that this mode might be adopted of valuing a policy. You might select another company with premiums the same as the *Albert*, with capital and guarantee proprietary fund as nearly as possible the same as the *Albert* had, or, at all events, professed to have. You might then go to that company and find for what premium the company would now insure the life of a person insured in the *Albert*, after examining, if necessary, his state of health, and then, when you had found the premium which the person insured would have to pay to this new company at the present time, and had compared that with the premium he before paid for the same amount to the *Albert*, and had taken the difference between the two premiums, and had capitalized that difference, having regard to the expectancy of life, you would then have the amount which ought to be paid to the person insured to put him in as good a position as he would have been in if the *Albert Company* had not been wound up. Now, so far as that sum arises from the mere difference of age, introducing no other ingredient into the case, but taking, for instance, a person who was insured in the *Albert* at the age of twenty-five, and who is to be reinsured in this supposed company at the age of fifty, not looking at any difference in the state of health at one time and another, but merely looking at the difference of premium for a man of twenty-five and a man of fifty—so far as the decision was founded on those elements, the decision would have supplied a not inaccurate mode of arriving at an estimate of the value of the policy. If there was nothing more, then I should have been willing to have followed that principle. But the difficulty I feel is in following the decision so far as it introduces into the case other elements, namely, the re-examination and the consequences of the re-examination or the non-examination of the lives to be insured by the new company. Because the decision goes to this, that if the particular person whose life is proposed for reinsurance has in the meantime passed into a state of health which either makes him non-insurable or makes him insurable only upon more disadvantageous terms than formerly, or if the life to be reinsured cannot be produced for re-examination, in all those cases the person is under some peculiar and special damage arising in his own case —a special and peculiar damage separate from the question of the value of his policy, and practically provision is to be made out of the assets of the company for that particular and special damage.

Now I have considered *Bell's Case* (Law Rep. 9 Eq. 706) with great anxiety, as the decision of a Judge for whose opinion I have the highest possible respect, and who appears in that particular case to have taken very great pains to give weight to all the arguments which were addressed to him in support of the different views that were then proposed to the Court. I will state shortly the reasons which make me unable to follow the part of the decision which I have last adverted to.

In the first place I find, judging from the report, that neither in the arguments before the Court, nor in the judgment of the learned Judge, was any reference whatever made to the section of the Act which I have mentioned or to the 25th rule. In point of fact, the date of the winding-up order mentioned in the 25th rule was not taken at all as the date for valuation, but a period altogether different, namely, a period at the close of the time covered by the last premium that was paid. I cannot help thinking that, if the section and the rule had been prominently brought before the Court, the Court would scarcely have arrived at the conclusion it did in *Bell's Case*.

In the next place, in my opinion, the problem to be solved under the Act being the finding of a just estimate of the value of the policy, the special damage sustained by a particular individual has nothing whatever to do with the value of the policy. In the course of the argument I put a case that will illustrate sufficiently my meaning. Take the case of *A* effecting an insurance on the life of *B*. *B* is abroad. The object is that *A* should be re-insured in a new company. The new company will say: "You must produce the life, so that we may examine him." He being abroad will not come at all; or, being at a great distance from England, says: "I will come home to be re-examined if you pay my expenses home and back again, which will cost £200." The question that occurs to my mind is, what possible equity can there be in the distribution of assets to make the other policyholders pay this £200 (because it must be paid out of their money) in order to indemnify the particular policyholder *A* for the expense of bringing to this country the life that is to be inspected for re-insurance? In point of fact, special damage seems to me to be a thing relating entirely to a class of ideas different from the class of ideas entering into the question of the valuation of a policy for the purpose of paying its value out of assets. Special damage would apply to the case I supposed some time ago of an action brought against a going company for a breach of contract, in which case a jury may be directed to take into account the special damage done to the particular individual.

The next point in regard to *Bell's Case* (Law Rep. 9 Eq. 706) which I have to observe upon is this: the mode of valuation there adopted involves the question of what another company would think of the health of the person formerly insured in the company being wound up. Now, in the first place, that raises the inquiry not merely of what the state of health of the person is, but retrospectively what the state of health of the person now to be examined was at a period nearly two years past and gone, as to which the state of health of any person must be merely a matter of speculation and conjecture. Further than that, it introduces not merely opinion evidence as to the state of health, which is never satisfactory, but opinion evidence of the worst possible kind, because it is to be the opinion evidence of another company, or the officers of another company, who are not going actually, or may not be going, to insure the life; for there is to be no obligation after all upon the other company to insure the life, and no obligation upon the person making the claim to insure himself in the other company. It is opinion evidence, again, which you have no means of checking, because the enquiry proposed by the Court is, what would another company do if it were asked to insure the life? The only answer to that can be from the other company, that they would either insure the life or not, or would insure it for such an additional sum, and from the very nature of the case that is evidence which cannot be controverted, because it is not evidence of a fact, but evidence merely of a speculative opinion.

Further, the test proposed in *Bell's Case* substitutes a solvent for an insolvent company, and under the guise of attempting to put the person insured in as good a position as he was in before, the mode adopted would put him in a very much better position.

Then this alone would be to my mind an insuperable difficulty in adopting the principle of *Bell's Case*,—the principle of that case is not homogeneous with the mode of valuing annuities adopted by the Court of Chancery in this winding-up. Annuities are proposed to be valued precisely on the principle of taking the value of the annuity as a piece of property, and not on any question of special damage sustained by the annuitant. Therefore, in dealing with assets of which there must be a distribution between annuitants and policyholders, and which ought to be distributed on homogeneous principles as between the two classes of claimants, the Court would be dealing with one class of claimants in one way and with the other in a perfectly different way, and instead of dividing the assets equitably would be dividing them inequitably.

I am, therefore, unable to follow the mode of valuation suggested in *Bell's Case*, and, after the best consideration which I am able to give to the subject, and after fortifying myself with the best advice which I could procure from the best authorities, I have come to the conclusion that there must be what is termed and well known as a pure premium valuation as at the date of the winding-up order, the rate of interest assumed being four per-cent, and the tables being the Seventeen Offices Experience Tables. The principle will be adopted in this way. There must be determined on the one hand the present value of the reversion in the sum assured at the decease of the life, and on the other the present value of the future annual premiums. The difference between these two sums represents the value of the policy. But the annual premium payable is divisible into two parts,—first, the part which it is calculated will provide for the risk, called the pure premium, and secondly, the addition for office expenses and other charges, which is sometimes called *the loading*. The pure premium only is to be taken into account. The value of the policy, therefore, will be the difference between the value of the reversion in the sum assured and the value of a life annuity of an amount equal to the pure premium.

The observations I have made have put out of the case altogether the question of any peculiar mode of valuation of profit policies. That I consider to have been properly dealt with by the Court of Chancery. The portion of the premium attributable to the sharing of the profits is altogether out of the case.

## *On the History and Tendency of Past Legislation with reference to Friendly Societies.*

[Extracted by permission of the publishers (Messrs. Longmans, Green, & Co.) from the article on "The Savings of the People" in the *Edinburgh Review* for July 1873.]

IN 1793 the first Act "for the encouragement of Friendly Societies" was passed, and its preamble gave as the reason for passing it, that the encouragement of such societies was likely to be attended with very beneficial results, "by promoting the happiness of individuals, and at the same time *diminishing the public burdens*". The encouragement given by the State consisted of exemptions from fees and stamps, facilities in recovering debts and settling disputes, and exemption of members from removal under the Poor Law until they became actually chargeable to the parish. The next Act of any consequence was that of 1819. Another Act had become needed. Since the passing of the former Act the population had increast from 9,000,000 to 11,500,000, but the amount annually expended in poor-law relief had risen to more than £7,500,000—within half a million of the amount annually expended now with nearly a doubled population. In other words, while since 1750 the population had increast by one half, the amount of poor-law relief had increast tenfold, or twenty times as fast as population.

The following is the preamble of the Act of 1819*:—

> "Whereas the habitual reliance of poor persons upon parochial relief rather than upon their own industry, tends to the moral deterioration of the people and to the accumulation of heavy burdens upon parishes, and it is desirable, with a view as well to the reduction of

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* 59 Geo. iii, cap. 128.

the assessment made for relief of the poor, as to the improvement of the habits of the people, that encouragement should be afforded to persons desirous of making provision for themselves or their families out of the fruits of their own industry . . . . and it is desirable *to protect such persons against the effects of fraud or miscalculation*," &c.

The prominent object of the State was undoubtedly to lessen the poor rates by inducing the working classes to become dependent on their *own* class by mutual insurance, instead of upon the rates levied on real property; and the practical way in which this was promoted was by the encouragement of benefit clubs—a mode of investment of savings peculiarly liable to *fraud* and *miscalculation*.

Now, laudable as the policy of the State may have seemed from the point of view of the ratepayers, we wish to look at it here from the point of view of the working classes; and to follow the action of the State as it sought, in the words of the Act of 1819, "to protect" the members of the clubs "from the effect of fraud and miscalculation".

And first as to *miscalculation*. Not only the rules but the tables of these clubs were to be submitted to the justices, and no tables or rules connected with calculations were to be confirmed by them, until they had been approved by two persons at least, known to be professional actuaries or persons skilled in calculations, as fit and proper, according to the most correct calculations of which the nature of the case would admit. In 1829 the necessity for the sanction of actuaries was abolisht, not because it was not needed, but because it was ineffectual. The statute of that year recited, that the existing data for the calculations of tables had been found imperfect and inefficient, and it enacted that quinquennial returns should be made by the clubs, so that correct data might be secured for the future. But, strange to say, the poor justices, who, with the aid of actuaries, had not succeeded in the past, were now in the future to be thrown upon their own guesses as to what payments were needful for insurance against sickness and death. They were henceforth to "*satisfy themselves* that the tables of payments and benefits proposed to be used were such as might be adopted with safety to all parties concerned". In other words, the State, in continuing to patronize the clubs, still undertook to guard them against miscalculations, while it acknowledged that their tables were at best very precarious. At length, in 1834, after further experience, the State backt out of all responsibility for the calculations, and relieved the justices from a duty which they were totally unable to discharge.

Now this legislation, with all its vacillation, disappointment, and ultimate retractation, was undoubtedly good-intentioned; but it was from the working man's point of view singularly unfortunate. The patronage of the State had caused an undue popular reliance upon clubs, the majority of which, owing to flaws in the data on which the tables were based, were necessarily and inherently rotten. The certificates of actuaries and the confirmation of their tables by justices, could not, from the nature of the case, secure their solvency. Instead of withdrawing its patronage, the State simply withdrew these precautions, and so, while technically freeing itself from responsibility in case of the insolvency of the clubs, it practically surrendered them to insolvency as their inevitable destiny.

That this uncertainty in the calculations was almost certain to produce insolvency, may be seen at once, when it is realized that the provision by insurance for old age and death, depends upon the regular payment and accumulation at compound interest of weekly or monthly payments during early and middle life. Every deficient *penny* per week in a member's payments continued at compound interest during the forty years, between twenty and sixty, must thus amount at the end of that period to a deficiency in the funds of £15 or £16 for that member alone. So precarious is the system of benefit clubs as a provision for old age, that if (without anyone's fault) fifty years ago, the weekly payments were fixed by the tables a few pence too low (as they almost universally were), a society with thousands of pounds safely invested, having husbanded all its moneys with perfect honesty and made no losses, may yet find itself today, when its accounts are investigated by an actuary, insolvent and able to pay only a few shillings in the pound of its future liabilities. And when to the chances of the payments being a few pence too low, are added the chances of the actual expenditure in dinners and salaries being a few pence too high, it will be seen at once how exceedingly precarious a mode of providing for sickness and old age the general run of clubs must inevitably be at best, leaving out of view all chances of fraud, speculation, or gross mismanagement. Could there have been a greater failure on the part of the State in the attempt to secure the working classes from the effects of miscalculations in the tables of the clubs?

And now as to the State protection of the clubs from *fraud.*

The chief chance of losses by fraud was found to be connected with the keeping and investment of the funds. By the statute of 1819, the State offered to take care of any funds of the clubs, giving Government security for their safety; and in order to tempt them to accept the offer, the rate of interest was fixt at a little more than 4½ per-cent. In 1828 the interest was lowered for all future clubs to a little above 3¾ per-cent. In 1850 the rate for future societies was reduced to little more than 3 per-cent, at which it now remains. In 1855 it was enacted, that the funds from fresh members who had entered or should enter even the old societies after 1850 should only receive the last and lowest rate.

The reasons for these reductions in the rate were obvious. The higher rates have, it is calculated, involved a loss to the State of not far short of £1,000,000 sterling; and had they been continued, the tax upon the country would have been far more serious. No doubt the reduction of the rates was necessary from the point of view of the Chancellor of the Exchequer, but it was singularly unfortunate from the working men's point of view. For by reducing the rate of interest, after offering a higher rate for a considerable period, the State, however unintentionally, greatly increast the chances of the insolvency of the clubs. It is true that notice was given, and an increase ought at once to have been made in the scale of payments for all future members of the clubs. But, as we have seen, the amount of the payments was at best a guess—a matter of rule of thumb. A certain rate, from long usage, had become general—a sort of pattern rate. One club copied another. Old clubs went on and new ones were started without any clear realization on the part of those who managed them, of what difference in the long run the reduction of the rate from 4½ to 3 per-cent would make in their solvency; whereas, in fact, in a calculation dealing with lifetimes, and thus embracing periods of forty or fifty or even sixty years, the difference in the accumulations of weekly and monthly payments at the higher and the lower rate was really immense. The same sum which commenced at 20 and accumulated at 4½ per-cent would amount to a given sum at fifty-nine, at 4 per-cent would not produce the same sum till sixty-one, and at 3 per-cent not till sixty-five. But illiterate members of clubs could not be expected to see this. They expected the same benefits for the same payments at the same ages as before. The solvency of the clubs was left in most cases to the chapter of accidents. And thus the action of the State, having for its object the protection of clubs from fraud, and really succeeding in securing the safety of the funds placed under its charge, unintentionally tended in another way to increase the chances of their general insolvency.

In a word, well-intentioned State patronage of benefit clubs for a century, has diverted the savings of the working classes to an enormous extent, from safer and better investments, into one which from its necessary conditions is almost the most precarious of all—so precarious that it is said that 50 or 70 per-cent or even more of the clubs are hopelessly insolvent. Those which are suspected to be rotten and insolvent are at least much more numerous than those which are known to be solvent and sound.

In ten parishes comprized in one division of a Hertfordshire Union, whose guardians made the inquiry, it was found that out of 130 adult males receiving parochial relief at the end of 1871, eight only were members of clubs; seventy more had been in clubs (making more than half of the whole number), but of these seventy, sixty-six ceased to be members of the clubs by their breaking up, and only four had otherwise withdrawn from them! This is the story of the State patronage of benefit clubs—a story of good intentions, leading to mistaken and vacillating action, and ending in failure and disaster.

This break-down of the benefit club system has become so complete and notorious, that a commission of inquiry is now sitting upon the subject. The whole of the evidence is not yet publisht, but it has already gone far enough to place some leading points in a clear light:—

(1) It has become manifest that the attempt to provide adequate allowance in sickness or a fixt annuity *during old age*—say after sixty-five—by means of benefit clubs has failed. In very few cases has it been realized in practice; and when attempted on sound priciples, it has been found to be so expensive as to make it a bad investment in a financial point of view.

(2) If provision for old age in the shape of annuities and pensions be a good object in itself, it can now be provided, with absolute Government security, through the Annuity Department of the Post Office.

(3) Were sick clubs establisht confined to securing adequat allowances during sickness—say *up to the age of sixty,* and no longer (with the addition, perhaps, of burial money in case of death under that age)—not only might they be made perfectly safe, but the payments might be, of course, much smaller than those required in a club providing for old age also. Let such clubs be as far as possible confined within a small area, and as much as possible to a circle within which the members are well known to one another,

so as to prevent imposition; and then to members entering in early life, say under thirty, the payment of 3*d.* per week would probably secure from 10*s.* to 15*s.* per week during sickness, and £5 burial fee in case of death up to the age of sixty.*

But sick clubs (in this restricted sense) standing alone would leave provision for old age to be otherwise made. They would only fulfil the proper function of co-operative insurance against sickness *up to* old age. They would be an aid to, not a substitute for, saving. They would secure a man from contingent disaster *while he was saving,* and so prevent his savings being dissipated by temporary ill-health or accident.

There remains, therefore, the question how the provision for old age ought to be made.

We have already insisted that the attempt to provide sickness allowances or pensions in old age, through the agency of the ordinary class of benefit clubs, is disastrous and must end in disappointment; and that if made at all, it had much better be made under Government security through the medium of the Post Office. But we wish to go one step farther, and to ask whether, after all, this kind of provision be in itself a good one, regarded as a general mode of saving, for adoption by the working classes? Is the benefit club or pension system in principle and in practice a mode of providing for old age on which an independent, honest working-man ought to be encouraged to rely?

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Looking at the system from the point of view of the working classes, instead of the ratepayers, it is impossible to regard it as satisfactory. A member of a benefit club may have been saving for years; but in case of his death, whether at thirty, forty, or sixty, it is as though he had not saved at all. Beyond the mere burial fee spent on the day of his funeral, his family are left as destitute as if he had not saved at all. Is it, indeed, a very laudable and meritorious thing for a man to throw his savings into such a shape, that, whilst affording to himself a precarious security against having to end his days in the workhouse, his family are left to the tender mercies of the Poor Law after his death? Would it be thought a sound and right mode of saving for the richer classes? And if not, why is it for the poor? While rich men have their settlements and entails and life insurances to keep their family property together, poor men are forsooth so to invest their savings that they may die with them! Could a scheme be devized more adapted to the purpose it has so fatally secured, namely, the perpetuation from generation to generation of a "proletariat"?

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* Very useful and simple clubs for this restricted object may be constructed on the principle of co-operative insurance, either with the same payment for all members who enter under a certain age, or with graduated payments. And such a club may be so constituted as to divide its surplus between its members at the end of every year, keeping a very small sum in hand to equalize the averages of good and bad years. It never could be insolvent for more than a month or two at most, because each fresh year would start with a clean balance-sheet; and if from special circumstances 4*d.* instead of 3*d.* was needed, the payment could easily be raised for a while to meet emergencies, or lowered if found to be more than was needed.

Again, if the system be tried by the test of how far it places the working man in an independent position in case of his being thrown out of work, or in connection with disputes between capital and labour, or if perhaps it should be desirable for him to emigrate, or, lastly, in enabling him to take advantage of opportunities for bettering his position and rising in the world, the conclusion is the same. However much he may have saved, it is, in all these cases, just as tho' he had not saved anything. If thrown out of work, his club does not help him. If he considers himself entitled to higher wages, he cannot, without combining with others in an organized strike, press his demand, because he has no capital to fall back upon. If he wishes to emigrate, not only has he no money saved which is available to pay his passage, but also by emigrating he will lose all his savings: he must, in most cases, leave behind him the provision for his old age for which he may have been subscribing to his club for years. Lastly, if the chance should offer of his becoming his own master, and a little capital needs to be invested in a workshop, materials, or tools, the money he may have been putting for years into a club is wholly out of his reach. Valuable, therefore, as might be sick clubs, restricted to the special and most useful object of insurance against the risks of sickness *while people are saving*, the system of benefit clubs, *as a mode of saving*, stands condemned on all hands: (1) as notoriously and almost necessarily insecure; (2) as uneconomical; (3) as vicious in principle, being a system of saving whereby no capital is created in the hands of the working classes. It is a system, which, if relied on by the working classes in the future to the same extent as it has been in the past, must tend to perpetuate a proletariat, and to keep the working classes in that state of famine as to capital, which has been pointed out as the great blot on their economic condition.

# *On Rates of Mortality and their Causes. By* W. Lazarus, *of Hamburg, Actuary of the General Insurance Company of Trieste. Translated by* T. B. Sprague, M.A., *Manager of the Scottish Equitable Life Assurance Society.*

A CORRECT knowledge of the rate of mortality among mankind is of such importance for answering many interesting questions, that we might hav expected it would hav been diligently studied from the earliest times. This, however, appears not to hav been the case, the reason probably being that questions as to the expectation of life arise in the first instance with reference to particular persons, and not to men in general. The proverbial uncertainty of human life—so often seen in the occurrence of death when least expected—is unfavourable to the notion of a general law of mortality, but naturally suggests the search after a means of predicting the duration of individual life. As thus understood, the question of the duration of life has at all times attracted great attention. Soothsayers and magicians claimd to hav the power of discerning from signs and omens of many kinds every event of life, its termination not excepted, and astrologers pretended to read in the stars the precise hour of each man's death.

It was reservd for a comparativly recent period to perceiv that the prediction of future events cannot be the result of a special prophetic gift; but that the only means we hav of lifting the veil of the future consists in a knowledge of the laws of nature that determin that future. If we wish to predict the length of human life, we must investigate the natural law which regulates life and death.

But so long as enquirers attempted to construct a law of mortality from observations on individuals, they must necessarily be unsuccessful, because the human organism thro' its complexity modifies a thousand-fold every law of nature that influences it. Whenever we hav recognized in nature the regular sequence of cause and effect, so as to refer the effect to a cause that has calld it into existence, we hav always found this effect to depend upon a definit set of conditions, which accompany the cause in such a way that a change in the conditions is followd by a change in the effect, the conditions themselves thus really constituting a part of the cause. If, however, we mentally distinguish between the conditions and the cause, this is solely for the sake of simplicity of conception. The various states and relations of the human organism, deviating on both sides from a certain medium (or normal) state, form such conditions; and when we have observd the effect that a given cause has produced on the normal organism, we hav no right to expect that it will produce the same phenomenons on an organism that deviates from the medium; for the conditions in the two cases are different. Now in making observations as to mortality, we find it utterly impossible to select for the purpose individuals who can be considerd as completely normal and agreeing in all respects with the medium; for not only is that medium itself imperfectly known to us, but even if it were known, agreement with it would be extremely difficult to define and to test, because the living organism is in a constant state of development and change. In order, therefore, to determin the typical man to which our observations are to refer, we must adopt the expedient of classing together a very large number of individuals who are as nearly as possible in the same circumstances, and observing this aggregat. The larger the number of the persons combined in such a group, the greater becomes the probability that deviations in one direction among the individuals composing it will be compensated by equal deviations in the contrary direction; and altho' the individual men observd will not coincide with the typical or normal man, yet, if the group is sufficiently large, the average represents an ideal type, so that by observing the aggregat of the phenomenons of this group, we may trace the law they follow under normal conditions. Various circumstances concur to make the collection of a sufficiently extensiv number of exact observations on the rate of mortality, one of the most difficult of statistical undertakings; and thus it happens that the knowledge of human mortality is even at the present time far from complete. The results hitherto obtaind are, as regards their details, far from certain and well establisht; much remains to be done in this direction, and the work is still being carried on with the greatest zeal.

Altho' the determination of the numerical death-rate is certainly the most important part of the enquiry, the following pages are devoted to a totally different branch of it, their aim being to consider the manner in which the observd facts of mortality may be referd back to the causes that produce them. No apology, however, will be required for the investigations, if they either lead to a satisfactory theoretical conclusion, covering the whole observd phenomenons, or are used for the discovery of new truths. Either of those results, tho' of no present practical utility, would be sufficient justification of the course of reasoning that led to it. Moreover, experience teaches us that, sooner or later, such labours are rewarded by useful practical results.

The author of these remarks, conscious that he has gained no such end, and that his work therefore requires some apology, hopes that a sufficient one may be found in his wish to direct the attention of more able enquirers to some noteworthy points well deserving their consideration; and that, if his remarks should giv an impulse to more successful investigations in the same direction, and to the attainment of more valuable results, they may lay claim to indulgent consideration.

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The very slightest consideration of the rate of mortality shows that it exhibits a certain regularity and conformity to law; and even in early times it appears to have been seen that the death rate among mankind must stand in some relation to the age. The fact that the death rate, which is very considerable at birth, diminishes pretty rapidly until it reaches a minimum; and then increases, at first slowly, but afterwards with constantly increasing rapidity; is so obvious that it could not long remain undiscoverd. It required, however, closer observation to investigate the numerical changes in the rate of mortality in passing from one age to another, and the actual rate at each age, and at the same time to determin whether the above is a complete description of the general course of the curve of mortality, or whether there are, besides the minimum mortality just mentiond, other less conspicuous minimums and maximums. We hav already seen that this problem can be solvd only by means of observing a large body of persons. If we wish to determin the rate of mortality at the various ages, we must collect into a group, as large a number as possible of persons of the same age, and we must observ how many of these persons die in a given time, making this observation for each age separatly.

But when observation has furnisht the answer to this enquiry, the further question arises, whether it is possible to survey from a common point of view, and to combine into a whole, the series of separat results so obtaind, in such a way that the observd ratios shall appear as the consequences of a law which governs the mortality. This question is quite independent of the former, and, to answer it, we must attempt an explanation of the observd facts and argue back from them to their causes. The following remarks hav to do with this enquiry, which, it will be seen, belongs to the recognized domain of speculativ investigation. Taking for granted the facts, as determind by observation, we shall seek to elucidate them, to combine them into a whole, and explain them as a consequence of a law of nature, which is the real object of our search.

We hav only to bear in mind how difficult it is to survey and take in at a glance the results of observations on the rate of mortality, and we at once see that the solution of the above problem would not only be of the greatest importance to the more thoro' understanding of the law of mortality, but still more as a stepping-stone to further conclusions. The rate of mortality being different for each year of age, so that if we wish to represent the mortality of a particular country we must state the rate at every age, and consequently employ a numerical series of more than 90 terms; it is obvious that such a series is neither a good representation of the actual rate of mortality there prevailing, nor a convenient standard of comparison with the mortality prevailing elsewhere. That, for such purposes, we shall not make much progress with the results of observation, is shown very clearly by the numerous attempts that have been made to condense these serieses of figures into some simple quantity, such as the "average lifetime", the "years of life", the "mean age", and other auxiliary quantities. These, however, are unsatisfactory substitutes for the serieses of numbers from which they are deduced, inasmuch as they are averages, in which the properties of the original numbers are not reproduced, since identical average values may result from elements of the most different kind. If, on the contrary, the problem should ever be solvd of representing the whole series by a simple formula in which its peculiarities are preservd, this would present a clear picture of the law of mortality, and could be easily compared with the formula similarly got from any other series of a similar nature.

If, however, we are still very far from possessing an exact and complete knowledge of the rate of mortality; if we may rather say it is known to us only in its general outlines and more prominent features—should not this be a reason for abstaining for the present from more general investigations? We think not; but certainly it greatly increases the difficulties to be encounterd. It should also lead to double caution in our remarks; for while we must never allow our theory to contradict facts establisht by observation, we cannot consider its complete agreement with them, as a proof of its truth and accuracy. For the results of observation can never be considerd absolutely correct, but must rather be regarded as approximations to the truth; and we must constantly bear in mind the possibility of their deviating from it, sometimes on one side and sometimes on the other.

Another branch of science may be referd to in support of this opinion, that the fact of our observations not being absolutely trustworthy is no reason for postponing our investigations. It is very well known that if the orbits of the heavenly bodies are calculated from the law of gravitation, they differ greatly from the actual orbits, because the mutual attractions have been left out of account; indeed, the disturbances which the orbits of comets and asteroids suffer thro' these attractions, may be so great as completely to change their form. Nevertheless, it is of inestimable value to us to know the law of gravitation and to be able to deduce from it the elliptical form of the orbit. We can during a long series of investigations wholly disregard the disturbances; indeed, we are often compeld to do so, because the present resources of analysis do not permit of our completely taking account of their effect. The law of gravitation, however, was discoverd when the observations in existence, tho' good, were far from absolutely correct.

Going back, however, to an earlier time, when the law of gravitation and the elliptical form of the planetary orbits were not yet known; and when the observations on the latter were represented by epicycles; it must be admitted that this representation materially contributed to simplify men's ideas, and was an important step towards the discovery of the truth. But at that time observations were faulty; and the hypothesis in question was erroneous.

The difficulties of collecting large numbers of observations as to mortality are greater than they appear at first sight. In these observations we require to know, not merely the number of persons of a given age who hav died in a certain period, say a year, but the ratio of that number to the number of persons of the same age who were alive at the beginning of the period, because the mortality is measured by the ratio of these two numbers. It is but seldom that we can succeed in ascertaining both terms of this ratio directly and wholly by means of observation; and we are mostly compeld to calculate one or other of them by the help of some hypothesis, which detracts from the exactness of the results.

The results of observations on mortality are embodied in mortality tables. But these tables do not give the values of the ratio we hav just explaind; they usually give in one column the number of survivers at each age out of a certain number of persons born; and in a second, the differences, that is to say, the number of deaths at each age. Thus the numbers given by observation are reduced to the same radix, and this calculation is usually not very exact. If we calculate from different mortality tables the rate of mortality at each age, and compare the several rates at the same age with each other, we find deviations, more or less considerable, which arise partly from errors committed in the construction of the tables, but principally from the particular circumstances of the persons who were observd. For deathrates vary in different countries; they are not the same for different positions in life and professions; they are different for men and women. Hence arises a further difficulty in investigating the true rate of mortality; for in consequence of these differences it is necessary that the group observd shall be composed of the same class of persons as the body to which the results obtaind are to be applied. If, moreover, we bear in mind that the rate of mortality is influenst by such temporary causes as epidemics, unusual heat or cold, abundant or scanty harvests, we see that a long-continued series of observations is required before we shall be able to deduce from them correct values of the medium (or normal) rate of mortality. It is rather a matter for wonder that the results obtaind are a tolerably good approximation to the truth than that they are not free from error.

All observations on the rate of mortality exhibit the same general features,—a mortality which is considerable immediatly after birth, but rapidly decreases to a minimum between the ages of 5 and 12, and then increases, at first slowly, afterwards rapidly. Many mortality tables show other maximums and minimums, which are, however, less important.

Bearing this in mind, it is an obvious idea to consider the mortality as a function of the age, to express this function by an analytical formula, and thus to regard the numbers that appear in the mortality table as particular values of this function. Many attempts of this kind hav been made,* but without much success. The mistake was made of attempting to find a formula for the number living at each age in the mortality table, insted of considering the rate of mortality, (or still better, its increment,) as a function of the age.* It is of course true that the number living is also a function of the age, if the mortality is so; but evidently the latter must be the simpler function, and it is better therefore to start with it.

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* We quote the following, where $x$ denotes the age and $l_x$ the number living.

De Moivre, $l_x = 86 - x$

Lambert, $l_x = 1000 \left\{ \frac{96 - x}{x} \right\}^2 - 6176 \left\{ \epsilon^{-\frac{x}{13 \cdot 682}} - \epsilon^{-\frac{x}{2 \cdot 431}} \right\}$

Babbage, $l_x = 6199 \cdot 8 - 9 \cdot 29x - 1 \cdot 5767 \frac{x(x-1)}{2}$

Littrow, $l_x = 598 \cdot 1673 - 8 \cdot 4175x + \cdot 2309x^2 - \cdot 0052x^3 + \ldots.$

Moser, $l_x = 1 - \cdot 2x^{\frac{1}{4}} + \frac{\cdot 7125}{10^5} x^{\frac{9}{4}} - \frac{\cdot 1570}{10^8} x^{\frac{17}{4}} + \ldots.$

As a rule, mortality tables giv the numbers of the living and the deaths for successiv integral years of age. If we denote the number of persons living at the ages $x, x+1, x+2, \ldots$ by $l_x$, $l_{x+1}$, $l_{x+2}, \ldots$, then the differences $l_x - l_{x+1}$, $l_{x+1} - l_{x+2}, \ldots$ are equal to the number of the deaths; and the mortality at the age $x$ is exprest by the ratio $\frac{l_x - l_{x+1}}{l_x}$. This ratio, however, is obviously equal to the probability that a person of the age $x$ will die in the course of the year. In what follows, we shall, consistently with this remark, use the word *mortality* as synonymous with the *probability of dying in a year;* and shall express the probability of the contrary event, $1 - \frac{l_x - l_{x+1}}{l_x} = \frac{l_{x+1}}{l_x}$ by the expression *vitality.* In the same way, the probabilities that death will, and that it will not, occur in a shorter or longer period than a year, may be termed the mortality and vitality of that period. If, then, we suppose that insted of observing the mortality during complete years among persons of integral ages, as $x$, we observ it during periods of $\frac{1}{2}$ year, $\frac{1}{4}$ year, or still shorter periods, the ages of the persons observd also proceeding by differences of $\frac{1}{2}$ year, $\frac{1}{4}$ year, &c.; and we then register the facts thus observd, in mortality tables of the usual form, we shall get a table from which would be deduced the probability, at each such fractional age, of dying in $\frac{1}{2}$ year, $\frac{1}{4}$ year, &c. Suppose that we proceed in this way to the limit, that is to say, to the mortality of the next instant, then we find the probability of dying in the next instant

$$\begin{aligned}\frac{l_x - l_{x+dx}}{l_x} &= -\frac{dl_x}{l_x}\\ &= -d\log l_x;\end{aligned}$$

and this probability givs us a suitable starting point for further investigations; $l_x$ being here the number of the living at any age $x$, supposed to vary continuously; and $-dl_x$ being the number of the dying in the instant $dx$.

So long as we confine ourselves, as hitherto, to the region of

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* The latter course has been very successfully adopted by Gompertz in 1825 and by Makeham more recently.

**purely numerical investigation, we have no difficulty in ascertaining the laws there prevailing, because these are of universal application; but as soon as we quit this region for another, we must ascertain whether the magnitudes we deal with can be subjected to numerical measurement, before we shall be justified in applying to them those laws. It is therefore necessary in the first place to examin our subject somewhat more closely.**

(*To be continued.*)

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# HOME AND FOREIGN INTELLIGENCE.

## LEGAL AND GENERAL LIFE ASSURANCE SOCIETY.

*Established* 1836.

### SIXTH DIVISION OF PROFITS, 1871.

### REPORT OF THE DIRECTORS, 16th April, 1872.

THIS MEETING is called, in pursuance of the provisions of the Society's Deed of Settlement, "for the purpose of declaring the amount of Profit to be set apart out of the Assurance Fund in respect of the period of five years" ending with the 31st December, 1871.

The amount of disposable Profit largely exceeds that declared at the close of any previous five years since the establishment of the Society.

Throughout the investigation, however, of which they now submit the results, the Board have looked to Security rather than to Profit: the valuation has accordingly been made upon the strictest and most cautious principles known to Actuaries.

1. The Table of Mortality employed throughout is that known as the "**17 Offices' Experience**" Table.
2. The rate of interest at which the liabilities have been discounted is **3 per cent.**, being the same as that originally assumed in determining the premiums charged by the Office.
3. The value of the pure or "**net**" **premium** only (*i.e.* the premium required exactly to meet the anticipated risk, on the assumption that there were no expenses and no profits,) has been brought into account against the corresponding liability under a Policy. All excess above the "net" premium, in the "gross" or Office Premium actually receivable *in the future*, has been rigidly reserved as a provision for *future* expenses, profits, and contingencies.
4. The assets of the Society have been estimated at their net market values on 31st December, 1871. They are fully detailed in the annexed Balance Sheet of the Society as at that date.

**The value of the protection** afforded to Policyholders, by the adoption of these conditions for the valuation, may be best shewn by a statement of the difference that would arise from the substitution of others less rigorous.

Thus (1) using the Carlisle Table of Mortality, (2) assuming 3½ per cent. as the rate of discount, and (3) bringing the gross premiums into account against the liabilities, the valuation surplus now to be declared would receive an apparent increase of £490,000. The mere difference in result thus amounts to one-third of the total Assurance Funds held by the Society.

**A short explanation of the principles** of valuation adopted, and of their effect in fortifying the position of the Society, may be acceptable to the Proprietors and the Assured.

*First.*—The enquiries upon which the "17 Offices' Experience" Table was founded had their origin in the conviction, that no conclusions drawn from the mortality amongst sections of the general population could afford so accurate a measure of the liability attaching to Life Assurance transactions as those drawn from the mortality actually occuring amongst *assured lives.* Accordingly, in the year 1843, the experience of seventeen leading Offices, including the "Equitable" and the "Amicable," was collected by a committee of the principal Actuaries, and Tables founded thereon were published. More recently, seeing how greatly Life Assurance had increased, the Institute of Actuaries determined upon the collection of a new "Experience." This has been published since the date of the Quinquennial Bonus Investigation made by the Society at the end of the year 1866. It extends to assurances upon 160,000 lives, contributed by 20 Companies, and confirms in an extraordinary degree the indications of the 1843 "Seventeen Offices' Experience" Table, which is found to necessitate a higher reserve against the death liabilities of an Office than any other in recognised use.

The Mortality actually experienced, however, by this Society, during the Quinquennium now concluded, has been considerably less than that anticipated by the Experience Table.

*Secondly.*—It will be borne in mind that the higher the rate of interest at which liabilities to mature in the future are discounted, the less is the present amount apparently required to meet them; and therefore the greater in the case of a Life Office is the surplus apparently disposable. By the assumption, therefore, that a rate of interest considerably higher than that used in the original calculation of the premiums will be realised during the whole continuance of the corresponding Policy-risks, an Office may for a time profess to retain sufficient reserves against liability, and yet give away unusually large sums by way of Bonus. But it has been authoritatively proved that another Office which, resisting the temptation thus to bid for popularity by the distribution of an artificially large Bonus, restricts the distribution to the smaller surplus arising from the assumption of a lower rate of interest, will ultimately be enabled, out of the improvement and accumulation of the larger reserve, to give the larger and more permanent Bonus, *whilst retaining in hand as additional security against its engagements* the additional sums prematurely lost to its competitor. Accordingly, in support of the Bonus to be now declared, the Society has during the past five years derived a considerable profit as interest upon its carefully husbanded funds; and this profit has been further increased by the fact that, in place of 3 per cent. assumed as the permanent rate of interest, the average rate at which the total assets were invested during the Quinquennium has been **£4 6s. 4d.** per cent., free of Income Tax.

*Lastly.*—The present value of the *excess* in the Office Premiums receivable in the future, above the net premiums, is shewn by the "Summary and Valuation of the Policies" to amount to **£330,000,** being nearly 26½ per cent. of the present value, £1,248,100, of such net premiums.

**Such being the principles** upon which the calculations are based, and the reserves provided for future Bonus, expenses, and contingencies, the Valuation of the total assurance transactions of the Society and the corresponding "Valuation Balance Sheet" are as follows:—

*Summary and Valuation of the Policies* (see p. 64.)

### VALUATION BALANCE SHEET.

*As at* 31*st December*, 1871.

("*The Life Assurance Companies Act*, 1870.")

| Dr. | £ |
|---|---|
| To net liability under Assurance and Annuity transactions (as per Summary and Valuation) | 1,280,963 |
| „ Claims announced, but not yet proved | 3,232 |
| „ Reserve for sundry accounts | 1,567 |
| | 1,285,762 |
| „ Disposable Surplus | 218,317 |
| | £1,504,079 |

| Cr. | £ |
|---|---|
| By Assurance Fund (as per Annual Balance Sheet annexed) | 1,487,698 |
| „ Accrued Interest (less tax) | 16,381 |
| | £1,504,079 |

**Of the disposable Cash Surplus** thus shewn the Directors recommend that the sum of £4,110 be carried forward, and that **£214,207** be set apart out of the Assurance Fund, in terms of the Society's Deed of Settlement, as profit in respect of the five years ending with the 31st December, 1871. This amount will, under the provisions of the Deed, be divisible between the Proprietors and the Assured in the following proportions:—

| | |
|---|---|
| To the Proprietors one tenth part, being ... | £21,420 |
| To the Assured nine tenth parts, being ...... | £192,787 |

**The Proprietors' share** is available towards an increase of the Dividend, to arise during the current five years from interest upon the Proprietors' Fund. The Directors accordingly propose to declare a dividend of **11s.** per Share, payable on the 1st July annually until the next Declaration of Profits. The Shares remain as before, with **£8** paid up on each.

## SUMMARY AND VALUATION OF THE POLICIES.

*As at 31st December*, 1871.

(*"The Life Assurance Companies Act*, 1870.")

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | | VALUATION. Value by the "17 Offices' Experience" Table, Interest 3 per cent. | | | |
|---|---|---|---|---|---|---|---|---|
| | Number of Policies. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Yearly Premiums. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Yearly Premiums. | Net Liability. |
| ASSURANCES. | | | | | | | | |
| I.—With Participation in Profits. | | £ s. d. | £ s. d. | £ | £ | £ | £ | £ |
| For Whole Term of Life | 2,755 | 3,877,405 18 2 | 114,665 12 11 | 91,120 | 2,297,608 | 1,470,429 | 1,153,277 | 1,144,331 |
| Extra Premiums thereon | .. | .. .. | 447 12 0 | .. | .. | .. | .. | 224 |
| Other Classes— | | | | | | | | |
| By Commuted Premiums | 30 | 61,121 0 0 | .. .. | .. | 43,753 | .. | .. | 43,753 |
| By Limited do. | 12 | 17,376 0 0 | 849 5 6 | 705 | 7,664 | 6,003 | 4,963 | 2,701 |
| Last Survivor | 17 | 28,412 0 0 | 390 19 2 | 278 | 15,566 | 5,687 | 4,020 | 11,546 |
| Joint Life | 21 | 24,517 10 0 | 1,021 12 6 | 791 | 15,813 | 11,642 | 8,986 | 6,827 |
| By Ascending Premiums | 4 | 13,000 0 0 | 231 14 2 | 186 | 6,072 | 7,348 | 5,853 | 219 |
| By Descending do. | 2 | 6,068 0 0 | 120 14 2 | 94 | 2,679 | 1,729 | 1,306 | 1,373 |
| Endowment Assurance | 1 | 1,078 0 0 | .. .. | .. | 1,004 | .. | .. | 1,004 |
| Extra Premiums thereon | .. | .. .. | 14 0 0 | .. | .. | .. | .. | 7 |
| Total Assurances with Profits | 2,842 | 4,028,978 8 2 | 117,741 10 5 | 93,174 | 2,390,159 | 1,502,838 | 1,178,405 | 1,211,985 |
| II.—Without Participation in Profits. | | | | | | | | |
| For Whole Term of Life | 233 | 430,250 0 0 | 13,253 8 5 | 11,809 | 237,755 | 184,498 | 164,163 | 73,592 |
| Extra Premiums thereon | .. | .. .. | 107 8 0 | .. | .. | .. | .. | 54 |
| Other Classes— | | | | | | | | |
| By Commuted Premiums | 1 | 500 0 0 | .. .. | .. | 326 | .. | .. | 326 |
| Last Survivor | 26 | 56,585 0 0 | 1,720 10 7 | 1,444 | 30,305 | 23,267 | 19,041 | 11,264 |
| By Ascending Premiums | 5 | 28,500 0 0 | 852 15 0 | 788 | 17,626 | 17,865 | 16,520 | 1,106 |
| Endowment Assurances | 5 | 3,400 0 0 | 210 16 6 | 190 | 2,450 | 1,694 | 1,525 | 925 |
| Endowments | 2 | 3,587 0 0 | 100 0 0 | 100 | 3,258 | 379 | 379 | 2,879 |
| Survivorships | 49 | 66,390 0 0 | 1,379 0 6 | 1,043 | 9,412 | 8,242 | 6,417 | 2,995 |
| Temporary Assurances | 28 | 40,550 0 0 | 534 16 0 | 379 | 839 | 713 | 507 | 332 |
| Extra Premiums thereon | .. | .. .. | 5 16 8 | .. | .. | .. | .. | 3 |
| Total Assurances without Profits | 349 | 629,762 0 0 | 18,164 11 8 | 15,753 | 301,971 | 236,658 | 208,552 | 93,476 |
| Total Assurances | 3,191 | 4,658,740 8 2 | 135,906 2 1 | 108,927 | 2,692,130 | 1,739,496 | 1,386,957 | 1,305,461 |
| Deduct Reassurances | .. | 354,581 6 5 | 10,760 9 2 | 9,285 | 182,272 | 161,462 | 138,943 | 43,329 |
| **Net Amount of Assurances** | **3,191** | **4,304,159 1 9** | **125,145 12 11** | **99,642** | **2,509,858** | **1,578,034** | **1,248,014** | **1,262,132** |
| Adjustment, viz.— For making all Claims payable *three* months after death | .. | .. .. | .. .. | .. | .. | .. | .. | 9,360 |
| | | | | | | | | 1,271,492 |
| ANNUITIES. | | Per Annum. | | | | | | |
| Immediate | 5 | 1,225 0 0 | .. .. | .. | 7,506 | .. | .. | 7,506 |
| Contingent | 4 | 925 0 0 | 22 14 6 | 14 | 1,568 | 137 | 83 | 1,485 |
| Deferred Temporary (no Life Contingency) | 1 | 550 0 0 | .. .. | .. | 480 | .. | .. | 480 |
| **Total of the Results** | .. | .. .. | 125,168 7 5 | 99,656 | **2,519,412** | **1,578,171** | **1,248,097** | **1,280,963** |

**The share allotted to the Assured** exceeds by about £40,000 that given at the last Division. The Directors have therefore the satisfaction of proposing to declare a Reversionary Bonus Addition **at the rate of £1 10s. per annum to each £100 assured by a Policy of not less than five years' duration,** (being £75 to each £1,000 for the Quinquennium), together with a proportionate addition upon all previous Bonus remaining attached to the Policy.

**The number of Policies participating** in the present Bonus Division is 2,842, assuring

| | |
|---|---|
| as original Principal Sums .................. | £3,638,882 |
| as existing Bonus Additions...................... | 390,096 |

As these previous Bonus Additions, which also will now receive proportionate additions, amount to more than one-tenth of the principal sums, the effect of this system of division is to increase by more than one-tenth the rate of Bonus, reckoned upon the original sums in the Policies only. Thus reckoned, the Reversionary Bonus now declared amounts **on an average** to the unusually high addition of **£82 10s.** for the five years, or **£16 10s. per annum on each £1,000.**

In addition to the vested Bonus a Prospective Annual Bonus at the same rate, and equally provided out of the profits now realised, will be assigned to all Bonus Policies becoming claims by death after not less than five years' duration before the next Division of Profits takes effect. This regulation secures to every Policy entitled to participate in the profits a full Bonus for each complete year it may have been in force when the life assured fails.

For all Bonus Policies which may not yet have completed the five years from the date of issue, qualifying for participation in the profits, a proportionate addition has been reserved, which will attach at the date of such completion.

**The total amount of Reversionary Additions** made to Policies as the result of the present Division is **£261,623.**

The addition made to an individual Policy is equal to the amount of Assurance, *entitled to participation Profits*, which the Cash Surplus falling to that Policy, viewed as a single Premium, would purchase at the present age. This mode of conversion best secures the maintenance of the present rule of treating vested Bonus Additions as sums assured, entitled in their turn to share in future Bonus Divisions.

**The various Returns and Schedules** required by "The Life Assurance Companies Act, 1870," to be lodged with the Board of Trade within nine months of the present Investigation will be sent in immediately.

**The Directors desire,** in concluding this Report, to record their opinion of the advantages to flow from the ample and precise information which the Schedules of the Act are calculated to render generally accessible. Publicity must eventually render impossible a continuance of the business of fraudulent companies, and at the same time prove the groundlessness of doubt and anxiety in regard to the safety of the principles on which Life Assurance is based. With increased knowledge will come increased recourse by the public to those Offices alone which demonstrate the soundness of their position.

## *BALANCE SHEET on the* 31*st December*, 1871.

| **LIABILITIES.** | | | | £ | s. | d. |
|---|---|---|---|---|---|---|
| Shareholders' Capital paid up | 160,000 | 0 | 0 | | | |
| *Interest, &c., at Credit of same* | 9,810 | 17 | 3 | | | |
| | | | | 169,810 | 17 | 3 |
| Assurance Fund | | | | 1,487,698 | 1 | 0 |
| Annuity Fund (*Nil*) | | | | | | |
| Total Funds | | | | 1,657,508 | 18 | 3 |
| Claims admitted but not paid | | | | 20,969 | 12 | 0 |
| Other sums owing by the Society, viz.:— | | | | | | |
| Dividends to Shareholders | 727 | 1 | 0 | | | |
| Due to Mortgagors | 2,158 | 18 | 5 | | | |
| | | | | 2,885 | 19 | 5 |
| | | | | £1,681,364 | 9 | 8 |

| **ASSETS.** | | | | £ | s. | d. |
|---|---|---|---|---|---|---|
| Mortgages on Property within the United Kingdom | | | | 960,207 | 8 | 11 |
| Ditto out of the United Kingdom (*Nil*) | | | | | | |
| Loans on Society's Policies | £35,257 | 14 | 8 | | | |
| Loan on a Policy and Shares | 500 | 0 | 0 | | | |
| | | | | 35,757 | 14 | 8 |
| INVESTMENTS:— | | | | | | |
| In British Government Securities | | | | 128,100 | 0 | 0 |
| Indian and Colonial Government Securities | | | | 19,875 | 0 | 0 |
| Foreign Government do., viz.:— | | | | | | |
| *Russian Railway Bonds* | £58,351 | 0 | 0 | | | |
| *United States* 5/20 *Bonds* | 10,033 | 0 | 0 | | | |
| | | | | 68,384 | 0 | 0 |
| Railway Debentures and Debenture Stocks, viz.:— | | | | | | |
| *English Debenture Stocks* | £91,450 | 0 | 0 | | | |
| *Indian Do.* (*Guaranteed*) | 77,900 | 0 | 0 | | | |
| *Indian Debenture Bonds* (*Guaranteed*) | 94,965 | 0 | 0 | | | |
| | | | | 264,315 | 0 | 0 |
| Railway Shares (*Nil*) | | | | | | |
| House Property (*The Society's Freehold Offices*) | | | | 12,800 | 0 | 0 |
| OTHER INVESTMENTS, VIZ.:— | | | | | | |
| Estate at Birkenhead | | | | 30,000 | 0 | 0 |
| Loans on Reversionary Charges | | | | 64,698 | 0 | 0 |
| Reversionary Annuities bought | | | | 45,250 | 0 | 0 |
| Rent charges bought | | | | 15,479 | 6 | 7 |
| Life Interest bought | | | | 3,373 | 0 | 0 |
| Shares in the Society bought | | | | 1,604 | 15 | 0 |
| Loans on Personal Security (*Nil*) | | | | | | |
| Agents' Balances (*Nil*) | | | | | | |
| Outstanding Premiums | | | | 7,425 | 16 | 10 |
| Ditto Interest | | | | 6,045 | 3 | 2 |
| CASH:— | | | | | | |
| On Deposit | £15,000 | 0 | 0 | | | |
| In hand, and on Current Account | 3,049 | 4 | 6 | | | |
| | | | | 18,049 | 4 | 6 |
| | | | | £1,681,364 | 9 | 8 |

*The following further particulars are extracted from the Returns under the Fifth and Sixth Schedules of the Life Assurance Companies Act:—*

The Deed of Settlement of the Society provides (Section 116) "that "at the expiration * * * of every * * * period (to be fixed

" by the Board of Directors) not exceeding Seven years * * * the
" Board of Directors shall cause a calculation to be made of the
" amount of the Profits * * * which, according to the existing
" knowledge of the principles and the approved practice of Life Assur-
" ance at the time, may, in the judgment of the Actuary of the
" Society, or of other the person or persons making such calculation,
" be with safety set apart * * *."

The principles adopted under the authority of this Section for the purposes of the present valuation were, as regards—

I. Mortality.—That the Mortality to be allowed for by this Society should prove identical with that indicated as occurring amongst Assured lives by the "Seventeen Offices' Experience" Table, recently confirmed by the New Experience collected by the Institute of Actuaries, and published in 1869:

That all Policies of this Society being payable three months only after death, a special reserve should be made on this account:

And that the Annual Premiums being receivable in advance should be assumed to be all due, one with another, six months hence.

II. Age.—That the age should be determined from the year of birth.

N.B. The manner in which this principle has been applied is best shown by an illustration: thus all lives born in the last half of 1841, or in the first half of 1842, were taken as born, one with another, on 31st December, 1841, and therefore to be of the exact age of 30 at 31st December, 1871; and so of other ages.

III. Interest.—That the rate of Interest at which the Assurance and Annuity Liabilities were discounted should be the same as that used in the original calculation of the Premiums (in the case of this office, 3 per cent.).

IV. Loading.—That the whole loading or excess in the Office Premiums actually receivable in the future above the Net (Experience 3 per cent.) Premiums should be excluded from the estimated Assurance Assets placed against estimated Assurance Liabilities, as being a provision for future Expenses and Profits.

On the Annual Premium-Income for Policies participating in Profits this Reserve amounts to Twenty-one per cent., and on that for non-Profit Policies to Thirteen per cent.

" . . . . *distribution of Profits.*"—The Deed of Settlement provides (Section 117) "that when * * * after every * * * period
" * * * to be fixed as aforesaid, an Extraordinary General Meeting
" specially called for the purpose * * * shall have declared the
" amount of the Profits * * * the Board of Directors shall cause
" * * * nine equal tenth parts * * * to be appropriated to those
" Life Policies issued by the Society * * * in respect of which the
" Premiums payable shall be calculated for persons desirous of partici-
" pating in the profits * * * and shall cause so much of the nine
" equal tenth parts * * * to be apportioned * * * to each of the
" said Policies as the Board of Directors shall consider fair and
" equitable * * *."

The distribution of the Profits ascertained at this valuation has been made on the principle of increasing by a uniform percentage not only the sum originally assured, but also any previous Bonus remaining attached to the Policy.

*CONSOLIDATED REVENUE ACCOUNT for* Five *Years, commencing 1st* January, 1867, *and ending 31st* December, 1871.

| | £ | s. | d. |
|---|---|---|---|
| Amount of Funds on 1st January, 1867, the beginning of the period | 1,405,040 | 2 | 0 |
| Premiums (after deduction of Reassurance Premiums) | 637,554 | 9 | 11 |
| Consideration for Annuity granted | 2,380 | 0 | 0 |
| Interest and Dividends | 333,349 | 4 | 6 |
| Other Receipts— | | | |
| Discount on Claims and Office Fees | 365 | 10 | 10 |
| Sundry Profits | 7,446 | 17 | 8 |
| | £2,386,136 | 4 | 11 |

| | £ | s. | d. |
|---|---|---|---|
| Claims under Policies (after deduction of Sums Reassured) | 570,605 | 2 | 4 |
| Surrenders | 17,660 | 19 | 3 |
| Annuities | 3,279 | 18 | 1 |
| Commission | 29,252 | 7 | 8 |
| Expenses of Management | 40,428 | 14 | 1 |
| Dividends to Shareholders | 47,500 | 0 | 0 |
| Other Payments:— | | | |
| Bonus Cash Payments | 19,900 | 5 | 3 |
| Amount of Funds on 31st December, 1871, the end of the period | 1,657,508 | 18 | 3 |
| | £2,386,136 | 4 | 11 |

The declaration of Profits is made upon Policies of not less than five years' duration. For those which may not at the time have completed the qualifying five years, a proportionate addition is reserved, to attach at the date of such completion.

The following Table shows the amount of Bonus so apportioned as the result of the present Valuation, viz.:—either, (**A**) as a reversionary addition to the sum originally assured; (**B**) as an equivalent cash payment; or (**C**) as a reduction during each of five years from the Annual Premium:—

| Effected at Age | NUMBER OF YEARS FOR WHICH POLICY FOR £100 IN FORCE. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 | | | 10 | | | 15 | | | 20 | | |
| | A | B | C | A | B | C | A | B | C | A | B | C |
| | £ | £ s. d. | £ s. d. | £ | £ s. d. | £ s. d. | £ | £ s. d. | £ s. d. | £ | £ s. d. | £ s. d. |
| 20 | 7 | 1 18 9 | 0 8 5 | 8 | 2 9 0 | 0 10 6 | 8 | 2 14 5 | 0 11 9 | 9 | 3 8 7 | 0 14 10 |
| 30 | 7 | 2 7 7 | 0 10 3 | 8 | 3 1 0 | 0 13 2 | 8 | 3 8 6 | 0 14 10 | 9 | 4 6 9 | 0 18 11 |
| 40 | 7 | 3 0 0 | 0 13 0 | 8 | 3 17 0 | 0 16 10 | 9 | 4 17 0 | 1 1 7 | 9 | 5 7 10 | 1 4 4 |
| 50 | 7 | 3 15 6 | 0 16 10 | 8 | 4 15 10 | 1 1 7 | 9 | 5 18 10 | 1 7 7 | 9 | 6 9 4 | 1 11 4 |

| Effected at Age | NUMBER OF YEARS FOR WHICH POLICY FOR £100 IN FORCE. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 25 | | | 30 | | | 35 | | |
| | A | B | C | A | B | C | A | B | C |
| | £ | £ s. d. | £ s. d. | £ | £ s. d. | £ s. d. | £ | £ s. d. | £ s. d. |
| 20 | 9 | 3 17 1 | 0 16 8 | 10 | 4 16 4 | 1 1 0 | 10 | 5 7 10 | 1 4 0 |
| 30 | 9 | 4 17 0 | 1 1 7 | 10 | 5 19 10 | 1 7 0 | 11 | 7 5 2 | 1 13 9 |
| 40 | 10 | 6 12 0 | 1 10 8 | 10 | 7 3 8 | 1 14 10 | 11 | 8 10 0 | 2 4 0 |
| 50 | 10 | 7 14 6 | 2 0 0 | 11 | 9 0 7 | 2 11 2 | 12 | 10 7 4 | 3 7 2 |

*Average Rate of Interest at which the Life Assurance Fund of the Company was invested (after deducting Income Tax) at 31st December.*

| 1867. | 1868. | 1869. | 1870. | 1871. |
|---|---|---|---|---|
| £ *s.* *d.*<br>4 6 3 | £ *s.* *d.*<br>4 5 4 | £ *s.* *d.*<br>4 6 8 | £ *s.* *d.*<br>4 6 8 | £ *s.* *d.*<br>4 6 8 |

For the surrender of non-profit Policies for the whole term of life and of Endowment Assurances is allowed as a minimum value one-third of the ordinary Premiums received as for the true age at entry. For Profit Policies is allowed:—

(1.) The same minimum value as though they had been on the Non-Profit scale.

(2.) The full cash value according to the published Bonus Table of any Profits attached to the Policy.

N.B. All Endowment Assurances granted by the Society are without profits.

If a Policy is granted on an "unhealthy" or under-average life, the Premium is charged as though the true age were as much higher as is supposed to correspond to the deterioration. This office age is then substituted for the true age for all valuation and reserve purposes. For purposes of surrender, whether of Policy or Bonus, the true age is adopted.

The Society does not transact business at other than European rates.

## PRACTICAL QUESTIONS.

"A, B, C, D, respectively aged 73, 68, 67, 40, are each entitled to "an equal share of the rents of a freehold estate during their respec-"tive lives, and as each life drops the survivers continue to share the "rents equally until there is only one life left, and then that life is "entitled to the whole fee of the estate. They are now all willing "to sell, and it is required to know in what proportion the proceeds of "the sale should be divided between them."

Taking 1 as the annual rental of the estate, and $\frac{1}{i}$ as the value of the fee simple of the estate, we may consider the interest of each of the persons A, B, C, D, as made up of two parts; namely, his interest in the reversion to the fee simple, and his life interest in the rental. Thus, employing the life contingency notation recommended by the Institute,

Value of 73's interest in the reversion to the fee simple = $A_{\overline{68.67.40}:\overset{4}{73}} \times \frac{1+i}{i}$; the assurance here being the value of 1 payable on the death of the last surviver of 68, 67, 40, provided that 73 be then alive, the 4 denoting that 73 is to die fourth. Also the value of his life interest in the rental of the estate

$$=\tfrac{1}{4}a_{73.68.67.40}+\tfrac{1}{3}(a_{73.68.67}-a_{73.68.67.40})+\tfrac{1}{2}(a_{73.68}-a_{73.68.\overline{67.40}})+\tfrac{1}{3}(a_{73.68.40}-a_{73.68.67.40})+\tfrac{1}{2}(a_{73.67}-a_{73.67.\overline{68.40}})$$
$$+\tfrac{1}{3}(a_{73.67.40}-a_{73.68.67.40})+\tfrac{1}{2}(a_{73.40}-a_{73.40.\overline{68.67}})$$

Now $a_{73.68.\overline{67.40}}=a_{73.68.67}+a_{73.68.40}-a_{73.68.67.40}$; substituting this in the above, and making similar substitutions for $a_{73.67.\overline{68.40}}$ and $a_{73.40.\overline{68.67}}$, and reducing, the above expression for the value of the life interest in the rental of the estate$=$ $\tfrac{1}{2}(a_{73.68}+a_{73.67}+a_{73.40})-\tfrac{2}{3}(a_{73.68.67}+a_{73.40.68}+a_{73.67.40})+\tfrac{3}{4}a_{73.68.67.40}$, and the value of 73's entire interest in the estate becomes

$$A_{\overline{68.67.40}:\overset{4}{73}}\times\frac{1+i}{i}+\tfrac{1}{2}(a_{73.68}+a_{73.67}+a_{73.40})-\tfrac{2}{3}(a_{73.68.67}+a_{73.68.40}+a_{73.67.40})+\tfrac{3}{4}a_{73.68.67.40}.$$

Similarly,

Value of 68's interest$=A_{\overline{73.67.40}:\overset{4}{68}}\times\dfrac{1+i}{i}+\tfrac{1}{2}(a_{68.73}+a_{68.67}+a_{68.40})-\tfrac{2}{3}(a_{68.73.67}+a_{68.73.40}+a_{68.67.40})+\tfrac{3}{4}a_{73.68.67.40}$;

Value of 67's interest$=A_{\overline{73.68.40}:\overset{4}{67}}\times\dfrac{1+i}{i}+\tfrac{1}{2}(a_{67.73}+a_{67.68}+a_{67.40})-\tfrac{2}{3}(a_{67.73.68}+a_{67.40.68}+a_{67.40.73})+\tfrac{3}{4}a_{73.68.67.40}$;

Value of 40's interest$=A_{\overline{73.68.67}:\overset{4}{40}}\times\dfrac{1+i}{i}+\tfrac{1}{2}(a_{40.73}+a_{40.68}+a_{40.67})-\tfrac{2}{3}(a_{40.73.68}+a_{40.68.67}+a_{40.73.67})+\tfrac{3}{4}a_{73.68.67.40}$.

For the purpose of verifying these formulas, we observe that the sum of the four values should be equal to the total value of the estate; that is, to $\dfrac{1}{i}$.

Adding the values of the separate interests, we get their sum equal to

$$A_{\overline{73.68.67.40}}^{3}\times\frac{1+i}{i}+(a_{73.68}+a_{73.67}+a_{73.40}+a_{68.67}+a_{68.40}+a_{67.40})-2(a_{73.68.67}+a_{73.68.40}+a_{73.67.40}+a_{68.67.40})+3a_{73.68.67.40}.$$

The first term is the value of the reversion to the estate and one year's rental on the death of the 3rd of the four lives whichever it may be; and we shall now shew that the other terms express the value of an annuity to continue so long as any two of the four lives are in existence, that is to $a_{\overline{73.68.67.40}^2}$.

We have

$$a_{\overline{73.68.67.40}^2}=a_{73.68.67.40}+a_{73.68.67.40|\overline{73.68.67.40}^3}+a_{\overline{73.68.67.40}^2|\overline{73.68.67.40}^2}.$$

Now

$$a_{73.68.67.40|\overline{73.68.67.40}^3}=a_{73.68.67.40|73.68.67}+a_{73.68.67.40|73.68.40}+a_{73.68.67.40|73.67.40}+a_{73.68.67.40|68.67.40}$$

$$=a_{73.68.67}+a_{73.68.40}+a_{73.67.40}+a_{68.67.40}-4a_{73.68.67.40}.$$

Also

$$a_{\overline{73.68.67.40}^2|\overline{73.68.67.40}^2}=a_{73.68.\overline{67.40}|73.68}+a_{73.67.\overline{68.40}|73.67}+a_{73.40.\overline{68.67}|73.40}+a_{68.67.\overline{73.40}|68.67}+a_{67.40.\overline{73.68}|67.40}+a_{68.40.\overline{73.67}|68.40}$$

$$=a_{73.68}+a_{73.67}+a_{73.40}+a_{68.67}+a_{68.40}+a_{67.40}-3(a_{73.68.67}+a_{73.68.40}+a_{73.67.40}+a_{68.67.40})+6a_{73.68.67.40}.$$

Hence

$$a_{\overline{73.68.67.40}^2}=a_{73.68}+a_{73.67}+a_{73.40}+a^{68.67}+a_{68.40}+a_{67.40}-2(a_{73.68.67}+a_{73.68.40}+a_{73.67.40}+a_{68.67.40})+3a_{73.68.67.40}.$$

*Q.E.D.*

We have now to calculate the values given by the above formulas; and the first thing is to decide on the rate of interest to be employed. The property being freehold, we may take it as worth 30 years' purchase, which would give the rate of interest $3\frac{1}{3}$ percent; but as $3\frac{1}{2}$ percent is the nearest rate at which we have annuity values tabulated, we will adopt that rate, thus valuing the fee simple of the estate at $\frac{1}{\cdot 035}$, or 28·571 years' purchase. The annuities on two joint lives are those of the Institute $\text{H}^{\text{M}}$. The annuities on three and four joint lives have been calculated (using the same mortality table) by means of Mr. Woolhouse's formula,

$$a=m\text{S}+\frac{m-1}{2}-\frac{m^2-1}{12}(\mu+\delta);$$

and we refer our readers to vol. xvii, p. 267, for an example of the process. The values thus obtained are

$$\begin{aligned} a_{73.68.67.40} &= 2{\cdot}9857, \\ a_{73.68.67} &= 3{\cdot}1068, \\ a_{73.68.40} &= 3{\cdot}7347, \\ a_{73.67.40} &= 3{\cdot}8407, \\ a_{68.67.40} &= 4{\cdot}5699. \end{aligned}$$

The next step is to calculate the values of the four contingent assurances; $A_{\overline{68.67.40}:\overset{4}{73}}$, $A_{\overline{73.67.40}:\overset{4}{68}}$, $A_{\overline{73.68.40}:\overset{4}{67}}$, $A_{\overline{73.68.67}:\overset{4}{40}}$. As there exists no method of calculating these values exactly, it is necessary to adopt some method of approximation, and we know of none better adapted for the purpose than that explained and exemplified in Griffith Davies's *Treatise on Annuities*, p. 354 *et seq.* We give here as an example the calculation of the value of $A_{\overline{73.68.67}:\overset{4}{40}}$.

We have

$$\begin{aligned} A_{\overline{73.68.67}:\overset{4}{40}} &= \Sigma \frac{D_{40+n}}{D_{40}} \left( {}_{n-1}p_{\overline{73.68.67}} - {}_{n}p_{\overline{73.68.67}} \right) \\ &= \Sigma \frac{D_{40+n}}{D_{40}} \frac{(l_{73}-{}^{n}l_{73})(l_{68}-{}^{n}l_{68})(l_{67}-{}^{n}l_{67})-(l_{73}-{}^{n-1}l_{73})(l_{68}-{}^{n-1}l_{68})(l_{67}-{}^{n-1}l_{67})}{l_{73}l_{68}l_{67}}. \end{aligned}$$

To obtain the value of this expression the method proposed by Griffith Davies is to take certain equidistant terms and express the value of the whole series of terms in terms of these selected terms. Thus, denoting the $m$th term of the above series by $\frac{{}^{m-1}K}{D_{40}l_{73}l_{68}l_{67}}$, and assuming that the terms between this and the $2m$th term, $\frac{{}^{2m-1}K}{D_{40}l_{73}l_{68}l_{67}}$, are in arithmetical progression, it can be easily shown that the sum of the entire series of terms, that is, the value of

$$\begin{aligned} &\Sigma \frac{D_{40+n}}{D_{40}} \cdot \frac{(l_{73}-{}^{n}l_{73})(l_{68}-{}^{n}l_{68})(l_{67}-{}^{n}l_{67})-(l_{73}-{}^{n-1}l_{73})(l_{68}-{}^{n-1}l_{68})(l_{67}-{}^{n-1}l_{67})}{l_{73}l_{68}l_{67}} \\ &= \frac{\frac{m+1}{2}\,{}^{0}K + m({}^{m}K + {}^{2m}K + {}^{3m}K + \dots\dots)}{D_{40}l_{73}l_{68}l_{67}} \quad \dots\dots \quad \text{(I)} \end{aligned}$$

See Griffith Davies's *Treatise on Annuities*, p. 357.

Now the oldest age in the table being 97, ${}_{31}p_{\overline{73.68.67}}$ will be $=0$; and there will be 31 terms in $\Sigma$. Taking $m=6$, we get

$$A_{\overline{73.68.67}:\overset{4}{40}} = \frac{\frac{7}{2}\,{}^{0}K + 6({}^{6}K + {}^{12}K + {}^{18}K + {}^{24}K + {}^{30}K)}{D_{40}l_{73}l_{68}l_{67}}.$$

The values of ${}^{0}K$, ${}^{6}K$, .... have now to be calculated, and we give as an example that of ${}^{6}K$.

We have $\frac{{}^6K}{D_{40}l_{73}l_{68}l_{67}} = \frac{D_{47}}{D_{40}} \cdot \frac{l_{73}-l_{80})(l_{68}-l_{75})(l_{67}-l_{74})-(l_{73}-l_{79})(l_{68}-l_{74})(l_{67}-l_{73})}{l_{73}l_{68}l_{67}}$.

| | | | | | |
|---|---|---|---|---|---|
| $l_{73}$ 30823 | $l_{68}$ 42717 | $l_{67}$ 44960 | $l_{73}$ 30823 | $l_{68}$ 42717 | $l_{67}$ 44960 |
| $l_{60}$ 13930 | $l_{75}$ 25691 | $l_{74}$ 28269 | $l_{79}$ 16068 | $l_{74}$ 28269 | $l_{73}$ 30823 |
| 16893 | 17026 | 16691 | 14755 | 14448 | 14137 |

| | | | | | |
|---|---|---|---|---|---|
| log $(l_{73}-l_{80})$ | 4·227707 | log $D_{40}$ | 4·317701 | log $(l_{73}-l_{79})$ | 4·168939 |
| „ $(l_{68}-l_{75})$ | 4·231113 | „ $l_{73}$ | 4·488875 | „ $(l_{68}-l_{74})$ | 4·159808 |
| „ $(l_{67}-l_{74})$ | 4·222482 | „ $l_{68}$ | 4·630601 | „ $(l_{67}-l_{73})$ | 4·150357 |
| | 12·681302 | „ $l_{67}$ | 4·652826 | | 12·479104 |
| | | | 18·09003 | | |

| | | | | |
|---|---|---|---|---|
| | 12·681302 | | | 12·479104 |
| log $D_{47}$ | 4·178463 | | | 4·178463 |
| | 16·859765 | ·058852 | | 16·657567 |
| | −18·090003 | −·036946 | | −18·090003 |
| | $\bar{2}$·769762 | ·021906 $=\frac{{}^6K}{D_{40}l_{73}l_{68}l_{67}}$ | | $\bar{2}$·567564 |
| $=\log^{-1}$ | ·058852 | | $=\log^{-1}$ | ·036946 |

In this way we obtain the values shown in the following table:—

| $m$ | $\frac{{}^{m-1}K}{D_{40}l_{73}l_{68}l_{67}}$ |
|---|---|
| 1 | ·000210 |
| 7 | ·021906 |
| 13 | ·038467 |
| 19 | ·019068 |
| 25 | ·003379 |
| 31 | ·000000 |

And finally from the formula (I), $A_{\overline{73.68.67}:\overset{4}{40}}=$·497655.

In a precisely similar way we obtain

$A_{\overline{68.67.40}:\overset{4}{73}}=$·020472, $A_{\overline{73.67.40}:\overset{4}{68}}=$·043211, $A_{\overline{73.68.40}:\overset{4}{67}}=$·049554.

Substituting these values, and those of the annuities as given above, in the expressions representing the values of the respective interests of 73, 68, 67, 40, we get

| | | |
|---|---|---|
| Value of | 73's interest | = 2·2935 |
| „ | 68's „ | = 3·5948 |
| „ | 67's „ | = 3·8159 |
| „ | 40's „ | =18·2019 |
| | | 27·9061 |

The sum of these respective interests ought to be $\frac{1}{i}$ or 28·571; but bearing in mind the approximat methods used in the calculations the result must be considered satisfactory. Thus, then, we arrive at the result, that if S be the sum realized by the sale of the estate, the respective shares of 73, 68, 67, and 40, should be

$$\frac{2\cdot2935}{27\cdot9061}S, \quad \frac{3\cdot5948}{27\cdot9061}S, \quad \frac{3\cdot8159}{27\cdot9061}S, \quad \frac{18\cdot2019}{27\cdot9061}S,$$

or 8·219, 12·882, 13·674, 65·226, percent respectively.

# CORRESPONDENCE.

*To the Editor of the Journal of the Institute of Actuaries and Assurance Magazine.*

Sir,—I have lately been reading some philosophical correspondence which took place between my namesake "the Learned John Ray and several of his Ingenious Correspondents," and have come across the following, which I think will interest your Fire Assurance readers.

1st, as regards the word "Petroleum."

I always imagined that this was a newly coined word, in use only since the comparatively recent discovery of the rock oil in America; but by the following letter from Mr. Willughby to Mr. Ray in 1662, it appears to have been then in use:—

"The first thing I saw considerable after I left Montpellier was a spring of Oleum Petroleum at Gabian; at the same place there is a kind of a black Pumice Stone, and a Medicinal Well . . . ."

2nd. A letter from Sir Hans Sloane to Mr. Ray is very interesting:—

"London, *Nov.* 10, 1685.

"I wrote a pretty while ago to you about the Hockesdon Earth; which, because I fear it miscarried, I now repeat, desiring your Opinion of it.

"Not far from Moorfields, near the new Square in Hockesdon [Hoxton Square], some Workmen digging a Cellar for a new House in the End of a Garden, when they were about 3 Foot below the Surface of the Ground, found a very strong Smell in the one half thereof. Passing that Way, and finding it very surprizing, and a thing that I had neither heard of, nor seen before, I thought it worth farther Enquiry.

"The Workmen having dug a Pit about 6 Foot deep, at about 3 Yards Distance from that End of the Cellar which smelt so strong, I there found three several Layers of Earth one over another, all of them, more or less, having the same Scent. The uppermost Stratum was Clay, or, as the Workmen call it, Loom: It did not smell till 3 Foot deep; but then was very strong, and something noisome. If one look earnestly on some Pieces of this Clay, there are easily discernable several small Quantities of a bituminous Substance, brownish Colour, and rough Consistence. I doubt not but this Substance gives the Smell, and other Qualities, to this Layer. This Clay preserves its Scent a pretty while, tho' by Degrees it grows fainter; and being expos'd to the Air for about a Month, will lose it quite. Eight Pounds of this Clay distill'd in a Retort, plac'd in a Sand-Fire ($3^d$ Degree of Heat), yielded one Pound of Phlegmatick Liquor, and six Drams of Oil, of a quite different Smell from anything I have hitherto met with.

"The second Layer was Gravel, which reached from 3 and a half to about 4 and a half deep, or thereabouts. It very much resembles the other in all its Qualities, except the Noisomness of its Smell: It loses its Scent much sooner than the former.

"The third layer was an earthy Sand, which smells stronger than the other two, and withal is much more fragrant: The deeper you dig, it smells the stronger. I took 8 Pound of this Layer at 9 Foot deep, and fill'd a Retort with it, and plac'd it as the Clay; but it afforded only 6 Ounces of Phlegmatick Liquor, and 2 Drams of Oil. This sandy loose Earth quits its Scent in about a Fortnight, being expos'd to the Summer-Air.

"Considering that Waters owe their greatest Differences to the several Soils through which they pass, I was very desirous to see what Sort of Waters would be produced by their being percolated through such a Strainer as this strange Sort of Earth; and desiring the Owner to dig till he should find Water, he accordingly did; and when he came to about 18 Foot deep, Water came in very plentifully; condition'd as follows:—

"It had at Top a curiously colour'd Film, the Colours of it resembling those of the Rainbow: Under this was a whitish coloured Water, which upon standing in a Vial some Days, lets fall a brownish Sediment, and by that means becomes diaphanous: It smelt very strong, as the Earth did, was somewhat bitter and clammy, as one may see by putting his Hands in it, and suffering them to dry without wiping. If you put some powder'd Galls into a Glass of this Water, so soon, or a little after you take it out of the Well, it will turn of a purplish red; but if it stand a Day or two, it will not at all.

"Several Persons having drunk of this Well about 3 Pints, say, that it usually works about 3 times by Stool, and very much by Urine.

"From which I conclude it to be a natural Bitumen, perhaps *sui generis*, that impregnates both Water and Earth. I desire your opinion in it, and remain, &c."

And the manner in which Mr. Ray overturns Sir Hans Sloane's theory, in his reply to the above, is highly amusing.

"BLACK NOTLEY, *Nov.* 17, 1685.

"I thank you for the Account of the Hockesdon Earth, and the rather, because I was lately informed that it was no natural Bitumen mingled with it, but had its' Original from the *burning* of a Painting Shop standing over the Spot where the Earth was digged up, and that there was no such Earth round about, but just within that Compass. So that as the Corinthian Brass was made by an accidentall Mixture of Metals melted down and running into one Mass, at the Deflagration of that City; so this bituminous Earth became impregnated by a Mixture of Oils and Colours melted and mixed together, and soaking into the Ground at the *burning* of that Shop."

If you deem the above extracts worthy of the space in your valuable *Journal*, kindly insert them in your next Number, and oblige,

Sir,
Your obedient servant,

R. RAY.

*Atlas Assurance Office, Cheapside,*
11 *July* 1873.

## ERRATUMS.

The following have been communicated to us:—

Vol. xiii, p. 307, line 3 from bottom of page.
Add the term

$$+\left\{\frac{3(1+i)^{\frac{3}{4}}+2(1+i)^{\frac{1}{2}}+(1+i)^{\frac{1}{4}}}{16(1+i)}\right\}.$$

Vol. xv, p. 129, Formula (6):

For $(a_x-a_{xy})\left(1+\frac{i^2}{8}+\frac{5}{24}i^3\,\ldots\,\right)$

read $(a_x-a_{xy})\left(1+\frac{i^2}{24}-\frac{i^3}{24}+\ldots\right).$

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

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## *On the Apportionment, or Division by Mutual Consent, of a Fund between the Life Tenant and the Reversioner. By* T. B. SPRAGUE, M.A., F.I.A., *Manager of the Scottish Equitable Life Assurance Society.*

[Read before the Institute, 24 November 1873.]

THE subject of this paper has on two previous occasions been brought before the Institute; namely, by Mr. Jellicoe on 26 November 1855, and by Mr. Baden on 27 March 1871. The former of these gentlemen stated that, at the time his paper was written, a great diversity of opinion existed among actuaries, as to the proper method to be adopted in advising upon the proportions in which a settld fund should be divided by mutual consent between the life tenant and the reversioner; and so far as can be judged from the discussion that followd the reading of the latter paper, a similar diversity still existed then. One point, however, was clearly establisht by the discussion, namely, that Mr. Baden was quite mistaken in supposing that, because Mr. Jellicoe's conclusions had not been publicly calld in question, therefore they wer generally accepted by members of the profession.

So long as much difference of opinion exists on a subject of practical importance, it cannot fail to be useful to bring it forward for further discussion from time to time, until the whole of the arguments for the various methods advocated must be considerd to hav been exhausted. It is not my intention to go again over the ground traverst by Mr. Jellicoe and Mr. Baden respectivly, or to examin the various arguments used by them. It will suffice to say that I accept the principls laid down by Mr. Baden, and agree generally with the objections he has urged against Mr. Jellicoe's method. Nor shall I consider the various possibl methods of division, as described by Mr. Jellicoe and illustrated by Mr. Porter (see vol. xvi p. 285). The only two methods which appear to call for any notice ar those advocated by Mr. Jellicoe and Mr. Baden. The other methods described by the former do not appear at the present time to find any advocats; and it may perhaps be fairly assumed that the use of them has been tacitly abandond. I purpose therefore to take up the subject at the point where Mr. Baden has left it; and will first state some additional reasons which appear to me to make his method preferabl to Mr. Jellicoe's, and then consider the method that should be adopted in certain cases where his principls ar not sufficient in themselvs to giv us any result.

It will be convenient in the first instance to describe clearly the two methods as I understand them. Mr. Jellicoe proposes that, in the first instance, the market values of the separat interests of the life tenant and the reversioner should be found by the formulas $\frac{1}{P+d}-1$ and $1-d(1+a)$. The sum of these market values must necessarily be less than the amount to be divided; and Mr. Jellicoe urges that the excess should be divided in proportion to these market values, so that the sum to be receivd by each of the claimants shall be exactly in proportion to the market value of his interest. Mr. Baden, on the other hand, urges that the fund shall be divided without any reference to the market values of the separat interests. The value of each interest is to be found according to the actual probabilities of life, the rate of interest used in the calculation being determind by comparing the incom produced by the fund with its capital amount; and these values will be the proper sums to pay to the two parties, and their sum will be necessarily equal to the sum to be divided.

Now I consider Mr. Jellicoe's method open to the following objections in addition to those urged by Mr. Baden. Firstly, that the sums found by the application of the above-given formulas will not, in reality, be the market values of the separat interests.

As regards the life interest, we should hav to make allowance in ascertaining its market value for the extra premium to provide against the contingency of the life proceeding to a tropical climate; also for the expenses that may hav to be incurd annually in obtaining proof of the continued existence of the life from time to time; and for the legal expenses attendant on the purchas. Some deduction would also be made on account of the incom tax being deducted on the total incom, and therefore virtually on the premium as well as on the interest. Assuming the life interest to be well secured, and the purchaser to be satisfied with 5 percent interest on his outlay, the actual market value will depend on the above circumstances; and it is accordingly incorrect to speak of the value found by the simple application of the above formula as being the market value. As regards the reversion, I consider it very doubtful whether the value found by the formula can be in any sense considerd as the market value. This, so far as I am able to judge, will agree more nearly with the value found from the Carlisle table of reversions with interest at 6 percent. Again, the market value of a reversion depends on the legacy or succession duty payabl; and in arriving at it, deductions ar always made for the legal expenses attendant, firstly, on the purchas, secondly, on the ultimat realization; and even assuming the principl of the formula to be correct, the value found from it without consideration of these circumstances cannot with any propriety be calld the market value of the reversion. It may be urged that in the case supposed, where a fund is divided by mutual consent, the legal expenses that would be incurd and hav to be allowd for in selling the separat interests, will not be incurd and need not be allowd for. This is perfectly true; but those who use the argument must admit also, that in the case supposed, the expense of effecting an insurance to secure the purchas of the life interest, or of purchasing an annuity to provide incom until the reversion falls in, need not be incurd and may therefore be disregarded. To sum up, I maintain that if we ar to be guided by market values, we must take the actual market values and not what may be calld theoretical market values.

My second objection is based on the fact that Mr. Jellicoe's formula for the value of a reversion gives a negativ value for young ages. Taking *d* at 5 percent and the annuity at 3½ percent, the value of a reversion on a life of 60, as found by Mr. Jellicoe's formula, is nearly equal to the value found by the Carlisle 6 per-cent table; at older ages, the former value is greater; but at younger ages, it becomes less than the value found by the latter table, until when we arrive at the age 19, it becomes negativ. Now the truth of a principl is always best tested by applying it to an extreme case. If the principl is a true one, it will hold in every case that can be imagind. Let us then apply Mr. Jellicoe's method of apportionment to determin in what proportion we should divide a sum between a life tenant of the age 19, and the reversioner. The result it leads to is that the life tenant is to take the whole of the fund and to receiv something in addition from the reversioner. This result is of course absurd. In other words, Mr. Jellicoe's method fails entirely for ages of 19 or less; its principl therefore cannot be correct. If we reject the negativ value given by the formula, and say that the reversion has no market value, it will follow from Mr. Jellicoe's principls that the whole of the fund is to be paid over to the life tenant. It may be urged that the case of a life tenant, age 19, is one that could not possibly occur in practice, for this reason, if no other, that he could not legally concur in a sale of the property; but it is not difficult to give instances of cases that actually occur, where Mr. Jellicoe's method leads to the same unsatisfactory result. Suppose, for exampl, that A and B, whose ages ar 65 and 60, ar successiv life tenants of an estate, and that on their death C is entitld to the fee simple; and it is required to know in what proportion the proceeds of the sale of the estate should be divided between A, B, and C. Here the reversionary life interest of B has no market value; but it would be clearly incorrect to say that therefore the proceeds of the sale ar to be divided between A and C, to the exclusion of B. It is one great merit of Mr. Baden's method that it applies equally well, whatever the age of the life tenant, and whatever the number of successiv life tenants.

Now it is to be noticed that, even if we admit the correctness of the first principl laid down by Mr. Jellicoe,—that the life tenant and the reversioner ar to receiv in the first instance the market values of their respectiv interests, it by no means follows that in the case where one of those interests,—say that of the reversioner,—has no market value, the whole of the fund is to be paid to the life tenant. This is a result of the second principl laid down by Mr. Jellicoe, namely, that the excess beyond the sum of the market values, is to be divided in proportion to those values. It will be seen that Mr. Jellicoe's argument in support of this second principl is extremely weak and unsatisfactory; and it has been well pointed out by Mr. Edward Smyth (see vol. xvi, p. 386) that there ar two other methods of dividing the excess, which ar just as plausibl. One of these is the division of the excess equally between the life tenant and the reversioner; the other is a rather more complicated arrangement, and will be best understood by the use of algebraical symbols. Suppose that V and W are the full values of the respectiv interests of the life tenant and the reversioner, so that $V+W=$sum to be divided, $=1$, suppose. Also let $v$ and $w$ be the market values of the separat interests. Then the excess to be divided is $1-v-w$; and Mr. Smyth says, "It may be argued that in addition to the market value of his interest, each party should be credited with the difference between the market value of the interest of the other party and the value of it when computed as a part of the perpetuity; for this is the difference of which each deprives the other until both consent to effect the arrangement:" in other words, $v+W-w$ would by this arrangement be allowd to the life tenant, and $w+V-v$ to the reversioner. The application of either of these methods of division would get rid of the anomaly of assigning the whole of the fund to the life tenant. For if we put $w=0$, or suppose the interest of the reversioner to hav no market value; then if the excess, $1-v$, is divided equally, the sums payable to the life tenant and reversioner will be $\frac{1+v}{2}$ and $\frac{1-v}{2}$ respectivly; while if the second method of dividing the excess is adopted, the respectiv shares will be $v+W$ and $V-v$. Either of these results is more reasonabl than that to which Mr. Jellicoe's principl leads. For even assuming that the reversioner's interest has no market value, it cannot be supposed that he can be reasonably calld upon to give up the whole of the fund to the life tenant without receiving an equivalent. These considerations, I think, justify me in saying that even if we admit the principl of starting in the first instance from market values, we must reject as unsound in principl the other part of Mr. Jellicoe's process, which divides the excess of the fund in proportion to the separat market values.

In the ordinary case of apportionment between life tenant and reversioner, or between several successiv life tenants and the ultimat reversioner, there will, when Mr. Baden's method is employd, be no excess of value to divide; for, as already stated, the sum of the values of the separat interests is exactly equal to the sum to be divided. Ther ar, however, two cases which sometimes occur, where the sum of the separat values is less than the amount to be divided;—where in fact the life tenant and the reversioner can, by combining, obtain a greater interest than the sum of the two that they possess separatly. The first case is that of the tenant for life and the tenant in tail of an estate, the latter of whom, I will assume for the sake of simplicity, has no issue. If the tenant in tail should survive the life tenant, he will then be able to obtain the fee simple of the property; but if the life tenant and the reversioner combine to bar the entail, they can jointly obtain a power of disposal over the fee simple in the lifetime of the life tenant. So long as the two do not join, the interest of the reversioner is liable to be defeated by his dying before the life tenant; and if he sells his interest, a purchaser who wishes to secure himself from loss will be under the necessity of effecting an insurance on the reversioner's life against that of the life tenant; whereas, if the life tenant and reversioner combine, this insurance is unnecessary. If, then, the values of the separat interests ar computed according to the probabilities of life, the sum of these values will be less than that of the estate, altho' the rate of interest used in the calculation may be precisely that deduced from a comparison of the rental of the estate with its fee simple value. Ther remains therefore an excess to be divided, being, in fact, the value of the contingent reversion to the estate on the death of the life tenant, if the reversioner dies before him. The second case is that where A and B ar successiv life tenants of an estate, but hav a joint power of appointment over the whole or a portion,—a power, that is to say, which can only be exercised by the two combining for the purpose, so that if one of them dies before the power is exercised, it cannot be exercised at all. In this case, the additional interest which arises for division between A and B, is the reversion to the estate (or to the sum to be appointed) expectant on the death of the last surviver of A and B. I will suppose for the present that the power of appointment extends to the whole estate, which the successiv life tenants concur to sell.

In these two cases Mr. Baden's principles giv us no rule as to the ratio in which the property is to be divided; for when each of the consenting parties has receivd the full value of his separat interest, there remains still a sum availabl for division. In the latter of the two cases it is easily seen that it would be incorrect to divide this excess in proportion to the separat values of the interests. For suppose A to be on his death-bed, so that the real value of his life interest is insignificant, but that he is in possession of his full faculties; then the surplus to be divided is the reversion expectant on the death of B, and the result of dividing it in proportion to the values of the separat interests would be to give the whole to B to the exclusion of A. But this would clearly be unjust to A; for his consent to the exercise of the joint power of appointment is still just as necessary as if he were in perfect health; and if he declines to join in exercising it, B will lose for ever all share in the ultimat reversion.

In the former case, if we suppose the life tenant to be on his death-bed, so that we may neglect the possibility of the tenant in tail dying before him, the true value of the tenant in tail's interest becoms approximatly equal to that of the estate, and the supposition therefore dos not help us towards determining the propriety of dividing the surplus in proportion to the values of the separat interests. If, however, we suppose the reversioner to be on his death-bed, so that the value of his expectation is practically zero, the argument employd abov will apply. If the reversioner does not join in executing the disentailing deed, the tenant for life will, in the cases where there is no possibility of his being able to com to terms with the next tenant in tail, lose all chance of obtaining any portion of the reversion. It would therefore be unjust to giv the whole of the surplus to the life tenant, which would be the result of dividing it in proportion to the values of the separat interests.

We thus see that in both cases the principl of dividing the surplus in proportion to the values of the separat interests works unjustly under certain circumstances. That principl therefore cannot be a correct one, and we must seek for som other. Having, however, shown the inapplicability of the abov mentiond principl, I am unable to see any good reason why one of the parties interested should receiv a greater share of the surplus than the other;—in other words, my conclusion is that the surplus should be divided equally between them. The final conclusion then is that in both the cases under consideration each of the parties interested should receiv the full value of his separat interest and one half of the surplus.

It will now be convenient to giv formulas for the several shares. Suppose the value of the estate to be 1, and the annual incom to be $i$; $x$ to be the age of the tenant for life, and $y$ that of the reversioner, being the tenant in tail in the first case, and the next life tenant in the second case. Then the life tenant's interest in the estate is represented by an annuity of $i$ continued up to the day of his death, and its value is therefore $i\mathring{a}_x$. The tenant in tail's interest may be taken as the contingent reversion to the fee simple, if he should survive the life tenant; such reversion being payabl at the instant of the life tenant's death, and not (as supposed in the usual formulas) at the end of the year in which that death occurs. Its value will therefore be $\bar{A}^{1}_{xy}$, and the surplus remaining for division will be $1-(i\mathring{a}_x+\bar{A}^{1}_{xy})$. Dividing this equally between the life tenant and the tenant in tail, we get

$$\text{Life tenant's share} = \tfrac{1}{2}\{1+i\mathring{a}_x-\bar{A}^{1}_{xy}\};$$
$$\text{Tenant in tail's ,, } = \tfrac{1}{2}\{1-i\mathring{a}_x+\bar{A}^{1}_{xy}\}.$$

In the second case, the interest of the present life tenant (or A) will, as before, be represented by $i\mathring{a}_x$; and that of the next life tenant, or B, by $i\mathring{a}_{x|y}=i(\mathring{a}_y-\mathring{a}_{xy})$. The surplus to be divided is therefore $1-\{i\mathring{a}_x+i(\mathring{a}_y-\mathring{a}_{xy})\}=1-i\mathring{a}_{\overline{xy}}$; and dividing this equally between A and B, we get

$$\text{A's share} = \frac{1}{2}+\frac{i}{2}(\mathring{a}_x-\mathring{a}_y+\mathring{a}_{xy});$$
$$\text{B's ,, } = \frac{1}{2}-\frac{i}{2}(\mathring{a}_x-\mathring{a}_y+\mathring{a}_{xy}).$$

Let us now suppose that the object of the two parties in concurring, is not to sell the estate and divide the proceeds, but to raise a sum of money by mortgage of the fee simple, thus keeping the estate in the family; and this, I believe, will be found to be the more common case. Other things remaining the same, suppose that a sum S is to be raisd by mortgage of the estate, and that the interest to be paid on the loan is at the rate of interest $j$, which, it is to be observd, will always in practice be a higher rate than $i$. At the present time, the values may be fairly taken as ·0333 for $i$ and ·04 for $j$, supposing the property to be a large one. Then, in the first case, the incom of the life tenant will be diminisht by $Sj$, and the sum to be receivd by the tenant in tail will be diminisht by S. The values of the sums thus given up by the respectiv parties, in consequence of the proposed arrangement, must then be calculated at the rate of interest $j$, and the surplus divided equally. Thus we shall get, using dashes (′) to indicate that the annuity and assurance values ar to be calculated at the rate of interest $j$, and not, as before, $i$,

$$\text{Life tenant's share} = \frac{\mathrm{S}}{2}\{1 + j\mathring{a}'_x - \overline{\mathrm{A}}'^{1}_{xy}\};$$

$$\text{Tenant in tail's ,, } = \frac{\mathrm{S}}{2}\{1 - j\mathring{a}'_x + \overline{\mathrm{A}}'^{1}_{xy}\}.$$

In the second case, the incom of both A and B will be diminisht, in consequence of the arrangement, by S$j$; and the values of these reductions must be calculated at the rate of interest $j$, and the surplus, as before, divided equally between A and B, thus giving

$$\text{A's share} = \frac{\mathrm{S}}{2} + \frac{\mathrm{S}j}{2}(\mathring{a}'_x - \mathring{a}'_y + \mathring{a}'_{xy});$$

$$\text{B's ,, } = \frac{\mathrm{S}}{2} - \frac{\mathrm{S}j}{2}(\mathring{a}'_x - \mathring{a}'_y + \mathring{a}'_{xy}).$$

The legal and other expenses attendant on the transaction, may fairly be divided in proportion to the sums receivd by the two parties; but any succession duty payabl in consequence of the arrangement, must of course be borne by the reversioner.

I have supposed abov that the tenant in tail has no issue. If, however, he is married and has one or more children, the value of his interest may be considerably increast. He can at once obtain what the lawyers call a "base fee", that is to say, he or his assigns will be entitled to receiv the rents of the property as long as any of his children or their issue shall be living. I will not attempt to giv any formula for the value of the interest in this case, as there are som legal questions connected with the interest, which I am not prepared to answer. Thus, if the tenant in tail, after acquiring a base fee, dies in the lifetime of the tenant for life, leaving issue surviving, can the base fee (which we will suppose to hav been sold by the tenant in tail) be enlarged into a fee simple on the death of the life tenant? and if so, when and how? On the answer to these questions will of course depend the value to be assignd to the tenant in tail's interest. I leav this point to be discust by those members of the Institute who hav been led by circumstances or inclination to study the intricacies of real property law.

It may now be useful to giv som illustrations of the numerical application of the abov principls and formulas. I will first take the case, so fully discust by Mr. Jellicoe, of the division of £1000 3 per-cent Stock, between a life tenant of 60 and the reversioner. It is first to be observd that, tho' Mr. Jellicoe in his numerical illustration assumes the Stock will realize £850, he has very clearly laid down the principl that whatever sum the fund may realize, is to be divided in the same proportion; so that whatever sum the fund realizes, the ratio of the two shares will be unalterd. According to Mr. Baden's principls, however, we must compare the amount which the Stock will realize with the annual incom arising from it, in order to determin the rate of interest. I will consider the four cases of the Stock realizing £850, £900, £950, and £1000, respectivly. The rate of interest in these various cases will be respectivly

$$\frac{30}{850}=\cdot 03529,\ \frac{30}{900}=\cdot 03333,\ \frac{30}{950}=\cdot 03157,\ \frac{30}{1000}=\cdot 03.$$

We hav in the first place to find the values, at these four rates of interest, of the annuity of £30 enjoyd by the life tenant. This is easily don by means of interpolation. It will probably be useful to the junior members of the Institute to hav so much of the actual work given as will enable them to verify it without trouble.

Taking the common correction, in order to make the annuity complete, the quantity we hav to calculate is $\mathring{a}_{60}=a_{60}+\frac{1}{2}A_{60}$. Using the Carlisle mortality table, we hav the values of the annuities and assurances tabulated for the rates of interest, 3, 3½, 4, 4½, 5, percent. Writing down and differencing the annuity values, we get

| Rate of Interest percent. | $a_{60}$ | $\Delta$ | $\Delta^2$ | $\Delta^3$ |
|---|---|---|---|---|
| 3 | 10·491 | | | |
| | | −·428 | | |
| 3½ | 10·063 | | +·028 | |
| | | −·400 | | −·001 |
| 4 | 9·663 | | +·027 | |
| | | −·373 | | −·004 |
| 4½ | 9·290 | | +·023 | |
| | | −·350 | | |
| 5 | 8·940 | | | |

Now the well known formula for interpolation is

$$u_{x+n}=u_x+n\Delta u_x+\frac{n.\overline{n-1}}{1.2}\Delta^2 u_x+\frac{n.\overline{n-1}.\overline{n-2}}{1.2.3}\Delta^3 u_x+\ldots.$$

In the present instance, $x$ denotes the rate of interest, and we hav

$$u_x=10\cdot 491,\ \Delta u_x=-\cdot 428,\ \Delta^2 u_x=\cdot 028,\ \Delta^3 u_x=-\cdot 001.$$

Also the values of $n$ corresponding to the abov mentiond rates of interest, ar as follows:

| Rate of Interest percent. | $n$ |
|---|---|
| 3·000 | ·0 |
| 3·157 | ·314 |
| 3·333 | ·667 |
| 3·529 | 1·058 |

Substituting in the formula for $u_{x+n}$, we get the values of the annuities

| Price of Stock. | Rate of Interest. | $a_{60}$ |
|---|---|---|
| 100 | 3·000 | 10·491 |
| 95 | 3·157 | 10·354 |
| 90 | 3·333 | 10·203 |
| 85 | 3·529 | 10·038 |

In interpolating the values of $A_{60}$ for the different rates of interest, it will be sufficient to use first differences. Thus

| Rate of Interest. | $A_{60}$ | $\Delta$ |
|---|---|---|
| 3 | ·66531 | —·03942 |
| 3½ | ·62589 | ·03602 |
| 4 | ·58987 | ·03296 |
| 4½ | ·55691 | ·03024 |
| 5 | ·52667 | |

whence,

$$A_{60} \text{ at } 3{\cdot}157 \text{ percent} = {\cdot}66531 - \frac{{\cdot}157}{{\cdot}5} \times {\cdot}03942$$

$$= {\cdot}6530.$$

Proceeding thus, we hav finally,

| Price of Stock being | 100 | 95 | 90 | 85 |
|---|---|---|---|---|
| $a_{60}=$ | 10·491 | 10·354 | 10·203 | 10·038 |
| $\tfrac{1}{2}A_{60}=$ | ·333 | ·326 | ·319 | ·312 |
| $\mathring{a}_{60}=$ | 10·824 | 10·680 | 10·522 | 10·350 |

The annuity enjoyd by the life tenant is in each case £30. Multiplying the abov complete annuities, we get the sums to be receivd by the life tenant; and the reversioner will receiv the remainder of the sum realized by the Stock. Thus

| | | | | |
|---|---|---|---|---|
| Life tenant's share = | £324·72 | 320·40 | 315·66 | 310·50 |
| Reversioner's " = | 675·28 | 629·60 | 584·34 | 539·50 |
| Total . . | £1000·00 | 950·00 | 900·00 | 850·00 |

On reviewing these calculations, it may be thought at first that greater accuracy would be obtaind, and the conditions of practice more nearly adhered to, by calculating the value of the annuity on the supposition that it is payabl halfyearly; but it must be borne in mind that, if we allow for the dividend on the Stock being payabl halfyearly, the rate of interest will be no longer 3 percent when the Stock is at par, but $(1{\cdot}015)^2-1$, that is, 3·0225 percent.

Next take the case of a life tenant, age 60, and a tenant in tail, age 30, and suppose that the sum to be divided between them is £1000 to be raisd by mortgage of the property at 4 percent interest. We hav here to calculate the values of the respectiv shares at 4 percent. The value of the life tenant's interest will be $40\mathring{a}_{60}=40(a_{60}+\frac{1}{2}\mathrm{A}_{60})$ nearly, which is found to be £398·3. The value of the tenant in tail's interest is $1000\overline{\mathrm{A}}^{\,1}_{60.30}=1000\mathrm{A}^{\,1}_{60\;30}(1+i)^{\frac{1}{2}}$ approximatly. The value of $\mathrm{A}^{\,1}_{60.30}$ can be found by either of the following formulas:

$$\tfrac{1}{2}\left\{\mathrm{A}_{60.30}+\frac{a_{59.30}}{p_{59}}-\frac{a_{60.29}}{p_{29}}\right\}, \text{ or } \tfrac{1}{2}\left\{1+\frac{v(\mathrm{N}_{59.29}+\mathrm{N}_{59.30})-(\mathrm{N}_{59.29}+\mathrm{N}_{60.29})}{\mathrm{D}_{60.30}}\right\}.$$

The values by the two formulas ar ·516 and ·51559 respectivly. Of these, the latter is to be preferd as more correct; and adopting it, we get the value of the tenant in tail's interest =525·8. The surplus to be divided is therefore 1000−398·3 −525·8=75·9; and dividing this equally, we get

Life tenant's share=398·3 + ½ × 75·9=436·2;
Tenant in tail's „ =525·8 + ½ × 75·9=563·8.

Lastly, taking the case of a life tenant, A, age 60, and a life tenant in reversion, B, age 30, let us suppose that £1000 is to be raisd by mortgage at 4 percent under a joint power of appointment. We hav value of A's interest $=40\mathring{a}_{60}=398{\cdot}3$, as found abov; and value of B's interest $=40\mathring{a}_{60|30}=40\left(1-\frac{i}{2}\right)a_{60|30}$ =314·8, using the correction given in my paper "On the Value of Reversionary Annuities payabl halfyearly, &c. according to the conditions of practice" in vol. xv, p. 127. The surplus, therefore, to be divided equally between A and B is 1000−398·3−314·8=286·9; and we get finally,

A's share=398·3 + ½ × 286·9=541·7;
B's „ =314·8 + ½ × 286·9=458·3.

⁂ The author of this paper is alone responsible for the spelling adopted therein.

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A few remarks may here fitly be made in explanation of the novelties of spelling adopted abov, which the Council of the Institute hav kindly allowd me to introduce, while, as will be seen by the appended note, they disclaim all responsibility for them. Inasmuch, however, as anything like a full discussion of orthografical principls woud be quite out of place in a publication like this *Jurnl*, I will be very brief.

It is undeniabl that whereas, when English was first reduced to riting, all words wer spelt fonetically, ther is at the present time a great, and, it may perhaps be said, a growing divergence between the receivd spelling and pronunciation of a very large number of English words. While som persons appear to think that this divergence is interesting from an antiquarian point of view, and shoud be carefully preservd, I hold the opinion very strongly that it is greatly to be deplored, as in the highest degree inconvenient, if not amounting to a national misfortune. It seems very doutful whether anything effectual can ever be don to remedy this defect in the ritten language, but this can only be determind by actually making the experiment. Mere speculativ essays in works or periodicals devoted to fonetical or orthografical subjects, will never influence in the least the opinion or practice of the public at large. It appears to me, therefore, that persons holding the views exprest abov, shoud no longer leav the spelling of their works to be settld by the printers, but shoud consider on what principls English spelling may best be reformd, and uze their utmost endeavors to carry those principls into practice in their publisht ritings. In this way will attention be best drawn to the subject, and public opinion formd. It may possibly be found that the public will prefer to leav matters as they ar, the printers being the only persons allowd to introduce changes in spelling, as dictated by caprice or fashon; or it may proov that some of the new principls of spelling suggested from time to time will commend themselvs to general approoval. This, however, can only be ascertaind by actual trial, and I hav to express my sincere thanks to the Council of the Institute for allowing me to put my views on record for the consideration of the readers of the *Jurnl*.

As regards the principls on which I hav arranged the spelling in the abov paper, I may state that I hav had no intention of proposing a complete system of fonetic orthografy, or of putting forward a system that shall rival Mr. Pitman's Fonotypy, or Mr. Ellis's Glossic. The general adoption of a complete and consistent system, such as those gentlemen aim at, woud be attended with very great advantages; but it woud be, in the truest sense of the word, a revolution, and I believe that it woud be utterly impossibl to get such a system ever accepted by the public. The principls by which I hav been mainly guided in introducing departures from the usual mode of spelling, are the two following.—Firstly, that whereas, in pronouncing the preterits and past participls of the so-calld regular verbs, ther ar three terminations which are uzed according to fixt rules, dependent upon the closing sound of the verb, these ar, according to the present mode of spelling, all represented by the same symbol *ed*. I consider it better that they should be represented by the three distinct symbols *t, d,* and *ed,* so far as this is possibl, without departing from the spelling of the verb itself. In conformity with this principl, I write *establisht,* and *considerd,* instead of *established* and *considered;* whereas a word like *ended* I spell in the usual way. In words like *prepared, induced,* and *proposed,* the correct pronunciation can only be represented by totally changing the spelling of the original verb, thus *prepaird, indeust,* and *propoazd.* This appears to me too revolutionary a change to be ventured on at present, and I therefore leav the spelling of the words unalterd.

Secondly—I propose to leav out certain letters that ar not sounded, and ar therefore useless; and, in particular, the mute *e* at the end of many words, when it is not only useless, but misleading, as suggesting a rong pronunciation. I thus leav out the final *e* in words like *positiv, negativ, exquisit, determin,* in order to show that those words ar not to ryme with *strive, site,* and *mine.* The same principl of course applies to such words as *hav* and *giv,* which, if pronounced according to their usual spelling, would ryme with *shave* and *hive.* It will be observd that, in former days, a great number of words wer spelt with a final *e,* which hav now lost it; and for proof of this, I hav only to refer the readers of the *Jurnl* to the articl "Life Insurance in the Time of Queen Elizabeth", in vol. xvi, p. 419. In proposing, therefore, to omit a number of mute final *e's,* I am only carrying out further a principl that has alreddy been extensivly acted on.

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The following abstract of the discussion which followed the reading of the paper is abridged from the *Insurance Record.*

The PRESIDENT said that the subject of the paper is probably more useful to the actuary in his professional than in his official capacity. Nevertheless, it is not without interest to most of us, and he hoped it would be fully discussed. Before proceeding to the discussion, he thought it right to call attention to one point in connection with the subject. Mr. Sprague appeared, while accepting the principles laid down by Mr. Baden and rejecting those advocated by Mr. Jellicoe, to lay rather too much stress on the method recommended by Mr. Jellicoe, as if that method was to be used in all cases and under all circumstances. He did not think that Mr. Jellicoe intended anything of the kind, and he would read the concluding lines of Mr. Jellicoe's paper, which was read eighteen years ago. (See *Journal*, vol. vi, p. 66.)

"From what has been said, it will be apparent that, in questions of this kind, a great deal of judgment and discretion must always be required from the actuary, and that it is not possible to lay down any general rule which shall altogether obviate such requirement. Little else, after all, can be done, than to bring under notice the various arguments which apply in such cases, leaving it to those who are concerned to make the selection proper to the one they may have in hand. That they may be enabled to do this, it is obvious that the fullest information should be given, not only as to the security to be dealt with, but as to the position and objects of the parties concerned; and, thus provided, the actuary must execute his task as best he may, bearing in mind the ancient precept,—'Est modus in rebus; sunt certi denique fines, quos ultra citraque nequit consistere rectum.'"

Mr. B. NEWBATT was of opinion that the question of the division by mutual consent of property between the tenant for life and the remainder-man, is one which is so often complex and so varied in its incidents, that actuaries are justified in differing with regard to it. He had not a very strong opinion as to the relative merits of the two theories propounded by Mr. Baden and Mr. Jellicoe respectively—having, indeed, used them both; but he preferred, on the whole, Mr. Jellicoe's method—to the extent at least of taking the market values of the two interests as the basis of the arrangement, making other arrangements in the division of the surplus in accordance with what Mr. Sprague has suggested, or not, as may be thought well.

Mr. SAMUEL BROWN wished to allude to that part of Mr. Sprague's paper in which he speaks of Mr. Jellicoe's method as not really giving the market values. He says, "As regards the life interest, we should have to make allowance in ascertaining its market value for the extra premium to provide against the contingency of the life proceeding to a tropical climate." Under these circumstances, he (Mr. Brown) thought it quite clear that the letter P, which indicates the premium, is intended to represent the full premium for whatever the risk may be, and no objection of that kind can fairly be made to it. Then, with regard to the value of the reversion, Mr. Sprague says, "I consider it very doubtful whether the value found by the formula can be in any sense considered as the market value. The market value of a reversion depends on the legacy or succession duty payable, and in arriving at it deductions are always made for the legal expenses attendant, firstly, on the purchase, secondly, on the ultimate realization." In practice, he (Mr. Brown) believed that it is always customary to consider the question of value to relate to what can be produced by a certain amount of public stock, or any other property, reduced to cash in possession. So far, he thought Mr. Sprague's objections to Mr. Jellicoe's proposal a little hypercritical. His experience led to the conclusion that a greater number of the class of cases referred to by Mr. Baden, as they are placed before an actuary, will be judiciously met by the view which Mr. Baden has taken rather than by that which Mr. Jellicoe has taken. There is no doubt that other methods may be adopted under varying circumstances, and Mr. Jellicoe's method is one which applies to a great number of cases; but there are certain other cases in which Mr. Baden's may apply more effectually and more completely. But the essential point in Mr. Baden's proposal, in which it differs from any other, is to confine the valuation of all the shares to be divided exclusively to the rate of interest produced by the fund in question. The objection to the ordinary method of Mr. Jellicoe's being applied to these special cases, is that you assume the parties are compelled, the one to purchase an annuity, or the other to effect an insurance at the rates at which the assurance offices will allow them to accomplish these objects. Now, when the parties mutually give consent to realize the property between them, so that no assurances are necessary, it is rather hard upon them that they should respectively be supposed to be saddled with that proportion of a premium which indicates merely the expenses of carrying on a business, a business which in these cases they are not called upon to avail themselves of. The function of the ordinary charged premium is not merely to cover the pure risk, but to enable the company to guarantee the payment of claims, and to meet the necessary expenses of carrying on the establishment. That premium is not a fair premium which, as between the two parties concerned, it is necessary to charge. On the other hand, the purchase of an annuity can only be effected at a low rate of interest, and generally also with some slight additional charge for expenses, and in that case the price of the annuity is not a fair representation of what the reversioner requires in this case. But if we take the rate of interest for both sides—the life tenant and the reversioner—at the same fair market rate, and if we use the same table of mortality for both, then Mr. Jellicoe's formula which applies to the market value will also apply to the ordinary formula for obtaining the value of a life annuity and of a perpetuity afterwards. Under these circumstances, he was disposed to think, without coming to Mr. Baden's view of always exhausting the fund, that we might take the ordinary market rate of interest of the day for the security in question, and if under special circumstances any excess of value should then be found, it might be divided between the parties according to these respective values, without reference to assurances. That will be found to be adopting as the basis one main feature of Mr. Baden's proposal, at the same time it will more generally and accurately meet the varying state of the money market and the value of the property on the division of the fund at the time it is made.

Mr. ADLER thought that Mr. Sprague had gone into an unnecessary exactness in showing how to interpolate and arrive at the value of the annuity, where the rate of interest is ·03157, for instance. If it is essential to be so exact in this particular problem, seeing that it is to be an equitable distribution, further exactness might be introduced by interpolating between the ages. In the example taken, if the life tenant is not exactly 60 years of age, but is 60 years old and so many months or so many days, we still do not interpolate, but take the nearest age; and in the same way we might be content to take the nearest rate of interest. Then, while the paper is headed "The apportionment of a fund between the life tenant and the reversioner," Mr. Sprague considers the apportionment also where there are successive life tenants, and he deals not so much with funds as with landed estates. Now, the whole of Mr. Baden's method, which Mr. Sprague advocates, is based upon the fixed revenue which the property annually yields. In the case of estates, there is some difficulty in arriving at the annual revenue of such property, and without your knowing this the whole of that method of arriving at the results will be disturbed. In the case of Consols, there is, we know, 3 per-cent actually guaranteed; but how about shares or stock, the dividends of which fluctuate, or even 5 per-cent guaranteed Indian railway stocks? They may give rates somewhat higher in future, and a difficulty would be introduced in valuing such property. Certainly in cases of parks and other landed property, which yield no absolute revenue, there will be great difficulty. In that case we should still, he presumed, have to fall back upon the old method of arriving at the market value of the separate interests. The case where the necessity of partitioning the property comes from out of doors, where the parties interested are compelled to divide, has not been satisfactorily dealt with; such cases as compensation, &c. Suppose a railway takes a strip of land, and the question is how to divide the value of it between the life tenant and the remainder-man. Mr. Baden would have some difficulty in treating that question according to his method. How are you to estimate the life income derived from the land? Mr. Sprague was generally so happy in alluding to all sources whence light can be thrown on any subject he may have under consideration, that he was rather surprised he had not adverted to Mr. W. T. Thomson's work on "*The Pecuniary Interests of Heirs of Entail*", in which this very subject, and more especially the law and procedure as regards Scottish entails, was fully entered into and illustrated by full and complete tables.

Mr. C. Walford remarked that in early times, when monarchs bestowed estates upon their courtiers and followers, for services good, bad, or indifferent, it was by no means unusual to grant what may be termed a limited or qualified estate. To quote a few instances: in one case, an estate was granted for the period during which a particular tree should remain standing. It would be a curious problem for an actuary to ascertain the pecuniary interest of a tenant in such an estate. In another case, an estate was to be held until the marriage of a certain person should take place. Another grant was to continue until a certain debt should be paid; yet another until default be made in the payment of a certain debt, at a certain time; another, until an heir should attain his majority. Again, estates may be limited to peers of the realm, or to other persons occupying specific positions, as lords of certain manors, &c. Whatever the qualification, the possessors can only convey the estates with a like limitation—therefore any one of these contingencies might become the subject of a required valuation. If the condition of the grant or bequest was either realized, or failed to be realized, as the case might be, the estate was lost. The legislature has happily shown a tendency to lessen the creation of such qualified estates, but they are any and all lawful at the present moment. In ordinary practice, however, base fees have a much narrower signification, being limited to those cases where the tenant in tail conveys his estate by bargain and sale to another and his heirs. While by specific enactment,—the Fines and Recoveries Act already referred to,—a base fee means, for the purposes of that Act, really only the estate created by the alienation of the tenant in tail where the issue are barred, but those in remainder or reversion are not. It is in such cases that the services of the actuary are so frequently called into requisition.

Mr. EDWARD SMYTH pointed out that Mr. Jellicoe has not in his papers upon this subject absolutely restricted himself to the formula which Mr. Sprague has criticized; but in a paper "On the rationale of certain actuarial estimates" in the 8th volume of the *Journal*, he has, in a foot-note on the third page, given not only that particular formula, but also, as an alternative, the 6 per-cent single premium value, for the general use of which Mr. Sprague, in one of his papers, has shown what he (Mr. Smyth) conceived to be conclusive ground. Mr. Jellicoe may be considered to have thus saved himself from the possible suspicion that in every conceivable case he would have had us adopt the formula which he introduced. Still he has not told us in what cases to use the one formula, and in what the other; and especially he has not given any indication of the circumstance pointed out by Mr. Sprague, that what Mr. Baden calls the "rateable division" of the fund would, when based upon Mr. Jellicoe's formula, lead to the absurdity of assigning the whole of the fund to the life tenant in the imaginary case of his being very young. And, by parity of reasoning, in the practical cases of the life tenant being twenty-one or ranging up to say thirty, the adoption of such a calculation would result in giving much too small a sum to the reversioner. These, therefore, are points on what seems to have been unbroken ground, and he was obliged to Mr. Sprague for carrying on the subject to those less usual cases which had not been touched upon by Mr. Baden. He had done this with considerable elaboration; and, with regard to the formulas, however loosely actuaries may in actual business hold to them, it is hardly possible to have them too precise or too exact in the *Journal*, to fall back upon whenever need may arise.

Mr. HODGE thought that the formula which Mr. Jellicoe gave does not necessarily involve the adoption of the two different rates of interest which he assumes, and which lead to the absurd conclusion pointed out by Mr. Sprague. In fact, Mr. Sprague's remarks upon the accuracy of Mr. Jellicoe's mode of ascertaining the market value in the case of reversions are rather beside the mark, because the question is,—what is the proper mode of partitioning the fund, and whether or not the foundation of that mode should be the market values of the different parties' interests. Assuming Mr. Jellicoe's mode of ascertaining the market values not to be correct, still the principle upon which the fund ought to be divided may be perfectly so, only we should have to adopt some other method of ascertaining what the market values should be. Now Mr. Jellicoe assumes that, in the case of reversions, interest for the purchase-money should be at 5 per-cent, while he would invest the money to provide for the income until the reversion falls in by the purchase of an annuity at $3\frac{1}{2}$ per-cent; but those rates do not necessarily follow from Mr. Jellicoe's formula. He put that down partly, (Mr. Hodge thought,) for the purpose of example, and partly because he considered that it was a fair mode of valuation at the time he wrote, which was a good many years ago. But if you assume the same rate of interest both with regard to the valuation of the reversion and the purchase of the annuity, then you have no anomaly of the kind pointed out, and you leave the principle untouched. The formulas adopted by Mr. Jellicoe were in common use before he published his paper; nevertheless, that paper is a most valuable one as giving to those unacquainted with this subject complete information upon it. He had been very much struck with the ability which it displays, and the clear and comprehensive manner in which the different cases are explained. He was still of opinion that the market value and the mode of dividing the surplus in proportion to the market value is a fair and just mode of proceeding. These are cases which seldom occur. He did not think that actuaries are often consulted to make a partition of the sort contemplated. As to the two values not being together equal to the whole value of the fund, that is an objection not of the slightest consequence, because we constantly see cases where the values of all the separate parts of a property are not equal to the value of the whole. Take, for instance, the value of a house let at a ground rent for a long number of years. If the ground rent and lease were sold separately, they would not realize so much as the freehold of the house. He could not see that Mr. Sprague assigns any reason for dividing the surplus equally between the parties. He could understand Mr. Jellicoe's system, and also Mr. Baden's, being based upon a scientific foundation; but could not understand any scientific foundation for merely dividing the surplus between the two persons interested.

Mr. R. P. HARDY had been much struck with the completeness that Mr. Sprague had aimed at, and which he had, to a large extent, satisfactorily accomplished. He seemed to have raised almost every point which comes before actuaries in practice in connection with these matters, and to provide rules for almost every case; but, in the result, he gives rules as rigid as those which he erroneously assigns to Mr. Jellicoe. The latter is about the last man in the world to lay down a rigid and an unbending rule for the profession, and certainly the last man not to test his principles by what Mr. Sprague calls "extreme cases." It is, probably, through approaching the subject from this point of view that Mr. Sprague has been led to misinterpret what Mr. Jellicoe said with regard to the market value. Mr. Jellicoe never intended to take the fund as it was put down in the statement; but, like every other man of sense, he would ascertain what the fund would actually yield when realized, and that is what he valued, and what we value every day. Mr. Baden will, no doubt, admit that we cannot always carry out the principle which he advocates—namely, valuing the interest of the tenant for life according to the interest produced from the fund. (Mr. BADEN—"Hear, hear.") Supposing the fund is in trust, producing 4 per-cent, he believed that the reversioner can insist upon that fund being put back into Consols; as the Court will not allow either of the parties to benefit at the expense of the other, but will protect the reversioner's interest. He would ask Mr. Baden whether, in that case, he would rigidly apply the rule he has laid down, or whether he would consider that as a case that did not fall within his rule. He noticed that, not merely in this paper, but in all his papers on the subject, Mr. Sprague takes the benefits as complete. Now in practice he (Mr. Hardy) never did that, because the income is never complete. The most striking case is that of advowsons. A man when first inducted has to pay for dilapidations, first-fruits, tenths, &c., and the half-year's income will probably about meet the expenses. Mr. Sprague did not in this paper touch upon a large question which sometimes arises in this apportionment of a fund, when the question is not so much how the estate should be apportioned between the tenant for life and the tenant in tail, but what is a fair and reasonable consideration to the tenant in tail for re-settlement. The interests for life and remainder being thrown back into the estate, the reversioner really sells nothing but the contingent reversion. He would like to see how Mr. Sprague would treat that case.

Mr. C. J. BUNYON thought Mr. Sprague's paper an exceedingly useful one. It has brought forward many points which perhaps never occurred in the first instance to ordinary calculators. He (Mr. Bunyon) had long ago come to the conclusion that no one rule can be laid down for all cases, and that, useful as formulas may be, we must really ascertain the circumstances affecting any particular case before we can deal fairly with it. There are some very important cases of apportionment which do come before actuaries, although Mr. Hodge hazarded the supposition that they do not often do so. Not many weeks since he had had two very important cases before him, and in one of them a very large fund indeed was subject to division. It was a question where there was a tenant for life at an advanced age, and a contingent reversioner, of still more advanced age, who would be entitled to the whole reversion of the property if the tenant for life died in his lifetime. They were desirous of dividing the property between them, and the question was how the total corpus should be divided. The fund was very large indeed, and they wisely thought that the best thing they could do was to take the opinion of an actuary. In another case that he had seen quite recently, property to the extent of exactly £100,000 had been sold by the tenant for life and the tenant in tail and remainder-man. They certainly did not submit the case to an actuary, for they divided in equal shares the total amount of the corpus. The opinions of actuaries on these subjects may be exceedingly useful, and they ought to be prepared at all points to meet them as they arise. Now there are one or two cases to which Mr. Sprague has called attention which are especially worth adverting to. One is the case where father and son are able to exercise a joint power, acquire the fee simple, and divide it between them. But in what way are they to divide it? It may be said on one side that they should divide it in proportion to their interests, but their power is, after all, the motion of their own mind, which is equal on the part of each; and if you provide for the interests which they are entitled to, if they do not exercise the power, he was not prepared to say that Mr. Sprague's proposition—that the surplus should be divided equally—is not the right one. That is a case which probably had not occurred before to many gentlemen in this room—certainly not to the younger members—and it shows how useful it is to have such a paper as this, in which all these cases are considered.

Mr. FINLAISON was in a position to say that a fund is often divided between the life tenant and reversioner, particularly when the fund is in Stock. It is usually done by transferring a portion of the Stock and the annuity which the fund brings in. That is done continually under the sanction of the Court. A portion of the Stock is transferred, and if the life tenant is at all aged and the reversioner at all young, he is immediately in possession of a very large fund. Even landed estates are frequently dealt with in that way, with deeds giving the consent of the life tenant. The estate is sold, and a Government annuity is bought to the amount of the supposed net rental.

Mr. A. BADEN hoped that it would not be thought that the paper which he formerly read before the Institute (see vol. xvi, p. 269) had been intended as anything like an attack upon Mr. Jellicoe. He did not think either that Mr. Sprague could be charged with any intention of the sort. No one could entertain a higher estimate of the value of Mr. Jellicoe's services to the profession than he (Mr. Baden) did, and no one had profited more by them than he had. He had always found Mr. Jellicoe's papers extremely suggestive, and would have lacked one very great source of instruction if those papers had never been written. He thought there had been some little misunderstanding as to the scope of his paper. He had distinctly laid down that it was intended to meet only one peculiar class of cases. Mr. Jellicoe's paper was much more comprehensive. It showed the methods in vogue for many kinds of cases, whereas he had strictly confined himself to one. But, in that one, he thought the principle he had laid down admits of no compromise at all. When his paper appeared in print, Mr. Porter had been good enough to send a note to the editor of the *Journal*, giving examples of various modes of apportionment, and wound up by saying that "Mr. Baden appeared to be the reversioner's very good friend." That implied that he assigned a value to the interests of the reversioner which was more than equitable or more than he ought to receive. It appeared to him that Mr. Porter's conclusion was derived simply from a comparison of the results of various modes of apportionment that had been used; but he did not think that any such comparison would enable us to arrive at any conclusion at all. We must appeal to some standard of equitable apportionment. He had not heard any refutation of the principle and the standard laid down in his paper. The mode which he recommended for apportioning a fund is not, of course, to be looked at as a hard and fast method, to be employed under all circumstances. As an illustration of the way in which a case of the kind might be dealt with, he quoted a case in which he had been consulted:—"A, a lady aged 64, being possessed of a life interest in £1,750 Consols, and B being entitled to the reversion of the stock at A's death, and a mutual arrangement being contemplated whereby the stock may be reduced into B's immediate possession, I am of opinion that if both parties are equally desirous of effecting the arrangement, the equitable consideration to be paid by B to A is £500. But if B only desires it, A considering that the alteration as far as she is concerned is undesirable, then B could afford to give £670, or whatever intermediate sum would suffice to induce A to consent." Mr. Adler said we often have great difficulty in arriving at the annual value of the estate, and can not therefore in such cases fix the rate of interest at which the apportionment was to be made. But in all cases we have to use our best judgment, and make sometimes some very painful enquiries. We have to drag the information out of those who consult us by processes which are occasionally far from being successful; and, after all, we can only give a professional opinion, and take care to guard ourselves against the possibility of mistake in doing so. In these cases there is supposed to be some life income actually coming in, and it is for us to ascertain what the net income is; and having ascertained that, what really goes into the pocket of the life tenant is pure income after all outgoings are allowed for. We then arrive at the rate at which the apportionment of the fund should be calculated. He would only say, further, with regard to the cases brought forward by Mr. Sprague, in which he advocates the equal division of the surplus, that he thought that a right principle. It seemed to him that, having obtained an equitable value of the two interests in the first instance, and there being an actual surplus then remaining—a surplus which results from the equal concurrence of two wills—that surplus ought to be divided equally between them.

Mr. H. W. PORTER explained that his illustrations of the subject of Mr. Jellicoe's paper were not intended to exemplify all the possible ways of dividing a fund between the tenant for life and the remainder-man, but only those which, after some attention to the subject, he had reason to believe were actually in use amongst different actuaries at the time, which was three years ago. With reference to the remark that Mr. Baden is the reversioner's very good friend, that opinion was, of course, only deduced from the actual results, and meant nothing more than that. Of those methods, including Mr. Baden's, there were seven. Mr. Sprague, in his paper, had summarily disposed of five. But if he were entirely behind the scenes, he would find there are more than two methods now in use among actuaries; and we must remember that there are actuaries who are not members of this Institute and who are, perhaps, rather loth to accept the modern views on these matters so ably enunciated from time to time at its meetings. Of course, it is desirable to get rid of as many of these different methods of treating the case as we possibly can; but notwithstanding the remarks made with reference to treating each case as it arises upon its merits, unless some sort of uniformity can be arrived at, and unless actuaries will agree to give up a little of their own individual opinions and bow somewhat to the opinion of the majority, the Institute will not succeed in removing a stigma which has always, more or less, and very naturally so, been attached to actuarial calculations,—particularly in the minds of solicitors. A solicitor goes to two actuaries and lays a simple question, as he thinks, before them. He requires, say, £1000 divided between two persons, and he obtains opinions varying as much as 28 per-cent. He naturally concludes, therefore, that actuarial calculations are not founded upon a very scientific basis.

Mr. SPRAGUE, not having been able to be present at the Institute when the above paper was read, wishes to make the following reply to some of the remarks made in the course of the discussion. He says—"On reading over my paper in the light of the remarks that wer made by the various speakers, I am inclined to think that som of my objections to Mr. Jellicoe's formula ar prest rather too far, and possibly deserv the charge that Mr. Brown has made against them of being 'hypercritical'. In particular, I must admit that I quite overlookt the circumstance pointed out by Mr. Smyth, that Mr. Jellicoe, in a paper subsequent to that which I referd to, distinctly recognizes that the actuary may in certain cases prefer to value the reversioner's share by the 6 per-cent single prèmium instead of by the formula $1-d(1+a)$. I take this opportunity of saying that I fully concur in what Mr. Hodge and others hav said as to the great interest and value of Mr. Jellicoe's papers on this and other subjects. It is to be noticed that in writing the above paper, I wisht to exhaust the arguments upon the one side of the question, leaving it to those who espouse Mr. Jellicoe's views to reply. This being the case, it is not surprizing that the arguments I hav put forward ar not all equally conclusiv; but it must be admitted that free and open discussion, in which the arguments on both sides are fully stated and unreservedly examind, is the most effectual means of arriving at the truth and establishing correct principles.

"The case mentiond by Mr. Adler, where a railway company takes under its compulsory powers a strip of land from a park forming part of a settled property, is very interesting, and requires some further consideration. Mr. Adler appears to take a wrong view of this question in two respects. (1.) He appears to suppose that in the case stated the purchase money of the land taken would necessarily be divided between the parties interested; but this is by no means the case. The railway company, having taken and paid for the land, would usually pay over the purchase money to the trustees, who would invest it in Consols, and hold it upon the same trusts as the land was subject to; so that, if a division of the fund is made, it is by no means necessitated by the action of the railway company, but is, as in other cases, the result of the mutual agreement of the life tenant and the reversioner. (2.) He loses sight of the fact that, in all cases, the thing to be divided is a sum of money, and never a piece of land. Altho' the land may produce no income, yet when it is sold, and the proceeds invested in Consols, it will produce a definit income, and thus fall within Mr. Baden's rule.

"As regards the ordinary problem of apportionment by mutual consent, further consideration has only confirmd the opinion I formd when Mr. Baden's paper was first read, namely, that the principles laid down by him ar beyond all question correct, and that, as he says, in the case contemplated by him, no compromize can be admitted."

## *On some Legal Incidents affecting the Duration of Base Fees in the Hands of Purchasers.* *By* C. J. Bunyon, M.A., *Barrister-at-Law, a Vice-President of the Institute of Actuaries.*

[Read before the Institute, 18 November 1873.]

As a wish has been expressed for some further information as to the manner in which a base fee created by the conveyance of a tenant in tail may be converted into a fee simple, it is proposed to consider the question in the following pages.

It is well understood, that an estate in tail is an estate in fee inheritable by heirs special, namely by lineal descendants generally as heirs, or such descendants of a particular class, and that a tenant in tail may convert such an estate into a fee qualified to last so long as there shall be issue inheritable under the entail. Such a conversion may be made by any conveyance sufficient to pass a fee, and will in any such case bind the heir until entry by him,(*a*) but if enrolled in Chancery in the manner required by the Act for the Abolition of Fines and Recoveries will, notwithstanding the want of the consent of the protector of the settlement, wholly bar the issue in tail, leaving the estate absolute during their existence, and liable only to be defeated by the failure of issue, and then by the entry of the remainderman. When the protector consents in the manner prescribed by the Act, there is no base fee, but the estate tail is converted into a fee simple and the remainders are absolutely barred.

When the right of entry continues in the issue in tail, he must enforce it by ejectment within 20 years after the death of the tenant in tail by whom the conveyance was made,(*b*) or within the extended time allowed in case of disability. If he fails to do so, he is bound by the Statute of Limitations, and his own issue and the remaindermen with him. And here it is well to note that the bar depends upon the right of entry—directly that it becomes operative, time begins to run, but not before—hence when possession may be referred to a conveyance by the tenant in tail, until his death time does not begin to run against his issue, but if he is dispossessed, and he fails to enforce his right of entry by ejectment to recover possession, time begins to run against him under the statute from such dispossession, and against his issue and all claiming after them also.(*c*)

When the conveyance has been enrolled under the Substitution for Recoveries Act, and a base fee has been created, it is not only

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(*a*) Watkin's Conveyancing, p. 119.
(*b*) Cannon *v.* Rimington, 12 C. B., 1, 18.
(*c*) Austin *v.* Llewellyn, 9 Exch. 276.

indefeasible during the existence of issue inheritable under the estate tail, but it is liable to be expanded into a fee simple by the Statute of Limitations, or may be at once so converted by a further enrolled assurance or confirmation after the estate tail has fallen into possession by any person who would have been tenant in tail in possession if the first deed of disposition had not been executed.(*a*) This liability to expansion is the particular question of interest for our consideration on the present occasion.

This effect of the Statute of Limitations has been much discussed by Lord St. Leonards both in his treatise on "Vendors and Purchasers" and in his book on "Real Property Statutes," in which latter he has introduced many of his observations omitted from the later editions of his larger work.

He observes (p. 86) :—

> One of the propositions of the real property commissioners was, that on any alienation by tenant in tail by any assurance *not operating as a complete bar to the estate tail, and all estates, rights and interests limited to take effect on the determination or in derogation of the estate tail,* possession under such assurance should have the same effect in barring the estate tail, and all estates, rights, and interests limited to take effect on the determination or in derogation of the estate tail, as if such possession had been adverse to the said estate tail, or to the said estates, rights, and interests. In their preliminary observations their proposition was, that when there had been an adverse possession for 20 years, under a base fee, created by tenant in tail, the remainders over and the reversion should be barred in the same manner as if a recovery had been duly suffered.
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> These intentions were carried into effect by the 23rd section of the Statute of Limitations,(*b*) which provides, that when a tenant in tail has made *an assurance which does not operate to bar an estate, to take effect after or in defeasance of his estate tail,* and any person is *by virtue of such assurance,* at the time of the execution thereof, or at any time afterwards, in possession or receipt of the profits of such land, or in the receipt of such rent, and the same person, or any other person whatsoever (other than some person entitled to such possession or receipt in respect of an estate which shall have taken effect after or in defeasance of the estate tail) continues or is in such possession or receipt for the period of 20 years next after the commencement of the time at which such assurance, if it had then been executed by such tenant in tail or the person who would have been entitled to his estate tail if such assurance had not been executed, would, without the consent of any other person, have operated to bar such estate or estates as aforesaid, then at the expiration of such period of 20 years such assurance will be effectual as against any person claiming any right to take effect after or in defeasance of such estate tail.

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(*a*) 3 & 4 Will. IV., c. 74, s. 19.

(*b*) 3 & 4 Will. IV., c. 27.

It will be observed that the statute requires a possession or receipt for 20 years next after the commencement of the time at which *such assurance*, if it had been then executed by the tenant in tail or the person who would have been entitled to his estate tail if such assurance had not been executed, would, without the consent of any other person, have operated to bar such estate, at the end of which 20 years *such assurance* is made effectual against any claimant after or in defeasance of such estate tail. Such assurance then is the one made by the tenant in tail, but whether possession was taken at the time of its execution, or at any later time, is immaterial. It must also have been perfected under the Substitution for Recoveries Act, or it would have no such operation; hence the case will not fall within the section under consideration(*a*) unless duly made and enrolled according to the provisions of the later Act.(*b*)

This section has been twice if not oftener under the consideration of the Court of Chancery, and on two of these occasions it was considered by Lord Cranworth, C.,(*c*) and by Lord Romilly, M.R.,(*d*) that it applied only to the case in which the bar had been effectual against the issue, and imperfect only against the remainderman, and in this view Lord St. Leonards concurs.(*e*) The force of the section appears to be to make time run against the remainderman, although there is no adverse possession; for since they have no right of entry, while there is issue living of the tenant in tail, they can make no claim, and yet they may be barred by time, and although the possession of the purchaser is not inconsistent with their title.

In the meanwhile, although the issue or the heir in tail may create a complete bar, they have no power to prevent time from running; but if 20 years expire before the failure of issue, the base fee is expanded into a fee simple. On the failure of issue within the 20 years the right of entry of the remainderman accrues.

It does not seem necessary here to enter upon the question of base fees created before the Act, as to which there may be greater difficulty, and perhaps difference of opinion,—but we may mention Lord St. Leonards' remarks upon them. "Base fees are frequently " created, but it is seldom, if they do not quickly determine, " that they are not enlarged into pure fees. I do not remember

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(*a*) 3 & 4 Will. IV., c. 27, s. 23.
(*b*) 3 & 4 Will. IV., c. 74.
(*c*) Penny *v.* Allen, 7 De Gex, M. & G., 409.
(*d*) Morgan *v.* Morgan, 10 Eq., 103. See also Goodall *v.* Skerratt, 3 Eq. R., 295
(*e*) Real Property Statutes, 89.

" more than one instance of a conveyance by way of transfer of a " base fee actually in existence; and it rather seems that those " created before the Act by a tenant in tail in possession are " by another section rendered unassailable."(*a*) It will be more to the purpose to recall to our remembrance that although the interest of the tenant in tail who has created the base fee, or of his issue, is wholly divested, nevertheless they retain the power when filling the position of tenant in tail in possession but for the deed creating the base fee, of accelerating the bar against the remainderman by executing a further deed of disposition under the Act. Such a tenant in tail, although his interest in the estate is gone, nevertheless retains this power over it. He can complete the title against the remaindermen; and this may be a saleable interest. It is readily susceptible of valuation, being measured by the insurance premium which an insurance office would require, under the circumstances of the family, to insure the value of the land in the event of the total failure of issue —a contingency which may depend on one or any number of lives.

The power of the protector is uncontrollable even by a Court of Equity. The exercise of the discretion reposed in him cannot be a breach of trust, for he is not a trustee of the power. He cannot even be bound by an agreement not to exercise it.(*b*) This is the policy of the statute, and carries out the general practice, for on the resettlement of an estate it has always been a matter of arrangement or bargain with the heir in tail to determine the new limitations, and they cannot be impeached unless there has been an exercise of undue influence and a gross abuse of parental authority. The unbounded power of the protector has, however, called forth much observation. He may, it is said, if tenant for life, or for years determinable on his life, with remainder to his sons in tail, with remainder or reversion to himself in fee, sell and convey his life interest and reversion, and still remain protector, and in substance resell his reversion to the tenant in tail by consenting to his acquiring the fee simple, a covenant not to exercise this power of consenting being simply void.(*c*) In like manner, when his reversion is vested in his own assignees in bankruptcy, he may destroy their estate, and even acquire a great property against his creditors by consenting to a resettlement. There is, however, more appearance than reality of hardship here, for the proposition that the assignees in bankruptcy of a father should be entitled to

(*a*) Real Property Statutes, 90.
(*b*) 3 & 4 Will. IV., c. 74, ss. 37, 44.
(*c*) Real P. Stats., 204–225.

wring a consideration for a consent out of a son tenant in tail in remainder resettling his property on his marriage would be liable to much objection; and the assignees take the estate subject to all its incidents, one of which is the liability to be barred.

When the separate interests in the family property pass through the hands of an actuary, and are valued and tested by the actual requirements of the possessors, the merits or demerits of the system under which they are created stand out more prominently than under any other circumstances. At the present time, when there is a demand for the abolition, as it is said, of entails, founded in many cases in mere ignorance, but in others in a longsighted political design of altering the constitution of Society, it is interesting to consider the question as it actually appears in practice. The powers of sale and the management powers contained in all well-chosen settlements, and the Provisions of the Settled Estates Acts, provide, or seek to provide for the well-being of the estate independently of the successive possessors. It is argued, on the other hand, that an absolute owner can alone deal with the land so as to give the greatest stimulus to its powers of production, and the largest benefit from it to the community at large.

I do not propose on the present occasion to fall back upon first principles, and consider the foundations upon which property in land is admitted at all. When, however, it is conceded that land is property, it may be thought impossible to contend that the owner shall not be permitted to limit a life estate in it, while the possessor of personal property retains the power to do so. All that then really remains for consideration is the system of limiting estates tail under the present law.

The advantages seem to be the following:—

(1). It enables the settlor to provide for the devolution of the family property in such a manner that it will always accompany an hereditary dignity, and such dignities represented by the House of Peers form an integral part of the constitution of the Country.

(2). The power exercised by the protector of the settlement, usually the tenant for life, and the limited power of the tenant in tail in remainder, practically leave the property at the disposal of the family rather than of the individual, and tend to its preservation. An improvident youth, who after running a career of riot dies in early manhood without descendants, and before possession brings with it the sense of responsibility, is prevented from dissipating it.

On the other hand the theory of base fees is not so defensible. It gives the remainderman one chance more. The tenant in tail may survive the tenant for life, but die before perfecting the disposition, although he may be bound to do so, and the Court of Chancery, if time were given, would compel it. In exchange for and as the consequence of this chance, the tenant in tail cannot borrow except upon onerous terms; not in general being able to do so without insuring for the whole term of life instead of insuring against the contingency of his death in the lifetime of the tenant for life—a difference, as is well known to actuaries, of very important moment. It is true that a mode of proceeding has been suggested for obviating this necessity,(*a*) but in the absence of judicial approval cautious practitioners not unfrequently hesitate to run a risk in important transactions. We then come to this curious result, that the only parties who benefit from the present system of base fees are the insurance offices, which divide among them the large premiums payable upon the insurances effected to cover reversionary loans; and the practical question follows, whether it would not be an amendment of the law to take away from the tenant in tail in remainder the power of creating a base fee, but to give to his conveyance, enrolled in Chancery, if you will, the power of operating upon the fee simple, provided only that he survives the tenant for life and his estate tail succeeds to the possession?

If the power to create a base fee were taken away, it would follow that during the lifetime of the tenants for life the tenant in tail could no more bar his own issue than those entitled to the succeeding estates. To take the common case where a father has settled his property upon himself for life, with remainder to his sons successively in tail male; the eldest son becoming a spendthrift, could no more during his father's lifetime bar his own children than he could his brothers and their issue. And there is no earthly reason, except that he could formerly effect this by a fine, why he should be able to do so. It would be much to the benefit of the family that his power in this respect should be restricted, and the only persons who benefit by the present law are his creditors, who, as we have shown, would be more than compensated by the alterations proposed.

At the present time, when legal reform has become a reality, and is likely to be pressed forward by vigorous hands, the amendment would carry with it the recommendation of being an

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(*a*) This is described by Mr. Sprague in the course of the discussion on his paper "On Reversionary Life Interests as Securities for Loans" (see vol. xvii, p. 247): the borrower insuring his life, not for the whole term, but against that of the life tenant, and for, say, one year longer, and a power of attorney being given for the execution of a deed of confirmation immediately upon the death of the tenant for life.—Ed. *J. I. A.*

important simplification of the law. The greater part of the cumbersome provisions of the Fines and Recoveries Act might be swept away. At the same time, the power of appointing a statutory protector might well be dispensed with, being rarely exercised and almost a dead letter, and principally interesting as presenting a *casus omissus* in the Act. When it was passed, fines and recoveries were not only venerable, but almost sacred in the eyes of conveyancers; but since that time we have got rid of many superstitions, and to the list of the defunct base fees should now be added.

The practical reason for this alteration of the law may not generally occur to conveyancers, but the reasonableness of the suggestion has not escaped them. Thus we find Lord St. Leonards observing, "If this be the true view of the operation of the section, it would "perhaps have been more consistent with the object of the Act, if "it had enacted that when, after the death of a tenant in tail "without his assurance having operated to bar the remainders, issue "in tail should live to be competent to bar the remainder, without "the consent of any other person, the remainders should be at once "barred."(*a*) And again, it may be observed that when, upon the bankruptcy of a tenant in tail, base fees are created or transferred under the bankruptcy clauses of the Substitution for Recoveries Act, they are enlarged by the Act by the event of there ceasing to be a protector of the settlement.(*b*)

The above observations apply to the case when the base fee has been created, and no further compulsion can be brought to bear against the tenant in tail. It may, however, happen, that he has contracted to execute his powers of confirmation, and it is considered that although the Substitution for Recoveries Act expressly provides that no contract shall have any effect under it, yet that a Court of Equity may compel specific performance of such a contract. We say may, rather than will, for this jurisdiction is at the discretion of the Court. Again, when the tenant in tail is under covenant for the payment of a reversionary charge, or any sum beyond his means, he may be made bankrupt and the remainder barred by the operation of the statute; but these latter cases involve future proceedings, and when the object is to make a complete security, a purchaser or lender may well decline to rely upon them.

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(*a*) Real Property Statutes, 89.

(*b*) 3 & 4 Will. IV., c. 74, ss. 60, 61. Bankruptcy Act, 1869 (32 & 33 Vict., c. 71, s. 25).

## *On the Method used by Dr. Price in the construction of the Northampton Mortality Table. By* WILLIAM SUTTON, B.A., *of the London and Provincial Law Assurance Society.*

[Read before the Institute, 29 December 1873.]

THE Northampton Table has played such an important part in the life insurance transactions of this country, and is still used to such a considerable extent, that a short explanation of the method by which it was constructed, may not even at the present day be without interest to the actuarial profession. In many ways we are greatly indebted to Dr. Price, and in the writer's opinion anything emanating from him is well worthy of our consideration. It is to be hoped, therefore, that no further apology need be made for the following remarks on one of his most important works.

In the 1st edition of Dr. Price's work on *Reversionary Payments*, published in 1771, there is given on p. 317 a table showing the "probabilities of life at Northampton," and on pp. 255 and 256 is given an account of the data employed, and an explanation of the method by which the corresponding mortality table was constructed. An examination and comparison of these with what is generally understood to be the Northampton Table, as given in the 4th and succeeding editions, at once shows us that the two tables and the data on which they are based are different. In fact Dr. Price, in a footnote on p. 358 of the 4th edition, has himself pointed this out. He says:—"The table printed in the " first three editions having been formed from the Northampton " Bills for 36 years, this table was rendered a little more correct " in consequence of being formed from the same Bills for 46 years. " * * * In consequence of these alterations, and also of " increasing the *radix* from 1165 to 11,650, in order to adjust the " decrements with greater regularity and precision, this table, in " my opinion, gives the *mean* probabilities and values of lives at " every age with more accuracy than any other table extant."

It will be seen, however, that Dr. Price gives the same explanation *mutatis mutandis* in the 4th edition of the method by which the 2nd table was constructed as he had already given in the three preceding editions of the method by which the first table was constructed; and we are led to the conclusion that the only material alteration in the two tables was the extension of the data; and this I shall presently show was substantially the case.

Bearing in mind the enormous importance which the Northampton Table acquired, it is a conspicuous fact how comparatively little was thought of it by its author when he first published it. In the preface to the 1st edition, he does not even mention it by name, but only speaks of it indirectly as one of "several new tables." In that edition no annuity values based upon the table are given, although it is mentioned in the headings of the table of annuity values according to De Moivre's hypothesis as giving (along with the Breslau and Norwich Tables) nearly the same probabilities of life as De Moivre's hypothesis. In the 3rd edition, the only calculations based on it are given in the Appendix, and consist of tables of values of deferred increasing annuities. The calculation of annuity values was at that time, however, a matter of considerable labour, and Dr. Price doubtless considered that the table, although useful for purposes of comparison, was not sufficiently trustworthy to justify any extensive calculations.

In the 4th edition, published in 1783, Dr. Price, as already pointed out, had reconstructed the Northampton Table, having an additional 10 years of observation for his basis. Before this 4th edition, containing Dr. Price's new Northampton Table, was published, the annuity rates and premiums had been already calculated by him, and adopted by the Equitable Society, in the place of the premiums calculated by Dodson, and in use up to that time. The second Northampton Table differs but slightly from the first, and it is difficult at first sight to account for the prominent part the Northampton Table has since taken in the insurance and annuity business of the country. Much of this prominence is undoubtedly owing to the fact that the table was adopted as the basis of the rates of the Equitable Society; and it seems by no means improbable that the success of that Society was one of the reasons which led the Government, in the Life Annuity Scheme passed in 1808, to make the Northampton Table the basis of the annuity rates, entirely ignoring the fact that what is profitable to the buyer of annuities will be unprofitable to the seller. No doubt Dr. Price's acknowledged ability and reputation carried great weight with Mr. Pitt's Government, but Dr. Price had been dead some 17 years, and it is by no means unlikely that could he have been consulted he would have advised against the adoption of the Northampton Tables. It is still more remarkable when we remember, as Mr. Frederick Hendriks has explained in a paper read before the Statistical Society in 1856 "On the Financial Statistics of Government Life Annuities," that Mr. Pitt had so far back as 1789 already at his command Kerseboom's Tables, and had contemplated their employment, these tables giving much higher annuity values than those of the Northampton Table —in fact differing little from those of Mr. J. Finlaison's Government Annuity Tables of 1829. However, whatever the cause, we know that the Northampton Table was used, and with what result, too, to our cost.

Enough has been said upon this part of our subject, but before bringing to your notice the immediate object of this paper—the method of construction employed by Dr. Price in deducing his table from the facts observed—it may be well to allude to the assumptions on which he based his calculations. In the first place, he held the opinion that, generally speaking, in towns where the deaths are nearly the same for a number of years as the births, that stationary state is not brought about simply by procreation. He says, page 242 in the 3rd edition, "In towns this very seldom happens, on account of the luxury "and debauchery which generally prevail in them. They are, "therefore, commonly kept up by a constant accession of strangers "or *settlers*, who remove to them from country parishes and "villages." This is, I take it, a theory which is by no means self-evident. But he goes still further, and argues that if the number of deaths for a series of years is in excess of the number of births, this excess arises from the deaths of immigrants. This would be correct if the observation extended over a century, and it was quite certain that *all the births had been duly registered as well as the deaths*. In framing the Northampton Tables, Dr. Price assumed that this was the case; but as is well known, this was incorrect. As Dr. Farr has stated in the 12th Report of the Registrar General, page v, "The town contained then, as it "does now, great numbers of Baptists, who repudiate infant "baptism, and thus, consequently, by reducing the ratio of the "christenings to the deaths, induced Dr. Price to believe that the "population was stationary."

¶ I do not, however, propose to go into the general question as to the correctness of the principles on which Dr. Price proceeded. It is now known without doubt that the population of Northampton was not stationary for any lengthened period in the last century, and that the excess of deaths over births as given by the facts at Dr. Price's disposal, and which he attributed to the immigration into Northampton of persons just 20 years of age, was in a very great measure brought about by the circumstance that many births (of dissenters) had taken place of which no account was taken. The incorrectness of his assumptions has been over and over again exposed, so that there is no necessity further to allude to them. The object of the following paper is of a different nature. It seems to be the generally received impression that altho' Dr. Price has given the rough results from which he deduced the Northampton Table, yet he has nowhere explained in detail the exact manner in which he made use of those results to obtain his table. For instance in the article on Mortality Tables in the *Penny Cyclopædia* it is stated that he "has not distinctly described the process by which he formed it."

Again, in the Eighth Report of the Registrar General, Dr. Farr has entered at considerable length into the question of "*How the Northampton Table was constructed by Dr. Price?*"* According to Dr. Farr, "he has only explained this indirectly in " his works; and the extent of the deviations from the facts is " nowhere stated." Now a careful examination of Dr. Price's explanation of the method he employed, and actual verification of the method as applied in the other tables mentioned by him, has led me to the conclusion that Dr. Farr's explanation hereafter given scarcely does Dr. Price justice, and in fact that the latter has indicated the method he employed with sufficient distinctness to enable us to reconstruct his table.

Before, however, proceeding to the explanation I propose to offer of the method he actually employed, it may be of interest to set before you the explanation suggested by Dr. Farr. It will be found in the 8th Report of the Registrar General, and I here quote as much of it as is necessary to make it intelligible. Dr. Farr says:—"The general principles of his (Dr. Price's) method are laid " down in the 'Observations on the proper method of constructing " 'tables for determining the rate of human mortality, the number " 'of inhabitants, and the value of lives in any town or district, " 'from bills of mortality, in which are given the numbers dying " 'annually at all ages.' He exhibits the alterations in the London " Tables (XIII, XIV, XV, XVI of his work); delivers the data " from which the Northampton Table was derived; and says 'the " 'XVIIth Table in this volume *is *framed in the same manner* " '*with Table XV for London: and this is *the *genuine Table of* " '*Observations for Northampton.*'" He then quotes part of the note which Dr. Price appended to the 4th and subsequent editions of his work, and which is as follows:—" The Bills give the numbers " dying annually between 20 and 30 greater than between 30 and " 40; but this being a circumstance which does not exist in any

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* The italics are Dr. Farr's.

"other register of mortality, and undoubtedly owing to some "accidental and local causes, the decrements were made equal "between 22 and 40; preserving, however, the total of deaths "between 20 and 40 the same that the Bills give them. The "Bills giving only the totals of deaths under 2 years of age, and "between 2 and 5, the proportions of deaths for every particular "year between 2 and 5, and for every *quarter* of a year after "birth till 1 year of age, were made the same nearly that "the Chester Register makes them." Dr. Farr then takes Dr. Price's facts and proceeds "to show the steps of the process employed by Dr. Price," giving for the purpose a set of tables which are here reproduced.

TABLE A.—*Showing according to Dr. Farr the process probably employed by Dr. Price.*

| (1) Age. | (2) Dying at each Age and under the Age following (Dr. Price's facts). | (3) Numbers Living, if Population had been stationary. | (4) "Probably Dr. Price's *Original* Table." | (5) "Dr. Price's *Altered* Table." | (6) "Differences." | Age. |
|---|---|---|---|---|---|---|
| 0 | 1,529 | 4,689 | 10,000 | 11,650 | 1,650 | 0 |
| 2 | 362 | 3,160 | 6,735 | 7,283 | 548 | 2 |
| 5 | 201 | 2,798 | 5,963 | 6,249 | 286 | 5 |
| 10 | 189 | 2,597 | 5,535 | 5,675 | 140 | 10 |
| 20 | 373 | 2,408 | 5,132 | 5,132 | ... | 20 |
| 30 | 329 | 2,035 | 4,337 | 4,385 | 48 | 30 |
| 40 | 365 | 1,706 | 3,636 | 3,635 | ... | 40 |
| 50 | 384 | 1,341 | 2,858 | 2,857 | ... | 50 |
| 60 | 378 | 957 | 2,040 | 2,038 | ... | 60 |
| 70 | 358 | 579 | 1,234 | 1,232 | ... | 70 |
| 80 | 199 | 221 | 471 | 469 | ... | 80 |
| 90 | 22 | 22 | 47 | 46 | ... | 90 |
| 100 | ... | 0 | 0 | 0 | ... | 100 |
|  | 4,689 |  |  |  |  |  |

Dr. Farr then proceeds: "Dr. Price probably deduced Col. 4 "from Col. 3; the first number 4689 was raised to 10,000, and "all the following terms of the series in the same ratio, which "gave the numbers 6735, 5963, 5535, 5132, 4337, &c., of Col. 4. "He lastly added 1650 (nearly one-sixth part, or ·1650) to the "original radix 10,000; made corresponding alterations at the "subsequent ages up to 20; and to render the series more uniform, "took from the deaths returned at the age 20–30 to add to those "at 30–40. * * * Dr. Price intended the additions to the "Table under the age of 20 to be a correction of the original

" Table, for the immigrants who, he assumed, entered North-" ampton at the age of 20 * * *."

Now after carefully studying Dr. Farr's suggested explanation as above given, and referring to Dr. Price's work, it occurred to me that a better explanation of the method might be obtained with some little trouble from what Dr. Price tells us himself. To show this, let us in the first place take the Northampton Table, as given in the first three editions, and the facts from which it is deduced, and see how the one is obtained from the other. The number of deaths in All Saints' parish during the 36 years 1735-1770 was 3690, and the number of christenings 3242, the difference being 448, which difference Dr. Price, in accordance with the principles laid down in his work, ascribed to the deaths of persons who immigrated into Northampton at age 20. Proceeding exactly in the way indicated by him on pp. 73-97, vol. ii (7th edition), we easily deduce the Northampton Table as given in the first three editions of Dr. Price's work. The process is shown in the following table:—

TABLE B.—*Showing the process employed by Dr. Price in constructing the first Northampton Table.*

| (1)<br>Age. | (2)<br>Dying at each Age and under the Age following (Dr. Price's facts). | (3)<br>Numbers Living, if Population had been stationary. | (4)<br>Col. 3 reduced to radix 1000. | (5)<br>Col. 4 less 121 as far as Age 20. | (6)<br>Col. 5 raised in proportion of 515 to 394 as far as Age 20. | (7)<br>Dr. Price's First Northampton Table. | (8)<br>Difference between Cols. 7 and 6. |
|---|---|---|---|---|---|---|---|
| 0 | 1,206 | 3,690 | 1,000 | 879 | 1,149 | 1,149 | — |
| 2 | 276 | 2,484 | 673 | 552 | 722 | 722 | — |
| 5 | 155 | 2,208 | 598 | 477 | 623 | 625 | 2 |
| 10 | 155 | 2,053 | 556 | 435 | 569 | 570 | 1 |
| 20 | 297 | 1,898 | 515 | 394 | 515 | 515 | — |
| 30 | 257 | 1,601 | 434 | ... | 434 | 435 | 1 |
| 40 | 297 | 1,344 | 364 | ... | 364 | 365 | 1 |
| 50 | 300 | 1,047 | 284 | ... | 284 | 284 | — |
| 60 | 293 | 747 | 203 | ... | 203 | 203 | — |
| 70 | 285 | 454 | 123 | ... | 123 | 123 | — |
| 80 | 155 | 169 | 46 | ... | 46 | 46 | — |
| 90 | 14 | 14 | 4 | ... | 4 | 4 | — |
| 100 | ... | 0 | 0 | ... | 0 | 0 | — |
|  | 3,690 |  |  |  |  |  |  |

*Note.*—The differences in Col. 8 are simply the result of further adjustments.

Thus far, then, no difficulty whatever is found in ascertaining the method pursued by Dr. Price. Let us now proceed to apply the same method to the facts on which he founded *the* Northampton Table, as given in the 4th and subsequent editions of his work, being the table on which the monetary tables at present in use are based. The number who died in the 46 years 1735–1780 in the parish of All Saints was 4689, and the number of christenings was 4220; the number of deaths among immigrants at age 20 would therefore be taken, according to Dr. Price, as 469. Proceeding just as before, Col. 6 of the following table should give us the numbers living at the different ages according to Dr. Price's corrected Northampton Table—that is, the Northampton Table as now understood.

TABLE C.—*Showing how the Northampton Table would have been constructed if the process exhibited in Table* B *had been employed.*

| (1) Age. | (2) Dying at each Age and under the Age following (Dr. Price's facts). | (3) Numbers Living, if population had been stationary. | (4) Col. 3 raised to radix 10,000. | (5) Col. 4 less 1000 as far as Age 20. | (6) Col. 5 raised in proportion of 5135 to 4135 as far as Age 20. | (7) Dr. Price's Northampton Table. | (8) Difference between Cols. 7 and 6. |
|---|---|---|---|---|---|---|---|
| 0 | 1,529 | 4,689 | 10,000 | 9,000 | 11,177 | 11,650 | 473 |
| 2 | 362 | 3,160 | 6,739 | 5,739 | 7,127 | 7,283 | 156 |
| 5 | 201 | 2,798 | 5,967 | 4,967 | 6,168 | 6,249 | 81 |
| 10 | 189 | 2,597 | 5,538 | 4,538 | 5,635 | 5,675 | 40 |
| 20 | 373 351 | 2,408 | 5,135 | 4,135 | 5,135 | 5,132 | 3 |
| 30 | 329* 351 | 2,057 | 4,387 | ... | 4,387 | 4,385 | 2 |
| 40 | 365 | 1,706 | 3,638 | ... | 3,638 | 3,635 | 3 |
| 50 | 384 | 1,341 | 2,860 | ... | 2,860 | 2,857 | 3 |
| 60 | 378 | 957 | 2,041 | ... | 2,041 | 2,038 | 3 |
| 70 | 358 | 579 | 1,235 | ... | 1,235 | 1,232 | 3 |
| 80 | 199 | 221 | 471 | ... | 471 | 469 | 2 |
| 90 | 22 | 22 | 47 | ... | 47 | 46 | 1 |
| 100 | ... | 0 | 0 | ... | 0 | 0 | ... |

*Note.*—The 1000 deducted in Col. 5 =10 percent of annual deaths.

In Col. 7 are given the figures of the Northampton Table which, allowing for the changes introduced by graduation, ought to correspond with those of Col. 6. It is at once evident, however, that they do not, and the question is, what departures did Dr. Price make from his facts—in other words, what deduction from the numbers living up to but not including age 20 did he make as a correction for the immigrants who entered at age 20? It will be seen that we can readily ascertain this from the following equation:—

$$\frac{10{,}000-x}{5135-x}=\frac{11650}{5135}$$

$$\text{or}\quad 5135\times 1650 = 6515x$$

$$\therefore\quad x = 1300$$

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* The decrements between 20 to 30, and 30 to 40, have been made equal according to Dr. Price's note to that effect.

Hence we find that Dr. Price considered that out of a total of 10,000 deaths, 1300 or 13 percent were the deaths of persons who entered Northampton at age 20. The proportion given by the facts was 469 out of 4689 or 10 percent.

We can now without any difficulty deduce the Northampton Table, as will appear from the following tabular summary.

TABLE D.—*Showing the process actually employed in the construction of the Northampton Table.*

| Age. | (1)* Same as Col. 4 of Table C. | (2) Col. 1 less 1300 as far as Age 20. | (3) Col. 2 raised in the proportion of 5135 to 3835 as far as Age 20. | (4) Dr. Price's Northampton Table. | (5) Difference between Cols. 4 and 3. |
|---|---|---|---|---|---|
| 0 | 10,000 | 8,700 | 11649·2 | 11,650 | ... |
| 2 | 6,739 | 5,439 | 7283 | 7,283 | ... |
| 5 | 5,967 | 4,667 | 6249 | 6,249 | ... |
| 10 | 5,538 | 4,238 | 5675 | 5,675 | ... |
| 20 | 5,135 | 3,835 | 5135 | 5,132 | 3 |
| 30 | 4,387 | ... | 4387 | 4,385 | 2 |
| 40 | 3,638 | ... | 3638 | 3,635 | 3 |
| 50 | 2,860 | ... | 2860 | 2,857 | 3 |
| 60 | 2,041 | ... | 2041 | 2,038 | 3 |
| 70 | 1,235 | ... | 1235 | 1,232 | 3 |
| 80 | 471 | ... | 471 | 469 | 2 |
| 90 | 47 | ... | 47 | 46 | 1 |
| 100 | 0 | ... | 0 | 0 | ... |

*Note.*—The 1300 deducted in Col. 2 = 13 percent of annual deaths.

Why Dr. Price concluded that 13 percent of the total deaths were those of persons coming into Northampton at age 20 does not appear. The proportion taken in the first Northampton Table was, as we have seen, 12·1 percent, being neither more nor less than that given by the facts from Dr. Price's point of view; but the proportion given by the facts on which *the* Northampton Table is based is only 10 percent. Bearing in mind, however, the following passage of his work:—"If for a course of years there has been no sensible "increase or decrease in a place, the number of annual settlers "will be equal to the excess of the annual burials above the "annual births. If there is an *increase,* it will be *greater* than "this excess. If there is a *decrease,* it will be *less*" (7th edition, vol. ii, p. 76)—it would seem that he had reason to believe that an increase had taken place in the population of All Saints' parish in the course of the 46 years of observation, and that this increase was brought about by persons settling in Northampton at age 20. It is probable, therefore, that this was one of the reasons which led him to consider 13 percent, instead of 10 percent, of the total deaths as arising from immigration at age 20.

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* The previous cols. are the same as in Table C.

This however, appears to be a very large percentage, almost incredibly large; for it is equivalent to saying that out of 5135 persons, in a town, of age 20, 1300, or more than 25 percent, are persons who have just settled in the town.

At the same time there are several singular coincidences which we may here point out. Thus for a complete century, *i.e.*, from 1633 to 1734, we are told on p. 77, vol. ii, that in Breslau the annual medium of births was 1089, and of burials 1256, the difference being 13·3 percent of the burials. Again, in Norwich, for the 30 years, from 1740 to 1769, the corresponding numbers were 1057 and 1206, the difference being 12·4 percent of the latter. Another point worthy of notice is that according to the Northampton Table, if we take the total number living at all ages as 5136, being the number ascertained by enumeration, to be the population of Northampton in 1746, we get from the formula

$$\mathring{e}_0+\tfrac{1}{2}=\frac{l_0+l_1+l_2+\ldots\ldots}{l_0}$$

$$\left.\begin{array}{r}\text{or } l_0\\ \textit{i.e.}\text{ total annual deaths}\end{array}\right\}=\frac{l_0+l_1+l_2+\ldots\ldots}{\mathring{e}_0+\frac{1}{2}}$$

$$\therefore\ \text{Total annual deaths}\ =\frac{5136}{25{\cdot}68}$$

$$=200.$$

Dr. Price tells us that for the 10 years, 1741 to 1750, being just about the time when the number of inhabitants was 5136, the average annual number of deaths at all ages was 197·5. A similar calculation to the above, taking the Northampton Table of the first three editions of Dr. Price's work, gives the average annual number of deaths as 197·8.

In conclusion, it may not be out of place to point out that various passages in the fourth and subsequent editions of Dr. Price's work have reference not to the Northampton Table as there given, but to the first Northampton Table constructed by him and given in the first three editions. This, no doubt, in great measure arose from the fact mentioned by Mr. F. Hendriks in his paper in vol. i of the *Assurance Magazine*, that Dr. Price was overburdened with business and ill when the 4th edition was published, and compelled to leave to others the preparation of a portion of the work. It will, nevertheless, save trouble to those reading it to bear this always in mind.

The following summary of the discussion which followed the reading of the paper is abridged from the *Insurance Record.*

The PRESIDENT congratulated Mr. Sutton on the successful result of his investigation, which had enabled him to reconstruct Dr. Price's Northampton Table. That once famous table is gradually going out of use. It is now probably more useful for comparison than for office purposes. Nevertheless, anything relating to the Northampton Table must be of interest to members of the Institute.

Mr. A. H. BAILEY thought that the result of Mr. Sutton's researches into the Northampton Table was of considerable interest and value. He had taken the trouble to read and examine the early editions of Dr. Price's work, which are not readily accessible, and which probably had not been seen by Dr. Farr; and, in consequence, had been able to all intents and purposes to reconstruct the Northampton Table as it appears in our ordinary text-books, conformably with Dr. Price's own views; whereas Dr. Farr proceeded on certain conjectures as to what Dr. Price did. This must have been a work of some labour, and was creditable to Mr. Sutton's industry and acumen. But while it now appeared that Dr. Farr had not done sufficient justice to Dr. Price, so Mr. Sutton had, quite unintentionally, scarcely done sufficient justice to Dr. Farr. The paper by Dr. Farr in the Appendix to the Eighth Report of the Registrar General, is one of his most valuable contributions to our particular subject. Although, perhaps, Dr. Farr had not met with such success as Mr. Sutton in reconstructing the Northampton Table, he had proceeded one or two steps further. By researches into county histories, and books of that character, he showed sufficiently that so far from the population of the town of Northampton having been stationary during the 46 years over which Dr. Price's observations extended, it had been, as everybody now is well convinced, steadily increasing throughout that period. But he went further. From the materials in the Registrar General's office he constructed a table of the mortality of the parish of All Saints, Northampton, during the years 1838–44 inclusive, and also, from the same materials, a table founded on Dr. Price's hypothesis of stationary population. The former of these new tables corresponded sufficiently well with any of the standard tables of the present time; the second, constructed on the hypothesis of a stationary population, corresponded with Dr. Price's table, and therefore gave the strongest possible confirmatory evidence of the faultiness of the assumption on which Dr. Price's table was constructed. That is a curious instance of how needful it is, in investigations of all kinds, to stand out, above all things, for truth. Dr. Price was a strong politician, and held most decided opinions about the decadence of the kingdom,—amongst other things, the decline of the population; and in this book of his there are some perfectly ludicrous observations to prove how the population was decreasing. There are two or three essays assigning various reasons for such a state of things. We now know that the very reverse was the case. It is still more remarkable that Dr. Price's biographer and admirer—his nephew, Mr. Morgan,—without the same excuse, fell into the same mistake. Any one who has read Mr. Morgan's reports to the Equitable Society knows that he was in the habit of keeping records of how far the actual experience of the Equitable corresponded with the Northampton Table; and the errors of that table should have been apparent when, in stating these ratios, he found that the decrements between the ages of 20 and 30 were as 1 to 2, while the decrements between 60 and 80 were as 4 to 5. Nothing, however, apparently, would convince Mr. Morgan that anything was wrong in the Northampton Table, and in his comments in his works he "follows on the same side", as they say in the Court of Chancery, about the decadence of the population of this country. In one paragraph Dr. Price says:—"The reflection on these facts"—a variety of facts are given—"must be mortifying to this country (the richest upon earth), if it be indeed true that our population is declining. But we must comfort ourselves by considering that in this case *value* is of more consequence than number. . . . . 1st. the decrease of London. It is almost reckoned certain, till some other satisfactory reason can be given for a diminution since 1727, of more than 7,000 per annum in the registered burials, and near 2,000 in the registered births". He (Mr. Bailey) did not apprehend that at any time since the great fire there had been any decrease in the population of London. He observed that Mr. Sutton says in his paper, "Bearing in mind the enormous importance which the Northampton Table acquired, it is a conspicuous fact how comparatively little was thought of it by its author when he first published it". That may have been the case when it was first published, but the language of the later editions tells a different story. Dr. Price says in the sixth edition—and it is a passage often quoted—that certain new tables have been ordered to be calculated; and, after giving a catalogue of them, he proceeds:—"They have been calculated by Mr. Morgan with incredible care and industry, and are correct and complete to a degree never before attempted in any tables of this kind. They are to form the basis of the future business of the Society"—that is, the Equitable—"and must conduce much to its growing credit and usefulness". Again, "They are all founded on a table of the probabilities of the duration of human life at Northampton, which will be inserted among the other tables in the second volume of this work. This table made a part of all the former editions of this work; but it is in the present edition much improved, and gives, I believe, more correctly than any other, the mean probabilities of the duration of human life, and therefore seems to be more proper than any other for general use". That is hardly consistent with attaching little importance to the table. And then, when consulted about certain alterations in the Equitable Society, Dr. Price, in recommending the Chester Table, says:—"I had, in the introduction to Mr. Morgan's treatise, recommended to the Society the observations on human mortality at Chester; and I had procured a copy of them from Dr. Haygarth, the ingenious founder of them. But the Directors of the Society have judged very rightly that they carry the probabilities of life too high for their business. I have been enabled by them to make the Northampton Table of observations more complete". He thought that Dr. Price really did attach considerable importance to the Northampton Table, and thought it was more fit for the purposes of the Equitable Society than any other. Then, again, with regard to its use for the purpose of Government life annuities. Mr. Sutton had said a great many things about what Pitt should have done. He (Mr. Bailey) apprehended that Prime Ministers in all ages are far too much occupied to attend to matters of detail, such as what table should be used for Government annuities. But the fact is that the Government life annuities commenced in 1808; and as Pitt died in 1806, he could have had but little to do with it. He thought the capital improvement the late Mr. Finlaison introduced into the Government annuities was not so much a substitution of tables, as pointing out that, for the purpose of life annuities, a distinction must be made between the sexes. As a matter of fact, the bulk of annuities are bought above the age of 60. [Mr. BADEN—The average is 64.] Now, for male lives, the Northampton Table differs from the table now in use for the purchase of Government annuities nowhere so much as one year's purchase. In the case of female lives, it is very different; and as the majority of annuities are purchased for female lives, it was a frightful mistake to use the Northampton Table, although it would not have been so bad for males. Mr. Sutton's paper is further of interest, because there is a great deal to be learnt from the use and misuse of the Northampton Table. He thought that table had been responsible for two great fallacies, which had exercised some considerable influence. One of those is the supposed material improvement of the duration of human life in the last 100 years. He was not going to deny that some improvement had taken place, but he believed it was much less than was commonly supposed. When any modern observation of mortality has been compared with the Northampton Table, it has been supposed that the difference between the two is due mainly to the improvement in vitality; but it is now established that the Northampton Table never represented the true state of the case. The other fallacy is, that the Northampton Table is a safe one to use for life assurance purposes, because it represents too high a mortality. When the question was before the Equitable Society as to what table they should substitute for the London observations, and they selected the Northampton in preference to the Chester Table, no doubt what was in their minds was this, that the Northampton Table gave higher premiums than the Chester Table, which of course is the case; and when that was stated to them, it was considered sufficient. [The PRESIDENT—A good reason.] A good reason, but by no means the whole of the case, because there is another rather important matter, and that is the question of how the liabilities are to be estimated, which was not thought of at the time the Northampton Table was first adopted. If the Northampton Table were consistently wrong, there might have been something said for that; but such is not the case, for, as already stated, it is substantially correct above the age of 60; and under the age of 20, where, as Mr. Sutton has shown, it was most knocked about by Dr. Price, it has been of very little, if any, practical use whatever. From the age of 20 to about 50 is the period during which the Northampton Table is so erroneous. The ordinary formula for the value of a policy is

$${}_nV_x=1-\frac{1+a_{x+n}}{1+a_x};$$

As the late Prof. De Morgan, in a letter to the *Journal* (see vol. xiv, p. 69), many years ago stated, that is much better expressed by the equivalent

$$\frac{a_x-a_{x+n}}{1+a_x}$$

so that, as he says, the value of a policy is in proportion to "the fall of the annuity". Now, the bulk of the assurances effected are at young or comparatively early ages, say between 20 and 40, where the value of an annuity by the Northampton Table is too low. But the heavy liabilities are above the age of 60, where the Northampton Table is pretty right; so that, speaking broadly, $a_{x+n}$ is right, and $a_x$ is too low, and therefore $a_x-a_{x+n}$ is too low also. Although, then, the Northampton Table gives too high a mortality, the value of a policy by it, as Mr. Manly has shown in his essay, is one of the lowest of any of the known tables; so that some offices have been living in something like a "Fool's paradise", when they think they are right in using the Northampton Table, because the errors in it are on what is called "the safe side". In the Carlisle Table, which has hitherto been the chief substitute for it, the error is exactly the reverse. There the value of the annuities at the younger ages is right, and the value of the annuities at the older ages is too high. So that, again, the value of the policies is too small, although the values come out higher generally by the Carlisle than by the Northampton Table. All these things go to show the value of Mr. Sutton's investigation, and the danger of using tables known to be erroneous, when it is supposed that by so doing you get a result in your own favour, for that is not always the case.

Mr. C. **Walford** believed it was through an accident at the outset that the Equitable Society was induced to adopt the Northampton Table. The accident was of several parts. In the first place, the death of Mr. Dodson, who was the real founder of the Equitable Society, before the Society had got very far on its way, led to the abandonment of the rates from the London Table which he had constructed for the Society. The resignation of Mr. Dodson's son within twelve months after he had succeeded his father, was another accident. Then the Northampton Table came before the court of the Equitable Society; and no doubt Mr. Bailey's view is a sound one, that the directors selected that table simply because the rates by it were higher. [The **President**—There was, I think, 15 per-cent put on it at first.] For beer retailers, female, and other risks, they put on 15 per-cent. He agreed with Mr. Sutton that Dr. Price himself did not in the first instance attach very much value to this table. He had taken the trouble to read each edition of Dr. Price's work, and had marked the different views expressed in each; and he had no doubt that the outside attention which the Northampton Table obtained, caused the Doctor to form a higher opinion of it than he had originally. There were numerous purposes to which that table came to be applied—the value of leases, church property, and so on. If any one could know precisely the amount of actuarial work which flooded in upon Dr. Price, arising from the large number of annuity societies which then existed, he would get to know better why it was, in the opinion of the Doctor, that the table became so important. But the history of the Northampton Table, apart from this point of view, is a remarkable one,—and not the least so in the fate which befel the Equitable—he meant the accumulation of its funds, which led to the declaration of those large bonuses, and created that insurance fallacy which has given rise to so many curious results, and still continues to do so. The bonuses which were declared by the Equitable were the result of a simple accident. The care, and (one may say) fright, which the directors felt from time to time in the early days of the Equitable, caused them to accumulate a large surplus, which of course compounded and tended to swell bonuses which would have been large without that. Then another view set in. The policyholders in the Equitable saw this large fund accumulating, and were always anxious, as they are in most mutual societies, to divide it. That again made the directors, of necessity and in self-defence, conservative on the subject of surplus, and made them accumulate even more than they would have done but for this antagonism on the part of the policyholders. And then there were other accidents connected with the history of the Equitable, which tended to cause the directors to feel the importance of keeping up high rates, even when they were convinced that they were too high. The history of the Equitable, which is associated a great deal with the history of life insurance, was by no means a bed of roses in its early times; and the quarrels which enforced themselves upon the management amongst its policyholders were of a severe kind. He (Mr. Walford) looked upon it as a further accident that the Equitable Society has survived to this moment. There were many societies which began about the same time and prior to it, with as fair a chance of success, that have long ago disappeared; and although nothing could alter his view of the almost veneration one feels for the Equitable Society, still it must be known that it might have shared the fate, and nearly did share the fate, of a great many societies of that period. But having survived, people came to look upon it as a very great society, and regarded the Northampton Table probably as the cause of its success. This table, therefore, had a halo of glory thrown around it which it did not deserve. If any other table had been guilty of causing such a national loss as the Northampton did, it would have been discarded. It not only produced a direct loss in reference to annuitants, but possibly also a great loss in the revenue arising from legacy duties and so forth. The table which some gentlemen here were probably thinking of was the Government Table founded in 1808, under which Stock might be exchanged for annuities, in regard to which selection was, as a matter of course, made against the Government. The legacy duty was calculated upon the Northampton Table, adopted by Act of Parliament, in 1796, and did not get abrogated till 1853. Therefore a large item in the revenue was yearly affected by the use of the Northampton Table, as well as many values of leases. In fact, it would be very difficult to estimate the wonderful influence which this table had on the finances of this country for a period of more than half a century. He was glad that Mr. Sutton had brought this subject before the society, and he hoped some day himself to be in a position to say something upon it which even Mr. Sutton had not at present discovered.

Mr. P. Gray thought that Mr. Walford had overlooked the fact that Mr. Bailey quoted Dr. Price's own words, assigning the reasons why the Northampton Table had been employed to construct the tables of the Equitable. There was no doubt that Dr. Price thought it a very suitable table indeed. Mr. Walford seemed to have overlooked that. [Mr. Bailey—Dr. Price says it is "more proper than any other for general use".] And Mr. Morgan, Dr. Price's successor, employs the same phrase with regard to the Northampton Table over and over again. It seemed very odd that immigrants should be assumed as coming to Northampton at precisely the age of 20. He thought at first the meaning was that they had come in at all ages up to 20; but the statement was so definite that it could not be received in that sense. [Mr. Sutton—It is not altogether an absurdity, as apprentices to the shoe trade, who had served their time in the country, came to the town for employment in large numbers; and their ages would be 20 or 21.] There is another small point that comes up. There are three appellations given to this table which we take as the foundation for our calculations. It is called a Table of Observations, a Table of Mortality, and a Bill or Bills of Mortality. The table we use, and which we are now pretty well agreed to call a table of mortality, is not a table of observations. It is a table exhibiting the results of, or deductions from, observations, the earlier examples having been deduced usually from the weekly or annual bills of mortality. I observe that the followers of Mr. Morgan, to the present time, continue to employ the objectionable appellation. Mr. Sang also, I notice, calls the mortality table a Bill of Mortality.

Mr. R. P. Hardy thought that some little injustice had been done to the memories of both Dr. Price and Mr. Morgan, in assuming that they were not cognisant of the defects of the Northampton Table. He would like to know what table in their day they could have employed in its stead. The London Table could not be said to be applicable to the population at large. It was incredible that Mr. Morgan, who was a prolific writer on the financial and political subjects of the day, should have fallen into any such mistake as is supposed and have persevered in it. He did not think either of those gentlemen ever said that it was a proper representative mortality table, but only that it was a table applicable for office purposes as used. We know there were certain regulations in the Equitable Society, which to a large extent counteracted its tendency. For instance, at every distribution of profit one third of the surplus was retained in hand. Why should Mr. Morgan, with his great sagacity of mind, have allowed the directors of the Society to retain for the benefit of the future, one third of the surplus? Simply because he knew that the base table was not a true one. He (Mr. Hardy) admitted this was correcting in a clumsy manner, but it was the best means at command. Neither could he believe that the revenue lost so much by the adoption of the Northampton Table as had been alleged. As the legacy duty is levied on the whole fund, if the life interest was underrated, the reversion was overrated, and therefore the revenue does not lose. If one goes now to Somerset House to pay duty upon a reversion, how do they value it? They take off from the fund the value of the life interest at 3½ per-cent, and that they call the value of the reversion.

The PRESIDENT—There is no rule for their valuing reversions in that way, but they have taken to do it.

Mr. HARDY—And they won't listen to any actuary on the matter. They take the same rate of interest on both sides, but a result so obtained is not the proper value of a reversion.

Mr. BADEN—Are you sure they adopt 3½ per-cent, Mr. Hardy?

Mr. BAILEY thought it is 4 per-cent now. With regard to the loss to the revenue, so far as probate is concerned, he did not think the question of mortality tables enters into the matter, because probate duty is paid on the corpus. As regards the legacy duty, it is taken on the value of the annuity, which is a small matter. A gentleman leaves £100 a year to his butler, for instance, upon which duty has to be paid. These annuities of small amount are usually charged upon Consols, and constitute a small fraction of the annual legacy duty. Previously to 1863, an annuity to a butler was valued at 3½ per-cent; but now it is valued at 4 per-cent. He hardly understood Mr. Hardy. A man dies and leaves his property to his widow for her life and remainder to his children, which is a common form of will. It is not usual to pay duty on the reversion to the children until the children actually inherit it. In valuing reversions for purchase, the legacy duty is deducted from the amount in reversion; but the duty is not actually paid by the purchaser, and is usually not paid until the reversion falls in. It may be anticipated, but few people adopt such a course. [The PRESIDENT—Occasionally. Mr. HARDY—In half the cases.]

Mr. SUTTON observed that Mr. Bailey thought he had done Dr. Farr an injustice. But he had purposely kept to one point throughout his paper, and that was to endeavour to show the exact manner in which Dr. Price constructed the Northampton Table. He had read very carefully what Dr. Farr wrote about the table, and was very much struck, like Mr. Bailey, with its great value; but he did not think it was material to the object he had in view to bring it forward in any way, believing that doing so would only lead to more confusion where there has existed a great deal already. As to the importance attached by Dr. Price to the Northampton Table, he was certainly still of opinion that when Dr. Price published the Northampton Table, as given in the first three editions, he attached to it a sort of general value for purposes of comparison, but did not consider it of special importance. When, however, the fourth edition was published, the table had been adopted by the Equitable Society, and it had thus acquired an importance which it had not previously possessed, and undoubtedly Dr. Price thought more highly of it than previously. With regard to Mr. Pitt's government, he had adopted verbatim Mr. Hendriks's statement. He had an impression that the annuity scheme was brought forward a year or two before it was passed, so that it was quite possible the details of the Bill had come under Mr. Pitt's notice. He quite agreed with Mr. Walford, that to say all that might be said with regard to the Northampton Table would fill a good-sized volume.

## *On the Arithmometer of M. Thomas (de Colmar), and its application to the Construction of Life Contingency Tables.* *By* PETER GRAY, F.R.A.S., F.R.M.S., *Honorary Member of the Institute of Actuaries.*

(Continued from page 32.)

PROBLEM VIII.—To construct a table for the formation of Survivorship Assurances.

A survivorship assurance on $(x)$ against $(y)$ is an assurance payable at the end of the year in which the combination $(x.y)$ is dissolved, provided the dissolution is caused by the death of $(x)$.

The value of this assurance, in respect of the $n$th year, is the value of the sum payable (supposed a unit) into the probability of the concurrence of these two events, namely, that $(x)$ shall die in that year, and that $(y)$ shall live over half of it. This gives

$$\frac{d_x\lambda_y v^n}{l_{x.y}},$$

in which $\lambda_y=\frac{1}{2}(l_y+l_{y+1})$.
Summation of this expression, for the proper values of $n$, will give the value of the assurance with reference to the whole or any portion of the future lifetime of $(x)$.

Making $n=1$, and multiplying numerator and denominator by $v^x$ for the compartment of the table in which $x$ is not less than $y$, and by $v^y$ for that in which $y$ is not less than $x$, we have the following expressions for the value in respect of the first year in the two cases:—

$$\frac{d_x\lambda_y v^{x+1}}{D_{x.y}}, \text{ and } \frac{d_x\lambda_y v^{y+1}}{D_{x.y}}.$$

Consequently, $D_{x.y}$ having been already formed, we have now to form only the numerators of these expressions, of the first for every combination of $x$ and $y$ in which $x$ is not less than $y$, and of the second for every combination in which $y$ is not less than $x$.

The following is the commencement of the two series $d_x$ and $\lambda_y$, with their differences. The former is given in the mortality table, and the latter is formed in accordance with the relation $\lambda_y=\frac{1}{2}(l_y+l_{y+1})$.

| $x$ & $y$ | $d_x$ | $\Delta$ | $\lambda_y$ | $\Delta$ |
|---|---|---|---|---|
| 97 | 9 | | 4·5 | |
| 6 | 40 | 31 | 29·0 | 24·5 |
| 5 | 86 | 46 | 92·0 | 63·0 |
| 4 | 139 | 53 | 204·5 | 112·5 |
| 3 | 195 | 56 | 371·5 | 167·0 |
| 2 | 254 | 59 | 596·0 | 224·5 |
| 1 | 329 | 75 | 887·5 | 291·5 |
| 0 | 408 | 79 | 1256·0 | 368·5 |
| 89 | 495 | 87 | 1707·5 | 451·5 |
| 8 | 615 | 120 | 2262·5 | 555·0 |
| 7 | 773 | 158 | 2956·5 | 694·0 |
| 6 | 941 | 168 | 3813·5 | 857·0 |
| 5 | 1138 | 197 | 4853·0 | 1039·5 |
| 4 | 1346 | 208 | 6095·0 | 1242·0 |
| 3 | 1545 | 199 | 7540·5 | 1445·5 |
| 2 | 1719 | 174 | 9172·5 | 1632·0 |
| 1 | 1883 | 164 | 10973·5 | 1801·0 |
| 0 | 2015 | 132 | 12922·5 | 1949·0 |
| * | * | * | * | * |

It may be well to remark that, the terms being arranged in reverse order, it is necessary, for the purpose of securing the correspondence of the differences with the ages to which they individually belong, to place them each against the term which constitutes the minuend in its formation. And they are all properly negative.

The formation for each age of the functions $d_x v^{x+1}$ and $\lambda_y v^{y+1}$ is the next step. For this purpose the Arithmometer comes into use. The commencement of the formation is here shewn, but it is so plain as not to stand in need of explanation.

| $x$ & $y$ | $v^{x+1}$ | $d_x$ | $d_x v^{x+1}$ | $\lambda_y$ | $\lambda_y v^{y+1}$ |
|---|---|---|---|---|---|
| 97 | 5520164 | 9 | 496815 | 45 | 24841 |
| 6 | 5685769 | 40 | 2274308 | 290 | 164887 |
| 5 | 5856342 | 86 | 5036454 | 920 | 538783 |
| 4 | 6032032 | 139 | 8384524 | 2045 | 1233551 |
| 3 | 6212993 | 195 | 12115336 | 3715 | 2308127 |
| 2 | 6399383 | 254 | 16254433 | 5960 | 3814032 |
| 1 | 6591364 | 329 | 21685588 | 8875 | 5849836 |
| 0 | 6789105 | 408 | 27699548 | 12560 | 8527116 |
| 89 | 6992779 | 495 | 34614256 | 17075 | 11940170 |
| 8 | 7202562 | 615 | 44295756 | 22625 | 16295797 |
| 7 | 7418639 | 773 | 57346079 | 29565 | 21933206 |
| 6 | 7641198 | 941 | 71903673 | 38135 | 29139709 |
| 5 | 7870434 | 1138 | 89565539 | 48530 | 38195216 |
| 4 | 8106547 | 1346 | 91141226 | 60950 | 49409404 |
| 3 | 8349743 | 1545 | 129003529 | 75405 | 62961237 |
| 2 | 8600236 | 1719 | 147838057 | 91725 | 78885665 |
| 1 | 8858243 | 1883 | 166800716 | 109735 | 97205930 |
| 0 | 9123990 | 2015 | 183848399 | 129225 | 117904761 |
| * | * | * | * | * | * |

We are now prepared for the principal formations.

When $x$ is equal to or greater than $y$, we have, if $y$ vary,

$$\Delta_y d_x\lambda_y v^{x+1} = d_x\lambda_y v^{x+1}.\Delta\lambda_y;$$

whence

$$d_x\lambda_{y+1}v^{x+1} = d_x\lambda_y v^{x+1} + d_x\lambda_y v^{x+1}.\Delta\lambda_y;$$

and for the formation in reverse order, by transposition,

$$d_x\lambda_y v^{x+1} = d_x\lambda_{y+1}v^{x+1} - d_x\lambda_y v^{x+1}.\Delta\lambda_y.$$

Finally, changing $y$ into $y-1$,

$$d_x\lambda_{y-1}v^{x+1} = d_x\lambda_y v^{x+1} - d_x\lambda_{y-1}v^{x+1}.\Delta\lambda_{y-1}.$$

By this formula, which is of the form P$-$QR, the required values are to be constructed; but $\Delta\lambda_{y-1}$ being negative, the regulator has to be set for addition.

The following is a specimen of the process.

$x>y$

| $x$ | $d_xv^{x+1}$ | $\lambda_y$ | $x$ | Diff. 0. | Diff. 1. | Diff. 2. | Diff. 3. | Diff. 4. |
|---|---|---|---|---|---|---|---|---|
| 97 | 49682 | 45 | 97 | ·02236 | ·14408 | ·45707 | 1·01600 | 1·84569 |
| | | 245 | | | | | | |
| 6 | 227431 | 290 | | ·65955 | 2·09236 | 4·65094 | 8·44902 | 13·55483 |
| | | 630 | 6 | ·68191 | 2·23644 | 5·10801 | 9·46502 | 15·40052 |
| 5 | 503645 | 920 | | 4·63354 | 10·29956 | 18·71044 | 30·01730 | 44·69858 |
| | | 1125 | 5 | 5·31545 | 12·53600 | 23·81845 | 39·48232 | 60·09910 |
| 4 | 838452 | 2045 | | 17·14634 | 31·14849 | 49·9717 | 74·4126 | 105·3096 |
| | | 1670 | 4 | 22·46179 | 43·68449 | 73·7902 | 113·8949 | 165·4087 |
| 3 | 1211534 | 3715 | | 45·0085 | 72·2074 | 107·5236 | 152·1687 | 206·8694 |
| | | 2245 | 3 | 67·4703 | 115·8919 | 181·3138 | 266·0636 | 372·2781 |
| 2 | 1625443 | 5960 | | 96·8765 | 144·2582 | 204·1558 | 277·5446 | 367·7567 |
| | | 2915 | 2 | 164·3468 | 260·1501 | 385·4696 | 543·6082 | 740·0348 |
| 1 | 2168559 | 8875 | | 192·4595 | 272·3709 | 370·2813 | 490·6362 | 641·1342 |
| | | 3685 | 1 | 356·8063 | 532·5210 | 755·7509 | 1034·2444 | 1381·1690 |
| 0 | 2769955 | 12560 | | 347·9062 | 472·9696 | 626·7021 | 818·9369 | 1056·322 |
| | | 4515 | 0 | 704·7125 | 1005·4906 | 1382·4530 | 1853·1813 | 2437·491 |
| 89 | 3461426 | 17075 | | 591·0385 | 783·1476 | 1023·3706 | 1320·0148 | 1679·830 |
| | | 5550 | 89 | 1295·7510 | 1788·6382 | 2405·8236 | 3173·1961 | 4117·321 |
| 8 | 4429576 | 22625 | | 1002·192 | 1309·604 | 1689·219 | 2149·673 | 2699·827 |
| | | 6940 | 8 | 2297·943 | 3098·242 | 4095·043 | 5322·869 | 6817·148 |
| 7 | 5734608 | 29565 | | 1695·437 | 2186·893 | 2783·005 | 3495·244 | 4324·181 |
| | | 8570 | 7 | 3993·380 | 5285·135 | 6878·048 | 8818·113 | 11141·329 |
| 6 | 7190367 | 38135 | | 2742·047 | 3489·486 | 4382·529 | 5421·897 | 6595·365 |
| | | 10395 | 6 | 6735·427 | 8774·621 | 11260·577 | 1424·0010 | 17736·694 |
| 5 | 8956554 | 48530 | | 4346·616 | 5459·020 | 6753·690 | 821·5399 | 9828·475 |
| | | | 5 | 11082·043 | 14233·641 | 18014·267 | 22455·409 | 27565·169 |
| * | * | * | * | * | * | * | * | * |

Here, as in the example of the last problem, to facilitate the subsequent summations, $x$ is constant in the rows, and $x-y$ in the columns. The results, as they arise, have consequently to be written in the rows, commencing in each case in the first column, Diff. 0. There are in the example two columns headed $x$. The ages in the first of these columns are opposite the rows in the principal columns occupied by the successive terms of $d_x\lambda_y v^{x+1}$, and the ages in the second are opposite the rows occupied by the summations of the same function. The columns between those containing the values of $x$, are occupied, as shown, by the terms of two of the series whose formation has been exemplified.

The process of formation is as follows. To form the row corresponding to any given value of $x$, take the proper term of $d_x v^{x+1}$ for the in-factor, and multiply continuously by the adjoining term of $\lambda_y$, and the differences of this series, commencing with $\Delta\lambda_{y-1}$. The regulator is at addition throughout, since the differences of $\lambda_y$ are negative. A slip containing the differences should be placed on the machine in front of the operator; and to avoid error, each difference as it is used should be covered with a paper weight.*

Series for verification may be formed as follows:—

The function is

$$d_x\lambda_y v^{x+1}.$$

Hence if $x$ vary,

$$\Delta_x(d_x\lambda_y v^{x+1})=\lambda_y.\Delta(d_x v^{x+1});$$

$$\therefore \quad d_{x+1}\lambda_y v^{x+2}=d_x\lambda_y v^{x+1}+\lambda_y\Delta(d_x v^{x+1};$$

or, changing $x$ into $x-1$,

$$d_x\lambda_y v^{x+1}=d_{x-1}\lambda_y v^x+\lambda_y\Delta(d_{x-1}v^x).$$

The series thus indicated is one that, commencing in Column 0, ($x=y$,) ascends diagonally. The formation, commencing with age 85, is as follows:—

---

* A better plan, perhaps, will be to insert the differences, as in the example, between the terms from which they arise.

| $x$ | $\Delta d_x v^{x+1}$ | $\lambda_{85} d_x v^{x+1}$ |
|---|---|---|
| 85 | | 4346616 |
| 6 | 1766187 | 3489486 |
| 7 | 1455759 | 2783005 |
| 8 | 1305032 | 2149673 |
| 9 | 968150 | 1679830 |
| 90 | 691471 | 1344259 |
| 1 | 601396 | 1052401 |
| 2 | 543116 | 7888280 |
| 3 | 413909 | 5879575 |
| 4 | 373082 | 4069008 |
| 5 | 334807 | 2444194 |
| 6 | 276214 | 1103718 |
| 7 | 177749 | 241107 |
| | 49682 | 000000 |

Taking $\lambda_{85}$ for the in-factor, we have here for the initial term $\lambda_{85}\times d_{85}v^{86}$, $=4346616$; and the succeeding terms are formed by continuous multiplication by the differences of the series $d_xv^{x+1}$, the regulator being set for subtraction. Verification is had by the term beyond 97 coming out equal to 0.

We have now to attend to the construction of the values of the function for the combinations in which $x$ is equal to or less than $y$.

The function is,

$$d_x\lambda_y v^{y+1};$$

and if $x$ vary we have,

$$\Delta_x(d_x\lambda_y v^{y+1})=\lambda_y v^{y+1}.\Delta d_x.$$

Whence,

$$d_{x+1}\lambda_y v^{y+1}=d_x\lambda_y v^{y+1}+\lambda_y v^{y+1}.\Delta d_x;$$

and for the formation in reverse order,

$$d_x\lambda_y v^{y+1}=d_{x+1}\lambda_y v^{y+1}-\lambda_y v^{y+1}.\Delta d_x.$$

And changing $x$ into $x-1$,

$$d_{x-1}\lambda_y v^{y+1}=d_x\lambda_y v^{y+1}-\lambda_y v^{y+1}.\Delta d_{x-1},$$

which is the working formula.

The following is an example, showing the commencement of the process.

$x<y$

| $y$ | $\lambda_y v^{y+1}$ | $d_x$ | $y$ | Diff. 0. | Diff. 1. | Diff. 2. | Diff. 3. | Diff. 4. |
|---|---|---|---|---|---|---|---|---|
| 97 | 24841 | 9 | 97 | ·02236 | ·09936 | ·21363 | ·34529 | ·48440 |
|  |  | 31 |  |  |  |  |  |  |
| 6 | 164887 | 40 |  | ·65955 | 1·41803 | 2·29194 | 3·21531 | 4·18814 |
|  |  | 46 | 6 | ·68191 | 1·51739 | 2·50557 | 3·56060 | 4·67254 |
| 5 | 538783 | 86 |  | 4·63354 | 7·48909 | 10·50628 | 13·68510 | 17·72598 |
|  |  | 53 | 5 | 5·31545 | 9·00648 | 13·01185 | 17·24570 | 22·39852 |
| 4 | 1233551 | 139 |  | 17·14635 | 24·05423 | 31·33218 | 40·58381 | 50·32886 |
|  |  | 56 | 4 | 22·46180 | 33·06071 | 44·34403 | 57·82951 | 72·72738 |
| 3 | 2308127 | 195 |  | 45·00848 | 58·62643 | 75·93738 | 94·17158 | 114·2523 |
|  |  | 59 | 3 | 67·47028 | 91·68714 | 120·28141 | 152·00109 | 186·9797 |
| 2 | 3814032 | 254 |  | 96·87643 | 125·4817 | 155·6125 | 188·7946 | 234·5630 |
|  |  | 75 | 2 | 164·34671 | 217·1688 | 275·8939 | 340·7957 | 421·5427 |
| 1 | 5849836 | 329 |  | 192·4596 | 238·6733 | 289·5668 | 359·7649 | 452·1922 |
|  |  | 79 | 1 | 356·8063 | 455·8421 | 565·4607 | 700·5606 | 873·7349 |
| 0 | 8527116 | 408 |  | 347·9063 | 422·0922 | 524·4176 | 659·1460 | 802·4015 |
|  |  | 87 | 0 | 704·7126 | 877·9343 | 1089·8783 | 1359·7066 | 1676·1364 |
| 89 | 1194017 | 495 |  | 591·0384 | 734·3205 | 922·9751 | 1123·570 | 1358·791 |
|  |  | 120 | 89 | 1295·7510 | 1612·2548 | 2012·8534 | 2483·277 | 3034·927 |
| 8 | 1629580 | 615 |  | 1002·191 | 1259·665 | 1533·434 | 1854·461 | 2193·414 |
|  |  | 158 | 8 | 2297·942 | 2871·920 | 3546·287 | 4337·738 | 5228·341 |
| 7 | 2193321 | 773 |  | 1695·437 | 2063·915 | 2495·999 | 2952·209 | 3388·680 |
|  |  | 168 | 7 | 3993·379 | 4935·835 | 6042·286 | 7289·947 | 8617·021 |
| 6 | 2913971 | 941 |  | 2742·047 | 3316·099 | 3922·204 | 4502·085 | 5009·116 |
|  |  | 197 | 6 | 6735·426 | 8251·934 | 9964·490 | 11792·032 | 13626·137 |
| 5 | 3819522 | 1138 |  | 4346·615 | 5141·076 | 5901·161 | 6565·757 | 7192·159 |
|  |  |  | 5 | 11082·041 | 13393·010 | 15865·651 | 18357·789 | 20818·296 |
| * | * | * | * | * | * | * | * | * |

The formation here is in such entire accordance with that of $d_x\lambda_y v^{x+1}$, already given, that explanation of it would seem to be superfluous. Comparison of the two examples will show that they differ only by the interchange of $x$ and $y$, $d_x v^{x+1}$ and $\lambda_y v^{y+1}$, and $\lambda_y$ and $d_x$. The process in both is the same. The leading term of each row is the product of the values of the two functions in line with it; and the succeeding terms are produced by the continuous employment as multipliers of the series of differences in the column adjoining that headed Diff. 0.

The formation of the verification series here is also in accordance with that of the series for the verification of $d_x\lambda_y v^{x+1}$.

| $y$ | $\Delta\lambda_y v^{y+1}$ | $d_{85}\lambda_y v^{y+1}$ |
|---|---|---|
| 85 | | 4346615 |
| 6 | 905551 | 3316099 |
| 7 | 720650 | 2495999 |
| 8 | 563741 | 1854461 |
| 9 | 435563 | 1358791 |
| 90 | 341305 | 9703857 |
| 1 | 267728 | 6657112 |
| 2 | 203581 | 4340369 |
| 3 | 150590 | 2626649 |
| 4 | 107458 | 1403780 |
| 5 | 69477 | 613136 |
| 6 | 37389 | 187642 |
| 7 | 14005 | 28269 |
| | 2484 | 00000 |

The formation of the functions $d_x\lambda_y v^{x+1}$ and $d_x\lambda_y v^{y+1}$ having been completed, there remain the summations of the terms to be performed. This is rendered a very simple matter in consequence of the arrangement adopted, which is that suggested by General Hannyngton in the case of the construction of $\mathrm{D}_{x\cdot y}$. To ensure accuracy the columns should first be added in groups of ten or fifteen terms; and the sums being carried forward, will serve as checks in the final summation.

It is the results of these final summations that are tabulated, each of them having reference to the entire after-lifetime of the combination opposite which it is placed.

To suit the form of the table, it is requisite now to make a change in the nomenclature that up to this point has been employed. The symbols $x$ and $y$ have been used hitherto to denote the ages of the life assured and the life assured against, respectively. This appropriation of the symbols was convenient so long as we were concerned with merely the construction of the terms; but it is otherwise when we come to consider the arrangement which will most facilitate the use of the table. The only tables of this kind that have been published are the valuable and extensive tables of Mr. David Chisholm, founded on the Carlisle Table of Mortality; and we cannot do better than adopt the arrangement and notation of which Mr. Chisholm has set the example in his work. In the sequel, therefore, when the ages are other than equal, $x$ will stand for the older of the two, and $y$ for the younger; and the two sets of results will be arranged in parallel columns, designated respectively by the symbols $\mathrm{M}\frac{1\phantom{.y}}{x.y}$ and $\mathrm{M}\frac{\phantom{x.}1}{x.y}$, the analogy of which to those which denote the values of Survivorship

Assurances on ($x$) and ($y$) is apparent. We may also, if we please, form a column $M_{x.y}$, by adding together the corresponding terms in the two preceding columns.

The following is a specimen of the final arrangement.

Diff. 2.

| $x$ | $y$ | $M_{\overline{\overset{1}{x}.y}}$ | $M_{\overline{x.\overset{1}{y}}}$ | $M_{x.y}$ |
|---|---|---|---|---|
| * | * | * | * | * |
| 85 | 83 | 18014·27 | 15865·65 | 33879·92 |
| 86 | 84 | 11260·58 | 9964·49 | 21225·07 |
| 87 | 85 | 6878·05 | 6042·29 | 12920·34 |
| 88 | 86 | 4095·04 | 3546·29 | 7641·33 |
| 89 | 87 | 2405·82 | 2012·85 | 4418·67 |
| 90 | 88 | 1382·45 | 1089·88 | 2472·33 |
| 91 | 89 | 755·751 | 565·461 | 1321·212 |
| 92 | 90 | 385·470 | 275·894 | 661·364 |
| 93 | 91 | 181·314 | 120·281 | 301·595 |
| 94 | 92 | 73·790 | 44·344 | 118·134 |
| 95 | 93 | 23·818 | 13·012 | 36·830 |
| 96 | 94 | 5·108 | 2·506 | 7·614 |
| 97 | 95 | ·457 | ·214 | ·671 |

In vol. v of the *Journal*, pp. 107 to 118, will be found a prototype of the operation that forms the subject of the present problem.

It is in place to mention that the examples here given, like those in illustration of Problem VII. are pointed to correspond to a radix of 10,000.

I have now completed the task I proposed to myself when commencing the preparation of the present series of papers. I have, I think, shown that the Arithmometer possesses a singular adaptation to the construction of the tables required for Actuarial use. The examples I have given by no means exhaust the capabilities of the instrument in this department; but the necessity for here going further into the subject is precluded by the publication, since my papers were commenced, of Mr. Ralph P. Hardy's remarkable work, entitled *Valuation Tables, based upon the Institute of Actuaries' Mortality Experience* ($H^M$) *Table*. Mr. Hardy's work contains complete tables, at four rates of interest, of most of the functions which I have chosen for examples, and of others besides, nearly all of them having been calculated by the aid of the Arithmometer. It thus forms an extensive repertory of examples for the use of such as desire to master the working of the machine, and to elicit its further capabilities. Had the work referred to been published, or had I known that it was in contemplation, before I commenced the preparation of my papers, they would, if they had appeared at all, have assumed a different form from that which they now present. Mr. Hardy's work is one of unquestionable utility, and will form a lasting monument of the author's skill and enterprise.

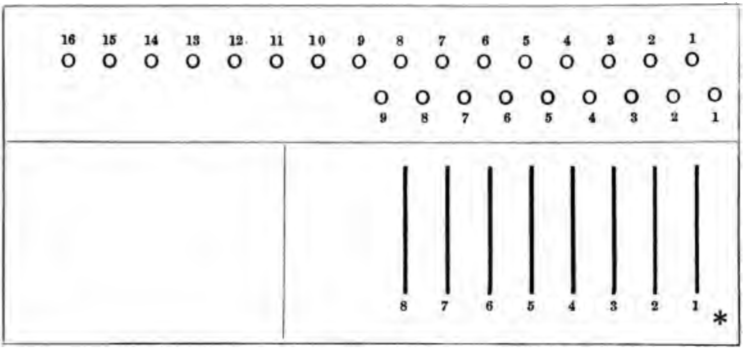
It will be sufficiently apparent, from what I have said, that I quite concur in the laudatory terms in which Mr. Hardy, in his preface, speaks of the Arithmometer. I also agree with him in what he says as to the existence of room for improvement in the strength and temper of certain of the materials employed. This refers chiefly to the springs, which form the weak point of the machine. Any improvement in form or material given to these, which should remove or diminish the tendency to give way that they sometimes exhibit, would be hailed as a boon by all users of the machine.

I hope it will not be deemed presumptuous if now, ere I close, I venture respectfully to invite the attention of the makers of the machine to two points, in regard to which I consider some modification is desirable.

The first has reference to the manner in which the result (when the process in use is multiplication) is presented. The figures composing it are seen at the bottom of a series of holes, that which I have designated $S_1$, being the upper series on the slide. These holes are of the form of truncated hemispherical cavities, a quarter of an inch deep (such being the thickness of the slide), and five-eighths of an inch in diameter; and a consequence of this form is, that in working, with the paper at the right hand, and especially if the numbers being dealt with are *long* ones, more movement of the body, and more stooping over the machine than is at all agreeable, become necessary, to avoid mistake in reading off the figures. A position and direction of the light, also, which, probably, cannot always be conveniently commanded, are required for a like reason.

The remedy for these inconveniences—for such I believe they will be admitted to be by all who have had occasion to use the machine extensively—would be, of course, a reduction in the depth of the cavities at the bottom of which the figures appear. I do not presume to suggest the manner in which this might be accomplished; I will only say that I see no insurmountable mechanical difficulty in the way.

The second point to which I refer, is the desirableness of having the openings through which the figures appear individually numbered. To render my meaning plain, I give here a plan of the machine, with the figures that I proposed inserted.



* *Note.*—The asterisk indicates the position of the handle.

I frequently feel the want of this provision in setting factors on the machine, having, in its absence, to count the holes; and also in performing multiplication from left to right.* I believe that those practically acquainted with the machine will agree with me in thinking that the additions I have suggested would facilitate its use.

* I find this the more convenient order, and I always practise it when it is *safe* to do so—that is, when the number on $S_1$ does not extend beyond the limit of the carrying power of the machine; and also even when it does so extend, provided that the leading figures change but slowly.

## *On the Solution of Problems connected with Loans repayable by Instalments.* *By* W. M. MAKEHAM, F.I.A.

THE only treatise of any value on this subject that I am acquainted with is a paper by Mr. Peter Gray, in vol. xiv of the *Journal of the Institute of Actuaries.* Considering the magnitude of the pecuniary interests frequently involved in transactions of the nature referred to, it is somewhat surprising that Mr. Gray's valuable labours have not, ere this, been followed up by contributions from other writers on the doctrine of Interest and Annuities. This, I think, is much to be regretted,—and must form my apology for the following attempt to add something to what Mr. Gray has given us.

---

To find the value of a loan repayable by instalments at stated periods of time, with interest in the meantime at the rate $i$ on the capital *receivable,* so as to yield to the purchaser a given rate of interest $i'$.

*Rule.* Find the *present value* of the capital receivable (at the given rate of $i'$) and deduct the same from the amount of the capital. Multiply the remainder by $\frac{i}{i'}$ and to the product add the present value of the capital. The result will represent the present value of the loan.

When the rate of interest allowed upon the capital receivable is the same as the rate assumed in the valuation, the present value of the loan will evidently be the capital receivable. Hence, in the case supposed, the present value of the interest is found by deducting the present value of the capital from the capital itself. And it is evident that the present value of the interest at any other rate $i$ will be in the proportion of $i$ to $i'$. Therefore, multiplying by $\frac{i}{i'}$ and adding the present value of the capital we have the present value of the loan, as stated in the rule.

The truth of this result may be demonstrated mathematically as follows:—

For every 1 of capital receivable we have:

Value of capital $=(1+i')^{-n}$. Deducting from 1 (the amount of capital) and multiplying by $\frac{i}{i'}$, we have $\frac{i}{i'}\{1-(1+i')^{-n}\}$, and adding the present value of the capital, $(1+i')^{-n}$, we have for the value of the loan $(1+i')^{-n}+i\cdot\frac{1-(1+i')^{-n}}{i'}$, which is evidently true by inspection.

The advantage of the method of solution herein proposed is the simplification of intricate cases. I give one or two examples.

*Example* 1. £125 is to be repaid as follows:—

One twenty-seventh at the expiration of 4 years,
" " " 5 "
and so on. Finally,
One twenty-seventh at the expiration of 30 years,

interest being payable in the meantime at the rate of 6 per-cent. Required the value of the loan to yield the purchaser $i'$ per £1.

The payments of capital evidently consist of an annuity of $\frac{125}{27}$ for 27 years, first payment at the end of 4 years. The value of the capital receivable is therefore:

$$\frac{125}{27}\cdot(1+i')^{-3}\cdot\frac{1-(1+i')^{-27}}{i'}.$$

Deducting from 125, the capital receivable, and multiplying by $\frac{\cdot 06}{i'}$, we have for the value of the interest,

$$125\left\{\frac{\cdot 06}{i'}-\frac{\cdot 06}{i'}\cdot\frac{(1+i')^{-3}}{27}\cdot\frac{1-(1+i')^{-27}}{i'}\right\},$$

and adding the value of the capital, as already found, we find the value of the loan under the prescribed condition to be:

$$125\left\{\frac{\cdot 06}{i'}+\left(1-\frac{\cdot 06}{i'}\right)\cdot\frac{(1+i')^{-3}}{27}\cdot\frac{1-(1+i')^{-27}}{i'}\right\},$$

a formula easily computed from the ordinary tables of interest and annuities.

*Example* 2. A loan of £10,000 is repayable (as follows) in 40 years with a premium of 25 per-cent.

£94 (*plus* 25 per-cent) first year,
102 „ second „
110 „ third „
and so on;

interest at the rate of 2·4 per-cent being payable in the meantime on the capital receivable. Required the value of the loan at 5 per-cent.

The formula for the value of the capital advanced is,

$$\overset{1}{V}_{40}\times 94+\overset{2}{V}_{40}\times 8. \quad \text{(see \textit{Journal}, vol. xiv, p. 195.)}$$

In the absence, however, of tables of $\overset{2}{V}_n$ we may use its equivalent, $\frac{1}{i'}\{\overset{1}{V}_n-n(1+i')^{-n}\}$. The formula then becomes:

$$\overset{1}{V}_{40}\left\{94+\left(40+\frac{1}{i'}\right)\times 8\right\}-\frac{40\times 8}{i'}.$$

$$\begin{array}{rl}
\frac{1}{i'}= 20 & \\
\hline
\times 8 = 160 & \\
40\times 8 = 320 & \\
94 & \\
\hline
574\times 17{\cdot}15909 & (=\overset{1}{V}_{40}) \\
\hline
8579{\cdot}545 & \\
1201{\cdot}136 & \\
68{\cdot}636 & \\
\hline
9849{\cdot}317 & \\
320\times 20 = 6400 & \\
\hline
\text{Value of capital} = 3449{\cdot}317 &
\end{array}$$

$$\begin{array}{rl}
10000{\cdot} & \\
3449{\cdot}317 & \\
\hline
6550{\cdot}683 & \times\ \frac{2{\cdot}4}{5}\,(=\cdot 48) \\
\hline
2620{\cdot}273 & \\
524{\cdot}055 & \\
\hline
3144{\cdot}328 & = \text{value of interest} \\
3449{\cdot}317 & \\
\hline
6593{\cdot}645 & \times\ 1{\cdot}25 \text{ (premium)} \\
1648{\cdot}411 & \\
\hline
8242{\cdot}056 & = \text{value of loan}
\end{array}$$

These examples will explain sufficiently the use of the method in finding the value of the loan. I proceed now to the inverse problem of determining the rate of interest corresponding to any given value.

*Rule.* Find the present value of the capital at the rate of interest allowed and deduct the same first from the capital and secondly from the purchase money. Divide the first remainder by the second and multiply by the rate of interest allowed. The result will be a first approximation to the rate of interest obtained by the purchaser.

To find a second approximation find the value of the capital at the rate obtained by the first approximation and repeat the process. In the same way, any number of successive approximations may be obtained to any requisite degree of accuracy.

The reason for this rule is shown thus:—Let C denote the capital, C′ its present value at the true rate, and A the purchase money. Then $C' + (C - C')\frac{i}{i'} = A$. Whence $i' = i \cdot \frac{C - C'}{A - C'}$.

All transactions involving the values of pecuniary benefits payable at future stated periods may be resolved into their essential elements,—viz., first, the capital advanced by the purchaser; and, secondly, the interest allowed as the consideration for the advance. Every farthing he receives over and above the capital advanced—under whatever name he receives it—forms a part of the interest realized on the investment. Thus an annuity certain of 1, the value of which at any given rate $i$ is $\frac{1-(1+i)^{-n}}{i}$, may be resolved into these two essential elements,—the $t$th payment consisting of

[1] $(1+i)^{-\overline{n-t+1}}$ capital repaid, and

[2] $1-(1+i)^{-\overline{n-t+1}}$ interest on capital outstanding.

We may therefore apply the rule for the determination of the rate of interest by successive approximations to the particular case of an ordinary annuity.

The amount of capital returned will in this case be the value of the annuity $\frac{1-(1+i)^{-n}}{i}$, which is the sum of the expression $(1+i)^{-\overline{n-t+1}}$ for successive values of $t$. The *present value* of the capital is, in like manner, the sum of the expression $(1+i)^{-\overline{n-t+1}} \times (1+i)^{-t}$, or $(1+i)^{-\overline{n+1}}$, which is evidently equal to $n(1+i)^{-\overline{n+1}}$.

Now, let us suppose that an approximate value (R) of the rate of interest has been obtained by inspection from the tables. Then in the formula above given $C=\frac{1-(1+R)^{-n}}{R}$ and $C'=n(1+R)^{-\overline{n+1}}$, and we have (A denoting the purchase money),

$$i'=R\cdot\frac{\frac{1-(1+R)^{-n}}{R}-n(1+R)^{-\overline{n+1}}}{A-n(1+R)^{-\overline{n+1}}}.$$

$$=R\left\{1+\frac{\frac{1-(1+R)^{-n}}{R}-A}{A-n(1+R)^{-\overline{n+1}}}\right\}$$

whence

$$i'-R=R\cdot\frac{\frac{1-(1+R)^{-n}}{R}-A}{A-n(1+R)^{-\overline{n+1}}},$$

which is identical with the formula usually given in works on Annuities Certain for determining the rate of interest by successive approximations. (See Baily *On the Doctrine of Interest and Annuities,* p. 129.)

It thus appears that the formula usually given for approximating to the rate of interest in annuities is only a particular case of the general formula described by the rule,—the capital in the one case being supposed to be returnable by a succession of instalments represented by the expression $(1+R)^{-\overline{n-t+1}}$, while in the other the successive instalments may be anything we please.

I now propose to investigate the same problem in a somewhat more scientific manner. Put $i'=r+h$, where $r$ is a near value of $i'$ and $h$ the difference $i'-r$. And from $C'+(C-C')\frac{i}{i'}=A$ we have $(C-C')\left(\frac{i}{i'}-1\right)=A-C$, or $\frac{C-C'}{i'}(i-i')-(A-C)=0=\phi i'$ (suppose) $=\phi(r+h)$. Expanding the last expression and solving for $h$ we have $h=-\frac{\phi r}{\phi' r}$ (nearly). Hence

$$h=\frac{\frac{C-C'}{r}(i-r)-(A-C)}{(C-C')\frac{i}{r^2}+\frac{1}{r}\frac{dC'}{dr}(i-r)}$$

$$=r\,\frac{\frac{C-C'}{r}(i-r)-(A-C)}{\frac{(C-C')}{r}\cdot(i-r)+(C-C')+\frac{dC'}{dr}(i-r)}.$$

Now, if we take $r=i$ as a first approximation to the true rate $i'$ we have

$$h=r\cdot\frac{C-A}{C-C'}=i\frac{C-A}{C-C'}.$$

Applying this formula to the case of an annuity certain, we have

$$i'-R=R\cdot\frac{\dfrac{1-(1+R)^{-n}}{R}-A}{\dfrac{1-(1+R)^{-n}}{R}-n(1+R)^{-\overline{n+1}}},$$

which will be found to agree precisely with Barrett's improvement of the formula previously given,—as described in the "Account of a correspondence between Barrett and Baily," see *Journal of the Institute of Actuaries*, vol. vi, p. 56.

The generalized (approximate) formula $h=r\cdot\frac{C-A}{C-C'}$, has been proved only in the particular case where the assumed near rate corresponds with the nominal rate allowed, and it is therefore open to question whether it can be applied to obtain more than one step further in the approximation. That it is perfectly applicable, however, to determine any number of successive approximations may be thus shown.

The expression $\frac{C-C'}{i'}$ represents the value of the interest *per each unit in the rate;* and $\frac{i'}{C-C'}$ = the rate of interest for which a payment of 1 down will provide. Hence $i'\cdot\frac{C-A}{C-C'}$ is the precise *extra* rate of interest for which the discount $C-A$ will provide,—and this extra rate added to the rate allowed ($i$) gives the true total rate $i'$. It is clear, therefore, that the nearer the assumed rate $r$ approaches to $i'$ the true rate the nearer will the expression $r\cdot\frac{C-A}{C-C'}$ approach to the true value of $h$ by which $i$ is to be increased, and this without limit.

As an illustration of the power of the method, I propose to apply it to the case given by Mr. Gray in the second part of the paper before mentioned (*Journal*, vol. xiv, p. 182).

In the Austrian Loan of 1865 the capital to be received by the purchasers of the Bonds amounted to £14,583,675·9—payable by 74 equal half-yearly instalments of £197070·8 each, commencing at the expiration of 2½ years,—with an additional payment of £436·7 at the termination of the period to make up the required amount. Interest at the rate of 2·4979 per-cent half-yearly being payable during the currency of the loan. The consideration paid by the lenders was £9,904,760.

The value of the capital in this case is

$$a_{\overline{74}|} \times v^4 \times 19070{\cdot}8 + v^{78} \times 436{\cdot}7.$$

Following Mr. Gray's example, I take for my first trial rate 4 per-cent. Dividing by 100, to save unnecessary figures, the calculation of the expression $r \cdot \dfrac{C-\dot{A}}{C-C'}$ is as follows:

$$\begin{array}{lrrll}
1{\cdot}37343 & 145837 & 46789 & 4{\cdot}67014 & \underline{{\cdot}44128 \times {\cdot}04} \\
\overline{1}{\cdot}93187 & 99048 & 106035 & \underline{5{\cdot}02543} & \\
\underline{3{\cdot}29462} & 39802 & & & {\cdot}01765 \\
 & & & \overline{1}{\cdot}64471 & \underline{{\cdot}02498} \\
\underline{\underline{4{\cdot}59992}} = \log C'^{*} & & & & \\
 & & & & \underline{\underline{{\cdot}04263}} = \text{1st Appn.}
\end{array}$$

From this it appears that our trial rate is too low,—and that the true rate is something over 4¼ per-cent. I therefore repeat the calculation, substituting the latter rate in lieu of the former:

$$\begin{array}{lrrll}
1{\cdot}35118 & 145837 & 108384 & 4{\cdot}67014 & \underline{{\cdot}43171 \times {\cdot}0425} \\
\overline{1}{\cdot}92769 & 37453 & & \underline{5{\cdot}03495} & \\
\underline{3{\cdot}29462} & & & & {\cdot}01727 \\
 & & & \overline{1}{\cdot}63519 & 108 \\
\underline{\underline{4{\cdot}57349}} = \log C' & & & & \underline{{\cdot}02498} \\
 & & & & \underline{\underline{{\cdot}04333}} = \text{2nd Appn.}
\end{array}$$

Having found two approximations we may proceed to find a third as follows:

$$\begin{array}{lccc}
 & 4{\cdot} & \text{gives} & 4{\cdot}263 \\
 & \underline{4{\cdot}25} & \underline{\quad,,\quad} & \underline{4{\cdot}333} \\
\text{Differences} & {\cdot}25 & & {\cdot}07
\end{array}$$

Hence $4+x$ gives $4{\cdot}263 + \dfrac{{\cdot}07}{{\cdot}25} \cdot x$ nearly; and equating these two values and solving for $x$ we have

$$x = \frac{{\cdot}263 \times {\cdot}25}{{\cdot}25 - {\cdot}07} = {\cdot}365$$

whence $4+x = 4{\cdot}365$, which is true to the last figure.

By Mr. Gray's process, with the use of two trial rates, the approximate value deduced is 4·373 (see *Journal*, vol. xiv, p. 186).

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[*] It is evident by inspection of the table that $v^{78} \times 4{\cdot}367$, the value of the final extra payment, has no significant effect in the value of C′.

Hence not only is the calculation greatly simplified and abridged by the method I propose, but a greater degree of accuracy is apparently attained.

The preceding examples are all taken from cases occurring in actual practice. I propose now to give solutions of the problems in a more general or elementary form.

C being, as before, the amount of the capital *to be received,* C′ its present value, A the purchase money or capital *advanced,* $i$ the nominal and $i'$ the true rate of interest allowed, we have seen that

$$A=C'+\frac{C-C'}{i'}\cdot i \quad . \; . \; . \; . \; . \; . \; . \quad [1]$$

or

$$=\frac{Ci+C'(i'-i)}{i'} \quad . \; . \; . \; . \; . \; . \; . \quad [2]$$

$$=C-(C-C')\left(1-\frac{i}{i'}\right) \quad . \; . \; . \; . \quad [3]$$

If the capital to be repaid be taken as unity, the three formulas for the value of A become

$$A=C'+\frac{1-C'}{i'}i \quad . \; . \; . \; . \; . \; . \; . \quad [1]$$

$$=\frac{i+C'(i'-i)}{i'} \quad . \; . \; . \; . \; . \; . \; . \quad [2]$$

$$=1-(1-C')\left(1-\frac{i}{i'}\right) \quad . \; . \; . \; . \quad [3]$$

The simplest and most usual cases are:—First, where the capital is repaid in one sum; and, second, where it is repaid in equal yearly instalments spread over the whole term of the loan. I will take these two cases in the order indicated.

Problem I.—A bond securing an advance of 1 payable in $n$ years, with interest in the meantime at the rate $i$, is to be sold to yield the purchaser interest at the rate $i'$. Required the price to be paid for the bond.

Here $C'=(1+i')^{-n}$. Hence the value of the bond is

$$(1+i')^{-n}+\frac{1-(1+i')^{-n}}{i'}\cdot i \quad . \; . \; . \quad [1]$$

$$=\frac{i+(1+i')^{-n}(i'-i)}{i'} \quad . \; . \; . \; . \; . \; . \quad [2]$$

$$=1-\frac{1-(1+i')^{-n}}{i'}(i'-i) \quad . \; . \; . \; . \quad [3]$$

PROBLEM II.—A bond similar to the last is to be repaid by equal annual instalments of $\frac{1}{n}$. Required the price to be paid to yield the purchaser $i'$.

Here $C'=\frac{1-(1+i')^{-n}}{ni'}=\frac{a_{\overline{n}|}}{n}$. Hence the value required is

$$\frac{a_{\overline{n}|}}{n}+\frac{1-\frac{a_{\overline{n}|}}{n}}{i'}\cdot i \quad . \quad . \quad . \quad . \quad . \quad [1]$$

$$=\frac{i+\frac{a_{\overline{n}|}}{n}(i-i)}{i'} \quad . \quad . \quad . \quad . \quad . \quad . \quad [2]$$

$$=1-\frac{1-\frac{a_{\overline{n}|}}{n}}{i'}(i'-i) \quad . \quad . \quad . \quad . \quad . \quad [3]$$

---

The general formula $1-(1-C')\left(1-\frac{i}{i'}\right)$ affords the simplest means of computing the value of the loan, by first determining the present value of the capital and then substituting such value in the formula. When the several payments of capital are made yearly, and are so regulated as to form a series with differences vanishing after a certain limited number of orders, the formula for finding the present value of the capital is

$$C'=\overset{1}{V}_n.u+\overset{2}{V}_n.\Delta u+\overset{3}{V}_n.\Delta^2 u + \ldots\ldots,$$

where $u$, $\Delta u$, &c., denote the first terms of the given series and its differences, and where

$$\overset{1}{V}_n=\frac{1-(1+i')^{-n}}{i'}$$

$$\overset{2}{V}_n=\frac{\overset{1}{V}_n-n(1+i')^{-n}}{i'}$$

$$\overset{3}{V}_n=\frac{\overset{2}{V}_n-n\frac{n-1}{2}(1+i')^{-n}}{i'}$$

and so on. See *Journal*, vol. xiv, p. 189, "On the Theory of Annuities Certain." Owing to the simplification introduced by the method herein explained as compared with the less artificial process adopted in the paper just quoted, it has been necessary to make but little use of the above formula for C′ in the preceding examples. In more complicated cases, however, the formula in question will be found of considerable utility, as I have shown in the paper last mentioned.

The effect of limiting the process of valuation to the payments of capital, which is the essential feature of the preceding method, is (generally) to reduce the series to be valued by one order of differences. Thus, suppose the capital (1) to be repayable in one sum at the expiration of $n$ years, the interest will be represented by a series of $n$ yearly payments of $i$ each. Again, if the capital is repayable by $n$ equal instalments of $\frac{1}{n}$ each, the series representing the interest will be

$$i, \quad \frac{n-1}{n}i, \quad \frac{n-2}{n}i, \ldots .$$

which has one order of differences, consisting of the constant term $-\frac{i}{n}$.

As a further illustration of the method proposed, I will take the last mentioned case, and value the loan by the direct process of determining the values of the capital and interest independently. Thus

$$\text{Value of capital} = \frac{1}{n}\overset{1}{V}_n$$

$$\text{Value of interest} = i\overset{1}{V}_n - \frac{\overset{2}{V}_n i}{n}$$

$$\text{Value of loan} = \overset{1}{V}_n\left(\frac{1}{n}+i\right) - \frac{\overset{2}{V}_n i}{n} \quad \ldots \ldots \quad (1)$$

Now the value of the loan by the formula given in this paper is

$$1-\left(1-\frac{\overset{1}{V}_n}{n}\right)\left(1-\frac{i}{i'}\right) \quad \ldots \ldots \quad (2)$$

where the function $\overset{2}{V}_n$, which is the coefficient of the first difference is eliminated. Of course the two formulas are in reality identical, as may be shown by substituting in (1) the value of $\overset{2}{V}_n$ in terms of $\overset{1}{V}_n$. Thus

$$\text{Value of loan} = \overset{1}{V}_n\left(\frac{1}{n}+i\right)-\frac{\overset{2}{V}_n i}{n}$$

$$=\overset{1}{V}_n\left(\frac{1}{n}+i\right)-\left(\frac{\overset{1}{V}_n}{i'}-\frac{n(1+i')^{-n}}{i'}\right)\cdot\frac{i}{n}$$

$$=\overset{1}{V}_n\left(\frac{1}{n}+i-\frac{i}{ni'}\right)+\frac{(1+i')^{-n}}{i}\cdot i$$

$$=\overset{1}{V}_n\left(\frac{1}{n}+i-\frac{i}{ni'}\right)-\overset{1}{V}_n i+\frac{i}{i'}$$

$$=\overset{1}{V}_{\frac{n}{n}}\left(1-\frac{i}{i'}\right)+\frac{i}{i'}$$

$$=1-\left(1-\frac{\overset{1}{V}_n}{n}\right)\left(1-\frac{i}{i'}\right).$$

Instead, however, of being repayable by a series of instalments regulated in accordance with a definite mathematical law, the payments of capital might be fixed by a purely arbitrary arrangement both as regards *time* and *amount*. The general formula for finding the value of the capital in this case will be:

$$C'=c_{n_1}(1+i')^{-n_1}+c_{n_2}(1+i')^{-n_2}+c_{n_3}(1+i')^{-n_3}+\ldots.$$

where $c_{n_1}$, $c_{n_2}$, $c_{n_3}$, &c., represent the instalments of capital payable at the respective periods $n_1$, $n_2$, $n_3$, &c. Calculating the value of C′, and substituting in the general formula, we obtain at once the value of the loan. But by the direct process of finding the values of the capital and interest separately, after determining the value of the preceding series, we should then have to calculate the value of a still more complicated one for the interest. This latter series would consist of the following terms:—

First, the value of $i$ per annum for $n_1$ years.

Second, the value of $i(1-c_{n_1})$ per annum for $n_2$ years, minus its value for $n_1$ years.

Third, the value of $i(1-c_{n_1}-c_{n_2})$ per annum for $n_3$ years, minus its value for $n_2$ years.

and so on. The trouble involved in this calculation would evidently in intricate cases be a considerable addition to the labour requisite for the solution of the problem.

As this case is of the utmost possible generality, it may be worth while to give, analytically, the reduction of the last mentioned formula of solution to the general formula $1-(1-C')\left(1-\frac{i}{i'}\right)$. The formula to be reduced is:

$$\mathrm{C}'+i\{\overset{1}{\mathrm{V}}_{n_1}+(\overset{1}{\mathrm{V}}_{n_2}-\overset{1}{\mathrm{V}}_{n_1})(1-c_{n_1})+(\overset{1}{\mathrm{V}}_{n_3}-\overset{1}{\mathrm{V}}_{n_2})(1-c_{n_1}-c_{n_2})+\ldots\ldots$$
$$+(\overset{1}{\mathrm{V}}_{n_{n-1}}-\overset{1}{\mathrm{V}}_{n_n})(1-c_{n_1}-c_{n_2}-\ldots\ldots-c_{n_n})\}.$$

Observing that the last term vanishes by reason of the equation

$$c_{n_1}+c_{n_2}+c_{n_3}+\ldots\ldots+c_{n_n}=1$$

the quantity within the brackets { } reduces to

$$\overset{1}{\mathrm{V}}_{n_1}c_{n_1}+\overset{1}{\mathrm{V}}_{n_2}c_{n_2}+\overset{1}{\mathrm{V}}_{n_3}c_{n_3}+\ldots\ldots+\overset{1}{\mathrm{V}}_{n_n}c_{n_n}$$
$$=\frac{1}{i'}\{(1-\overline{1+i'}^{-n_1})c_{n_1}+(1-\overline{1+i'}^{-n_2})c_{n_2}+\ldots\ldots+(1-\overline{1+i'}^{-n_n})c_{n_n}\}$$
$$=\frac{1}{i'}\{1-(c_{n_1}\overline{1+i'}^{-n_1}+c_{n_2}\overline{1+i'}^{-n_2}+c_{n_3}\overline{1+i'}^{-n_3}+\ldots\ldots+c_{n_n}\overline{1+i'}^{-n_n})\}$$
$$=\frac{1}{i'}(1-\mathrm{C}').$$

Hence the value of the loan becomes $\mathrm{C}'+\frac{i}{i'}(1-\mathrm{C}')$, and, finally, $1-(1-\mathrm{C}')\left(1-\frac{i}{i'}\right)$, the form required.

## HOME AND FOREIGN INTELLIGENCE.

### THE CLERGY MUTUAL ASSURANCE SOCIETY.

*Established* 1829.

EXTRACT FROM THE REPORT OF THE DIRECTORS FOR THE YEAR ENDING 31 MAY 1871.

*Comparative Statement of Claims, &c., &c., &c., during the last Five Years.*

| In Years ending 31st May. | Number of Life Assurance Members who died. | Number of Policies held by them. | Amount of Claims upon Death on account of such Members. | Total amount of existing Assurances. |
|---|---|---|---|---|
| | | | £ | £ |
| 1867 | 38 | 55 | 48,538 | 4,125,543 |
| 1868 | 51 | 63 | 56,984 | 4,293,610 |
| 1869 | 52 | 69 | 63,917 | 4,473,023 |
| 1870 | 46 | 58 | 56,250 | 4,689,458 |
| 1871 | 71 | 111 | 73,242 | 4,912,205 |

*Comparative Statement of Business done in the last Five Years.*

| In Years ending 31st May. | Life Policies issued. | Amount Assured by such Policies. | Proposals for Life Assurances declined. | Amount of such Proposals declined. |
|---|---|---|---|---|
| | | £ | | £ |
| 1867 | 266 | 208,395 | 17 | 18,500 |
| 1868 | 277 | 243,220 | 17 | 17,500 |
| 1869 | 315 | 274,540 | 13 | 7,500 |
| 1870 | 367 | 308,940 | 13 | 8,000 |
| 1871 | 426 | 353,345 | 28 | 21,270 |

The Eighth Quinquennial Bonus being due to the Members on the 1st of June last, a stringent valuation of the liabilities under all the Society's Policies has been made. The rate of interest used in the calculations has been three per cent., and in no case have the future profits of the Society been in any way anticipated. The result of the valuation shows a surplus of assets over liabilities of £325,453 15*s.* 5*d.*

Of this amount the Directors, having regard to the advanced age of some of the Members, and to the low rate of mortality hitherto experienced by the Society, have decided, on the advice of their Actuary and consulting Actuary, to reserve £45,453 15*s.* 5*d.*, and under Rule 31 to divide £280,000, a sum equal to 55 per cent. of the total premium income from assurances upon life for the past five years, being the largest Bonus declared since the commencement of the Society.

*Comparative Statement of Business done by the Society in the past Twenty Years, given in periods of Five Years:—*

| In the five years ending on 31st May. | No. of Life Policies issued. | Amount Assured. |
|---|---|---|
| | | £ |
| 1856 | 1,058 | 926,689 |
| 1861 | 1,132 | 909,645 |
| 1866 | 1,309 | 1,070,025 |
| 1871 | 1,651 | 1,388,440 |

*Comparative Statement of Claims by death paid for the past Twenty Years, given in periods of Five Years:—*

| In the five years ending on 31st May. | No. of Members who died. | No. of Policies held by them. | Amount paid. |
|---|---|---|---|
| | | | £ |
| 1856 | 107 | 137 | 125,434 |
| 1861 | 119 | 152 | 139,061 |
| 1866 | 187 | 235 | 210,807 |
| 1871 | 258 | 356 | 298,931 |

*Comparative Statement of the position of the Society as at 1st June 1856, and at the end of three successive periods of Five Years: —*

| | No. of Life Policies in existence. | Total amount of existing Assurances upon Life. | Total Annual Income. | | | Total Funds. | | |
|---|---|---|---|---|---|---|---|---|
| | | £ | £ | *s.* | *d.* | £ | *s.* | *d.* |
| 1 June, 1856 | 2,802 | 2,536,705 | 100,048 | 18 | 9 | 705,680 | 16 | 1 |
| 1 June, 1861 | 3,652 | 3,185,392 | 130,336 | 1 | 5 | 1,042,663 | 12 | 6 |
| 1 June, 1866 | 4,616 | 3,954,124 | 162,157 | 12 | 3 | 1,431,576 | 12 | 11 |
| 1 June, 1871 | 5,609 | 4,912,205 | 198,314 | 2 | 6 | 1,891,915 | 12 | 7 |

### No. 1.

Table showing what total "addition" would have been made to Policies of £1,000 each by means of Bonus up to 1st June 1871, inclusive, upon the supposition that *no reduction* of Annual Premium had been made by Bonus, but that the original Annual Premium had continuously been paid, and still remained payable, and that Bonus had in no case been paid in cash.

| Age at date of making the Assurance. | Number of years in which the Assurance has been in force. | | | | | |
|---|---|---|---|---|---|---|
| | Thirty-five. | Thirty. | Twenty-five. | Twenty. | Fifteen. | Ten. |
| | £ | £ | £ | £ | £ | £ |
| 25 | 523 | 408 | 329 | 242 | 186 | 118 |
| 30 | 600 | 461 | 346 | 260 | 201 | 130 |
| 35 | 716 | 531 | 393 | 270 | 215 | 139 |
| 40 | 885 | 650 | 458 | 311 | 229 | 147 |
| 45 | 1,133 | 797 | 578 | 371 | 259 | 157 |
| 50 | 1,510 | 1,038 | 716 | 487 | 316 | 183 |
| 55 | | 1,406 | 949 | 616 | 414 | 233 |
| 60 | | | 1,301 | 831 | 543 | 315 |
| 65 | | | | 1,154 | 733 | 408 |

For additions made to Policies in force 5 years, see Table No. 3, Col. 4.

### No. 2.

Table showing, in cases of Bonus being applied in *reduction* of Annual Premiums, *the probable number of years* to be passed through, according to past experience, before such Annual Premiums (*a gradual reduction* of which is quinquennially going on) will be finally extinguished, and addition will begin to be made to the original sum assured.

| Age at the date of making the Assurance. | Probable number of years in which the Premium will be extinguished, according to past experience, when additions will begin to be made to the Sum Assured. | Age at the date of making the Assurance. | Probable number of years in which the Premium will be extinguished, according to past experience, when additions will begin to be made to the Sum Assured. |
|---|---|---|---|
| 25 | 35 | 50 | 21 |
| 30 | 32 | 55 | 18 |
| 35 | 29 | 60 | 15 |
| 40 | 26 | 65 | 12 |
| 45 | 23 | 70 | 9 |

It is to be observed that the "probable number of years" it takes, as above, to extinguish the whole of the Annual Premium may be found to be given slightly in excess.

Note must also be taken that the above Table is based on the assumption that the end of the "probable number of years" will fall in a Bonus year, for, if not, the trifling sum paid since the previous Bonus will continue to be paid until the next Bonus, in which case, however, the addition to the sum assured at the next Bonus will be proportionately larger than it otherwise would have been.

### No. 3.

*TABLE showing, in a line with Ages in Column 1, the result of the single Bonus of 1871, arising from surplus capital accumulated between the years 1866 and 1871, in respect of Policies of £1,000 each, whether appropriated to "*Reduction of Annual Premium,*" (for which see Column 3,) or to "*Addition to the Sum Assured,*" (for which see Column 4,) or to a "*Payment in Cash,*" (for which see Column 5).

| 1 Age on 1st June, 1871. | 2 Sum Assured. | 3 Reduction of Premium. | | | 4 Addition to Sum Assured. | 5 Immediate Cash Payment. | | |
|---|---|---|---|---|---|---|---|---|
| | £ | £ | *s.* | *d.* | £ | £ | *s.* | *d.* |
| 30 | 1,000 | 1 | 6 | 3 | 57 | 25 | 7 | 11 |
| 35 | " | 1 | 13 | 8 | 64 | 30 | 9 | 6 |
| 40 | " | 2 | 2 | 6 | 69 | 35 | 15 | 10 |
| 45 | " | 2 | 13 | 1 | 73 | 41 | 0 | 1 |
| 50 | " | 3 | 7 | 3 | 77 | 46 | 13 | 1 |
| 55 | " | 4 | 11 | 5 | 85 | 55 | 11 | 5 |
| 60 | " | 6 | 18 | 9 | 102 | 71 | 18 | 2 |
| 65 | " | 11 | 18 | 2 | 137 | 103 | 10 | 5 |
| 70 | " | 20 | 2 | 8 | 178 | 142 | 15 | 10 |
| 75 | " | 35 | 18 | 8 | 239 | 201 | 17 | 0 |
| 80 | " | 67 | 7 | 7 | 328 | 289 | 8 | 7 |
| 85 | " | 134 | 6 | 1 | 475 | 423 | 2 | 8 |

* NOTE.—This Table has respect to all Policies upon which not less than Five Annual Premiums have been paid, and the Bonus is altogether independent of any previous Bonus or Bonuses.

### No. 4.

TABLE showing the amount of Surplus divided amongst the Members at each declaration of Bonus since the commencement of the Society:—

| Bonus year. | Divisible Surplus. |
|---|---|
| | £ |
| 1836 | 3,192 |
| 1841 | 13,089 |
| 1846 | 39,500 |
| 1851 | 79,844 |
| 1856 | 74,900 |
| 1861 | 148,300 |
| 1866 | 200,500 |
| 1871 | 280,000 |
| Total .. | £839,325 |

### *Balance Sheet on the* 31*st May* 1871.

**LIABILITIES.**

| | £ | *s.* | *d.* |
|---|---|---|---|
| Guarantee Fund (being old Subscriptions held in reserve) . . . . . . . . . . | 5,182 | 18 | 4 |
| Life Assurance Fund . . . . . . . | 1,885,154 | 1 | 6 |
| Sickness Assurance Fund . . . . . . . . | 1,578 | 12 | 9 |
| | 1,891,915 | 12 | 7 |
| Claims admitted but not paid . . . . . . | 18,958 | 0 | 0 |
| | £1,910,873 | 12 | 7 |

## ASSETS.

| | | | | £ | s. | d. |
|---|---|---|---|---|---|---|
| Mortgages on property, within the United Kingdom:— | | | | | | |
| Freehold Estates | £967,391 | 18 | 4 | | | |
| Long Leaseholds in the City of London | £39,000 | 0 | 0 | | | |
| | | | | 1,006,391 | 18 | 4 |
| Loans on the Society's Policies | | | | 131,657 | 0 | 0 |
| Do. of one-fifth Premiums | | | | 11,986 | 7 | 3 |
| INVESTMENTS:— | | | | | | |
| In British Government Securities | | | | 37,147 | 9 | 6 |
| Indian and Colonial Government Securities | | | | Nil. | | |
| Foreign Government Securities | | | | Nil. | | |
| Railway Debentures and Debenture Stock | | | | 90,000 | 0 | 0 |
| Railway Shares (Preference and Ordinary) | | | | Nil. | | |
| House Property | | | | Nil. | | |
| Other Investments—Gaslight and Coke Company's Mortgage Bonds | | | | 82,500 | 0 | 0 |
| City of London Bonds | | | | 10,000 | 0 | 0 |
| On security of Parochial and other Rates under Acts of Parliament | | | | 418,512 | 12 | 6 |
| Life Interests with Policies and Reversions secured by Real Estate and Funded Property | | | | 86,948 | 15 | 8 |
| Purchase of an Annuity | | | | 108 | 15 | 0 |
| Loans upon Personal Security | | | | Nil. | | |
| Agents' Balances | | | | Nil. | | |
| Outstanding Premiums | | | | 6,805 | 6 | 3 |
| Outstanding Interest | | | | 7,141 | 12 | 6 |
| CASH:— | | | | | | |
| On deposit | 15,000 | 0 | 0 | | | |
| In hand and on current account | 6,648 | 13 | 1 | | | |
| | | | | 21,648 | 13 | 1 |
| Other Assets.—Policy Stamps in hand | | | | 25 | 2 | 6 |
| | | | | £1,910,873 | 12 | 7 |

*The following additional particulars are taken from the Returns under the Fifth and Sixth Schedules of the Life Assurance Companies Act,* 1870.

The principles to be adopted in the valuation and in the distribution of Profits are, by the 31st rule of the Society, fixed by the Board of Directors acting under the advice of the Actuary or Consulting Actuary and approved by the Council of Reference.

Each Policy is valued separately. Policies for the whole of Life are valued by a prepared Table at the ages of entry for the number of years and months they have been in force. Reductions made in the premiums, and additions made to the sums assured by previous Bonuses, are valued at the ages attained by the members at the date of the valuation.

The Profits are distributed amongst the members entitled to participate in the following proportion:—

(*a.*) As regards Policies which were in force at the date of the last valuation.

The difference between the annual premiums which would be charged by the Society at the ages attained at the date of the last valuation, accumulated for five years at 3 per cent. compound interest, and the sums that would be reserved at the same rate of interest, and for the same period as the liabilities of such Assurances.

(*b.*) As regards Policies which were effected since the date of the last valuation.

The difference between the number of annual premiums paid accumulated at 3 per cent. compound interest, and the sums reserved at the same rate of interest as the liabilities of such Assurances.

The amounts so apportioned are either paid in cash, or appropriated to the reduction of the annual premium, or to the increase of the sum assured, according to the option exercised by the Policy holder at the time of making the Assurance.

The table or tables of mortality used in the valuation were

(*a.*) For assurances for the whole term of life,—a special Table derived from the annual premiums charged by the Society, such premiums being based upon the Carlisle Table of Mortality with a per-centage added for expenses of management.

(*b.*) For the other Classes in which the contingency of Life is involved,—the Carlisle Table of Mortality.

The rate of interest assumed in the calculations was three per cent.

Provision is made for future expenses and profits in the amount set aside as the reserved value of the Policies; the reserve being such a sum as, with the value of the annual premiums *payable* by the members, is equal to the single premiums which the Society would charge to assure the lives of the members *at their respective ages* at the date of the valuation.

Policies share in the profits which are in force at the date of the valuation.

The total amount of profit made for the five years ending with 31st May 1871 was £304,108 14*s.* 7*d.*, the surplus of £326,532 0*s.* 2*d.* including the sum of £21,423 5*s.* 7*d.*, the amount reserved at the Bonus of 1866; from this surplus, however, must be deducted £6,261 3*s.* 1*d.*, as explained in the Note to the Valuation Balance Sheet.

The amount of profit divided was £280,000. The number of Policies which participated was 5,561; and the original amounts of such Policies, £4,816,689 and £22,565, together £4,839,254.

Specimens of Bonuses, together with the amounts apportioned under the various modes in which the same might be received will be found in the following Table:—

| Number of years in force. | AGE AT ENTRY. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 20 | | | 30 | | | 40 | | | 50 | | |
| | Reduction of Premium. | Addition. | Cash. | Reduction of Premium. | Addition. | Cash. | Reduction of Premium. | Addition. | Cash. | Reduction of Premium. | Addition. | Cash |
| 5 | 0 2 1 | 5 0 0 | 2 3 8 | 0 3 4 | 6 0 0 | 3 0 11 | 0 5 3 | 7 0 0 | 4 2 0 | 0 9 2 | 8 0 0 | 5 11 |
| 10 | 0 2 7 | 6 0 0 | 2 10 9 | 0 4 3 | 7 0 0 | 3 11 7 | 0 6 9 | 8 0 0 | 4 13 3 | 0 13 10 | 10 0 0 | 7 3 |
| 15 | 0 3 4 | 6 0 0 | 3 0 11 | 0 5 3 | 7 0 0 | 4 2 0 | 0 9 2 | 8 0 0 | 5 11 2 | 1 3 10 | 14 0 0 | 10 7 |
| 20 | 0 4 3 | 7 0 0 | 3 11 7 | 0 6 9 | 8 0 0 | 4 13 3 | 0 13 10 | 10 0 0 | 7 3 10 | 2 0 3 | 18 0 0 | 14 5 |
| 25 | 0 5 3 | 7 0 0 | 4 2 0 | 0 9 2 | 8 0 0 | 5 11 2 | 1 3 10 | 14 0 0 | 10 7 0 | 3 11 10 | 24 0 0 | 20 3 |
| 30 | 0 6 9 | 8 0 0 | 4 13 3 | 0 13 10 | 10 0 0 | 7 3 10 | 2 0 3 | 18 0 0 | 14 5 7 | 6 14 9 | 33 0 0 | 28 18 |
| 35 | 0 9 2 | 8 0 0 | 5 11 2 | 1 3 10 | 14 0 0 | 10 7 0 | 3 11 10 | 24 0 0 | 20 3 8 | 13 8 7 | 48 0 0 | 43 6 |

### *Consolidated Revenue Account for Five Years, commencing 1st June* 1866, *and ending* 31*st May* 1871.

| | £ | s. | d. |
|---|---|---|---|
| Amount of Funds on 1st June 1866, the beginning of the period | 1,422,342 | 12 | 2 |
| Premiums | 533,493 | 14 | 9 |
| Consideration for Annuities granted | Nil. | | |
| Interest and Dividends | 332,746 | 4 | 4 |
| Other Receipts.—Fines on Renewal Premiums | 665 | 0 | 0 |
| | £2,289,247 | 11 | 3 |

| | £ | s. | d. |
|---|---|---|---|
| Claims under Policies | 319,289 | 9 | 9 |
| Surrenders | 19,348 | 12 | 1 |
| Annuities | 11,472 | 0 | 0 |
| Commission | Nil. | | |
| Expenses of Management | 43,191 | 14 | 1 |
| Other payments.— | | | |
| Bonuses paid in Cash | 10,791 | 13 | 10 |
| Amount of Funds on the 31st May 1871, the end of the period, as per Third Schedule | 1,885,154 | 1 | 6 |
| | £2,289,247 | 11 | 3 |

### *Summary and Valuation of the Policies as at* 31*st May* 1871 (see p. 150.)

### *Valuation Balance Sheet as at* 31*st May* 1871.

| Dr. | £ | s. | d. |
|---|---|---|---|
| To net Liability under Assurance and Annuity transactions (as per Summary Statement) | 1,558,622 | 1 | 4 |
| To Surplus | 326,532 | 0 | 2 |
| | £1,885,154 | 1 | 6 |

| Cr. | £ | s. | d. |
|---|---|---|---|
| By Life Assurance and Annuity Funds (as per Balance Sheet) | 1,885,154 | 1 | 6 |
| | £1,885,154 | 1 | 6 |

Note.—From the Surplus of £326,532 0*s*. 2*d*. must be deducted £6,261 3*s*. 1*d*. the difference between the Liability under 162 Sickness Assurance Policies, estimated at £7,839 15*s*. 10*d*. and the amount of the Fund, viz. £1,578 12*s*. 9*d*. as shown in Schedule 3, Account No. 2.

| | £ | s. | d. |
|---|---|---|---|
| On 31st May 1867 the average rate of interest was | 4 | 4 | 3 |
| 1868 ,, ,, | 4 | 4 | 10 |
| 1869 ,, ,, | 4 | 5 | 0 |
| 1870 ,, ,, | 4 | 4 | 10 |
| 1871 ,, ,, | 4 | 5 | 3 |

### *Summary and Valuation of the Policies as at* 31*st May* 1871.

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | VALUATION. Interest 3 per cent. | | |
|---|---|---|---|---|---|---|
| | Number of Policies. | Sums Assured and Bonuses. | Office Yearly Premiums. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Liability. |
| | | £ | £ s. d. | | | £ |
| ASSURANCES. | | | | | | |
| I. With participation in Profits. | | | | | | |
| For the whole term of Life . . . . | 5,526 | 4,856,739 | 117,057 15 8 | Not ascertained | separately. | 1,490,125 |
| Other Classes: | | | | | | |
| Commuted Premiums and Premiums in a limited number of Payments not yet paid up . . . . . . . | 35 | 25,096 | 243 10 8 | ... | ... | 15,633 |
| Extra Premiums payable . . . . . | ... | ... | 798 10 3 | ... | ... | ... |
| Total Assurances with Profits . . | 5,561 | 4,881,835 | 118,099 16 7 | ... | ... | 1,505,758 |
| II. Without participation in Profits. | | | | | | |
| For the whole term of Life . . . . | Nil. | ... | ... | ... | ... | ... |
| Other Classes: | | | | | | |
| Commuted Premium . . . . . | 1 | 400 | ... | ... | ... | 204 |
| On the first of two Lives . . . . | 2 | 650 | 37 7 7 | ... | ... | 64 |
| On the last of two Lives . . . . | 6 | 9,300 | 229 6 1 | ... | ... | 1,172 |
| Temporary . . . . . . . | 28 | 11,170 | 194 9 10 | ... | ... | 104 |
| Contingent . . . . . . . | 9 | 8,850 | 171 0 4 | ... | ... | 420 |
| Endowments . . . . . . . | 172 | 43,900 | 2,063 11 11 | ... | ... | 24,971 |
| Extra Premiums payable . . . . . | ... | ... | 14 10 0 | ... | ... | ... |
| Total Assurances without Profits . | 218 | 74,270 | 2,710 5 9 | ... | ... | 26,935 |
| Total Assurances (I. II.) . . . . | 5,779 | 4,956,105 | 120,810 2 4 | ... | ... | 1,532,693 |
| ANNUITIES. | | Per Annum. | | | | |
| Deferred, but now become Absolute for Life | 35 | 1,206 | ... | ... | ... | 10,415 |
| Ditto temporary, but now become Absolute for a Term of Years . . . . . | 29 | 1,150 | ... | ... | ... | 4,587 |
| Other Classes: | | | | | | |
| Deferred . . . . . . . . | 20 | 970 | 77 4 4 | ... | ... | 5,357 |
| Deferred temporary . . . . . | 35 | 1,490 | 645 4 9 | ... | ... | 5,537 |
| Contingent . . . . . . . | 5 | 160 | 32 8 10 | ... | ... | 33 |
| Total Annuities . . . . . . | 124 | 4976 | 754 17 11 | ... | ... | 25,929 |
| Total of Results . . . . . | 5,903 | ... | 121,565 0 3 | ... | ... | 1,558,622 |

Note.—The extra Premiums consist of £437 15s. and £14 10s.—together £452 5s. 0d. for Residence or Travel beyond the limits permitted by the Society, and are for the most part of short duration, and of £360 15s. 9d. for Assurances upon Lives which are considered not to be fully up to the average standard. Both sums are entirely

Specimens of Surrender Values of Policies for £100 for the whole of life, with Profits, will be found in the following Table:—

| Age at Entry. | At the End of | | | | | |
|---|---|---|---|---|---|---|
| | 5 Years. | 10 Years. | 15 Years. | 20 Years. | 25 Years. | 30 Years. |
| | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* |
| 20 | 3 15 5 | 7 16 5 | 12 1 8 | 16 13 5 | 21 15 11 | 27 11 9 |
| 30 | 4 14 6 | 9 16 2 | 15 9 10 | 21 18 1 | 29 0 1 | 36 6 4 |
| 40 | 6 9 6 | 13 15 9 | 21 17 6 | 30 4 3 | 38 2 2 | 45 15 11 |
| 50 | 9 15 6 | 19 16 11 | 29 7 9 | 38 13 6 | 47 5 9 | 54 18 9 |

The above values are exclusive of the values of any reductions that may have been made in the premiums, or any additions that may have been made to the sums assured.

The Surrender Value allowed for Endowments is the return of all the premiums paid without interest.

The Society does not grant business at other than European Rates.

In the case of Policies upon lives taken out at increased premiums, the difference between the premiums charged and the ordinary premium is not included in the valuation of premiums receivable.

## ERRATA.

The following have been forwarded to us by Mr. James Sorley, of the Life Association of Scotland, Edinburgh.

Vol. xv, page 109, lines 10 and 11,

For $$\frac{\mu+\delta}{12}t\left(\frac{1}{m}-t\right)\left(\frac{1}{m}-2t\right)-\frac{t^2}{24}\left(\frac{1}{m}-t\right)^2\frac{D''}{D}$$

read $$\frac{\mu+\delta}{12}\left(\frac{1}{m}-t\right)\left(\frac{1}{m}-2t\right)-\frac{t}{24}\left(\frac{1}{m}-t\right)^2\frac{D''}{D}$$

Page 109, lines 14 and 15,

For $$\frac{t^2}{2}\left(\frac{1}{2m}-\frac{t}{3}\right)-\frac{t^2}{24}\left(\frac{1}{m}-t\right)^2\left\{\delta(\mu+\delta)-\frac{D''}{D}\right\}$$

read $$\frac{t}{2}\left(\frac{1}{2m}-\frac{t}{3}\right)\mu-\frac{t}{24}\left(\frac{1}{m}-t\right)^2\left\{\delta(\mu+\delta)-\frac{D''}{D}\right\}.$$

Page 113, line 10 from bottom, For $\overline{A}_{\overset{1}{x}y}$ read $\overline{A}_{x\overset{1}{y}}$;

On following line, for $x$ read $y$, and for $y$ read $x$; and in each of the three succeeding formulas, (24), (25), and (26), for $\overline{A}_{\overset{1}{x}y}$ read $\overline{A}_{x\overset{1}{y}}$.

With reference to these, Mr. Woolhouse writes:—

"The errata on pages 109 and 113 (vol. xv) of my paper 'On an Improved Theory of Annuities and Assurances', which you were good enough to send me, are correctly stated, and I am obliged to your correspondent who has communicated them.

"Of those on page 113 I was aware immediately after the publication of the number of the *Journal*, but they were so obvious that I did not at the time think it necessary to make any special note of them. The formula in question was afterwards correctly stated by me on page 177 of vol. xvii.

"The formulæ on page 109 I find, on reference, are correct in my original manuscript; but, not having the same before me, when writing out the paper for the Institute, they were, at the time, reworked, and the omissions pointed out must then have occurred. The omissions would have been detected at once had the terms any effect whatever on the final results, but it will be observed that they severally vanish at both limits of the integral. The general reasoning given by me on pages 110 and 111 establishes the important proposition, that if the analytical expressions were carried to higher orders of development, all the succeeding terms would in like manner simultaneously vanish. The final results are therefore in the most rigid sense mathematically accurate, and are to be recognised as absolute principles."

# JOURNAL

OF THE

# INSTITUTE OF ACTUARIES

AND

# ASSURANCE MAGAZINE.

## *On the Rate of Mortality found to prevail among Residents in India, being Subscribers and the Male and Female nominees of Subscribers to the Uncovenanted Service Family Pension Fund, between the years* 1837–1872. *By* ALEXANDER JOHN FINLAISON, *of the National Debt Office, Fellow of the Institute of Actuaries.*

[Read before the Institute, 26 Jan. 1874.]

THE rate of mortality experienced by residents in India will supply an answer to an enquiry of the highest interest in an actuarial sense. The enquiry may even be considered of national importance so long as our countrymen seek their fortunes in a region where the effects of the climate on their prospects of health and longevity continue to be regarded as uncertain and hazardous.

A contribution to the general fund of knowledge on this subject, derived from precise and trustworthy records, may therefore be considered, perhaps, as not entirely devoid of value.

The mortality experience which is now brought under notice is that of the subscribers and the nominees of the subscribers to the Uncovenanted Service Family Pension Fund. This Fund is one of those societies the successful establishment of which has conferred so much honour on the officials of the late Honourable East India Company.

The Society was founded on the 1st of May 1837, for the maintenance of the widows and children of Christian men, in the Uncovenanted Service of the Honourable East India Company, residing in the Presidency of Fort William in Bengal.

The term "Presidency of Fort William" has been held by the Society's rules to include all the Provinces in India other than those under the Governments of Madras and Bombay.

The objects of the Institution are carried out by the receipt and accumulation of the monthly subscriptions of the members for the provision of pensions to their widows and orphans. The pensions are enjoyed by the widows during widowhood, by orphan daughters until marriage, and by the orphan sons until they attain the age of either 18 or 21 years.

Members are only admitted to the Fund under the condition that they have satisfactorily passed a strict medical examination. That this rule has been efficiently enforced will be allowed on the fact being stated that in some years as many as 25 per cent of the applications for admission have been refused—the candidates not coming up to the standard it has been sought to maintain.

The official occupations of the members, on the 1st of May 1872, are classified as follows:—

| | Per Cent. |
|---|---|
| In Public Offices | 32 |
| Public Works Department | 13 |
| Police and Judicial | 17 |
| Customs and Marine | 12 |
| Survey and Forest | 4 |
| Other Departments | 22 |
| | 100 |

From the foundation of the Society, records of the age at entry, marriage, and death, of the subscribers and their nominees, have been kept with precision and care. Every one of the cases included in the tables now published has been subjected to my own examination. The age at which each person passed from observation, together with the age of each survivor on the 30th June 1872, have been computed by myself.

## *Mortality Experience of Subscribers, Males, above the age of* 18 *years.*

Table A, for adult males, is constructed from the experience of 1964 subscribers, who entered the Fund between the 1st May

1837 and the 30th June 1871. Of these, 436 died while under observation, and 537 have, from one cause or another, quitted the Society, leaving 991 alive and on the books, on the 30th June 1872.

The number exposed to the risk of mortality at any year of age is assumed to be the number who were present during any period of that year of life, less one-half of those who entered, who withdrew, and who were living, at the age, at the close of the observation.

### *Mortality Experience of Nominees, Males, from birth to 21 years of age.*

Table B, for males under the age of 21, is constructed from the experience of 1492 sons who were nominated by their fathers between the 1st of May 1837 and the 30th of June 1871. Of these, 133 died while under observation, 298 were withdrawn, 282 attained their 18th birthday, and passed from observation under the conditions of one of the tables of the Society; 37 attained the age of 21 years, and left under the provisions of another table; and 742 were alive and on the books, on the 30th June 1872.

### *Mortality Experience of the Wives and Widows of Subscribers, above the age of 14 years.*

Table C, showing the mortality experienced by the married women, is derived from an observation of 1715 wives who were enrolled between the 1st May 1837 and the 30th June 1871. Of these, 330 died, 360 were withdrawn, and 1025 were alive and on the books, on the 30th June 1872, remaining either married or being widows in receipt of pension on that day.

### *Mortality and Marriage Experience of the Unmarried Daughters of Subscribers, from birth.*

Table D shows the mortality, and marriages, of the daughters of the subscribers, and contains the experience recorded in respect of 2031 girls, who were nominated by their fathers between the 1st May 1837 and the 30th June 1871. Of these, 187 died, 400 have married, and 280 have been withdrawn, leaving 1164 alive and on the books, on the 30th June 1872.

TABLE A.—MALE MORTALITY IN INDIA.—*The Experience of the Subscribers to the Uncovenanted Service Family Pension Fund, between the Years* 1837–1872.

| Age last Birth-day. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| | Entered. | Lived. | Died. | Discon-tinued. | Exist-ing. | Total of Cols. 3, 4, 5. | Half of Total of Cols. 1, 4, 5. | Exposed to Risk. | Died. |
| 18 | 2 | 2 | ... | ... | ... | ... | 1· | 1· | |
| 19 | 2 | 4 | ... | 1 | ... | 1 | 1·5 | 2·5 | |
| 20 | 6 | 9 | ... | ... | ... | ... | 3· | 6· | |
| 21 | 27 | 36 | ... | 2 | ... | 2 | 14·5 | 21·5 | |
| 22 | 37 | 71 | 1 | 2 | 1 | 4 | 20· | 51· | 1 |
| 23 | 43 | 110 | ... | 6 | 1 | 7 | 25· | 85· | |
| 24 | 77 | 180 | 2 | 8 | 5 | 15 | 45· | 135· | 2 |
| 25 | 81 | 246 | ... | 9 | 3 | 12 | 46·5 | 199·5 | |
| 26 | 112 | 346 | 3 | 20 | 11 | 34 | 71·5 | 274·5 | 3 |
| 27 | 92 | 404 | 2 | 14 | 16 | 32 | 61· | 343· | 2 |
| 28 | 114 | 486 | 5 | 21 | 12 | 38 | 73·5 | 412·5 | 5 |
| 29 | 92 | 540 | 7 | 15 | 25 | 47 | 66· | 474· | 7 |
| 30 | 119 | 612 | 4 | 21 | 30 | 55 | 85· | 527· | 4 |
| 31 | 108 | 665 | 6 | 23 | 42 | 71 | 86·5 | 578·5 | 6 |
| 32 | 107 | 701 | 5 | 16 | 27 | 48 | 75· | 626· | 5 |
| 33 | 90 | 743 | 15 | 20 | 53 | 88 | 81·5 | 661·5 | 15 |
| 34 | 88 | 743 | 13 | 27 | 37 | 77 | 76· | 667· | 13 |
| 35 | 85 | 751 | 11 | 22 | 34 | 67 | 70·5 | 680·5 | 11 |
| 36 | 69 | 753 | 10 | 22 | 33 | 65 | 62· | 691· | 10 |
| 37 | 67 | 755 | 13 | 23 | 37 | 73 | 63·5 | 691·5 | 13 |
| 38 | 64 | 746 | 14 | 19 | 30 | 63 | 56·5 | 689·5 | 14 |
| 39 | 57 | 740 | 19 | 16 | 30 | 65 | 51·5 | 688·5 | 19 |
| 40 | 51 | 726 | 10 | 11 | 28 | 49 | 45· | 681· | 10 |
| 41 | 43 | 720 | 14 | 24 | 44 | 82 | 55·5 | 664·5 | 14 |
| 42 | 41 | 679 | 9 | 20 | 45 | 74 | 53· | 626· | 9 |
| 43 | 37 | 642 | 13 | 18 | 30 | 61 | 42·5 | 599·5 | 13 |
| 44 | 36 | 617 | 17 | 12 | 25 | 54 | 36·5 | 580·5 | 17 |
| 45 | 36 | 599 | 18 | 9 | 34 | 61 | 39·5 | 559·5 | 18 |
| 46 | 18 | 556 | 16 | 17 | 29 | 62 | 32· | 524· | 16 |
| 47 | 26 | 520 | 15 | 16 | 28 | 59 | 35· | 485· | 15 |
| 48 | 23 | 484 | 9 | 5 | 27 | 41 | 27·5 | 456·5 | 9 |
| 49 | 17 | 460 | 20 | 12 | 34 | 66 | 31·5 | 428·5 | 20 |
| 50 | 15 | 409 | 10 | 10 | 24 | 44 | 24·5 | 384·5 | 10 |
| 51 | 13 | 378 | 20 | 15 | 26 | 61 | 27· | 351· | 20 |
| 52 | 15 | 332 | 7 | 13 | 29 | 49 | 28·5 | 303·5 | 7 |
| 53 | 7 | 290 | 17 | 4 | 13 | 34 | 12· | 278· | 17 |
| 54 | 11 | 267 | 12 | 8 | 18 | 38 | 18·5 | 248·5 | 12 |
| 55 | 6 | 235 | 10 | 4 | 20 | 34 | 15· | 220· | 10 |
| 56 | 8 | 209 | 10 | 7 | 11 | 28 | 13· | 196· | 10 |
| 57 | 9 | 190 | 2 | 3 | 14 | 19 | 13· | 177· | 2 |
| 58 | 2 | 173 | 4 | 8 | 11 | 23 | 10·5 | 162·5 | 4 |
| 59 | 1 | 151 | 8 | ... | 8 | 16 | 4·5 | 146·5 | 8 |
| 60 | 3 | 138 | 8 | 5 | 11 | 24 | 9·5 | 128·5 | 8 |
| 61 | 1 | 115 | 10 | 1 | 12 | 23 | 7· | 108· | 10 |
| 62 | ... | 92 | 8 | 1 | 10 | 19 | 5·5 | 86·5 | 8 |
| 63 | 2 | 75 | 7 | ... | 4 | 11 | 3· | 72· | 7 |
| 64 | 1 | 65 | 6 | 1 | 3 | 10 | 2·5 | 62·5 | 6 |
| 65 | 2 | 57 | 2 | 2 | 8 | 12 | 6· | 51· | 2 |
| 66 | 1 | 46 | 3 | 1 | 5 | 9 | 3·5 | 42·5 | 3 |
| 67 | ... | 37 | 6 | 2 | ... | 8 | 1· | 36· | 6 |
| 68 | ... | 29 | 1 | ... | 4 | 5 | 2· | 27· | 1 |
| 69 | ... | 24 | 3 | 1 | 1 | 5 | 1· | 23· | 3 |

TABLE A—(*continued*).

| Age last Birth-day. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| | Entered. | Lived. | Died. | Discon-tinued. | Exist-ing. | Total of Cols. 3, 4, 5. | Half of Total of Cols. 1, 4, 5. | Exposed to Risk. | Died. |
| 70 | ... | 19 | 1 | ... | 1 | 2 | ·5 | 18·5 | 1 |
| 71 | ... | 17 | 3 | ... | 2 | 5 | 1· | 16· | 3 |
| 72 | ... | 12 | 1 | ... | 3 | 4 | 1·5 | 10·5 | 1 |
| 73 | ... | 8 | 1 | ... | 1 | 2 | ·5 | 7·5 | 1 |
| 74 | ... | 6 | 2 | ... | 1 | 3 | ·5 | 5·5 | 2 |
| 75 | ... | 3 | ... | ... | ... | ... | ... | 3· | |
| 76 | ... | 3 | ... | ... | ... | ... | ... | 3· | |
| 77 | ... | 3 | 1 | ... | ... | 1 | ... | 3· | 1 |
| 78 | ... | 2 | ... | ... | ... | ... | ... | 2· | |
| 79 | ... | 2 | 1 | ... | ... | 1 | ... | 2· | 1 |
| 80 | ... | 1 | ... | ... | ... | ... | ... | 1· | |
| 81 | ... | 1 | ... | ... | ... | ... | ... | 1· | |
| 82 | ... | 1 | ... | ... | ... | ... | ... | 1· | |
| 83 | ... | 1 | ... | ... | ... | ... | ... | 1· | |
| 84 | ... | 1 | 1 | ... | ... | 1 | ... | 1· | 1 |
| | 1,964 | 19,038 | 436 | 537 | 991 | 1,964 | 1746· | 17292· | 436 |

TABLE B.—MALE MORTALITY IN INDIA.—*The Experience of the Sons of the Subscribers to the Uncovenanted Service Family Pension Fund, between the Years* 1837–1872.

| Age last Birth-day. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Entered. | Lived. | Died. | Discon-tinued. | Exist-ing. | Attained Age. | Total of Cols. 3, 4, 5, 6. | Half of Total of Cols. 1, 4, 5. | Exposed to Risk. | Died. |
| 0 | 319 | 319 | 22 | ... | ... | ... | 22 | 159·5 | 159·5 | 22 |
| 1 | 212 | 509 | 26 | 5 | 13 | ... | 44 | 115· | 394· | 26 |
| 2 | 178 | 643 | 15 | 9 | 13 | ... | 37 | 100· | 543· | 15 |
| 3 | 188 | 794 | 13 | 12 | 31 | ... | 56 | 115·5 | 678·5 | 13 |
| 4 | 123 | 861 | 5 | 12 | 42 | ... | 59 | 88·5 | 772·5 | 5 |
| 5 | 116 | 918 | 6 | 17 | 48 | ... | 71 | 90·5 | 827·5 | 6 |
| 6 | 78 | 925 | 3 | 12 | 42 | ... | 57 | 66· | 859· | 3 |
| 7 | 73 | 941 | 7 | 15 | 51 | ... | 73 | 69·5 | 871·5 | 7 |
| 8 | 65 | 933 | 6 | 12 | 50 | ... | 68 | 63·5 | 869·5 | 6 |
| 9 | 36 | 901 | 3 | 12 | 53 | ... | 68 | 50·5 | 850·5 | 3 |
| 10 | 33 | 866 | 5 | 14 | 46 | ... | 65 | 46·5 | 819·5 | 5 |
| 11 | 34 | 835 | 1 | 16 | 56 | ... | 73 | 53· | 782· | 1 |
| 12 | 16 | 778 | 2 | 13 | 50 | ... | 65 | 39·5 | 738·5 | 2 |
| 13 | 7 | 720 | 1 | 15 | 45 | ... | 61 | 33·5 | 686·5 | 1 |
| 14 | 10 | 669 | 2 | 22 | 45 | ... | 69 | 38·5 | 630·5 | 2 |
| 15 | 3 | 603 | 4 | 26 | 44 | ... | 74 | 36·5 | 566·5 | 4 |
| 16 | ... | 529 | 6 | 26 | 42 | ... | 74 | 34· | 495· | 6 |
| 17 | ... | 455 | 4 | 44 | 28 | 282 | 358 | 36· | 419· | 4 |
| 18 | 1 | 98 | 2 | 3 | 20 | ... | 25 | 12· | 86· | 2 |
| 19 | ... | 73 | ... | 9 | 16 | ... | 25 | 12·5 | 60·5 | |
| 20 | ... | 48 | ... | 4 | 7 | 37 | 48 | 5·5 | 42·5 | |
| | 1,492 | 13,418 | 133 | 298 | 742 | 319 | 1,492 | 1266· | 12152· | 133 |

TABLE C.—FEMALE MORTALITY IN INDIA.—*The Experience of the Wives and Widows of the Subscribers to the Uncovenanted Service Family Pension Fund, between the Years* 1837–1872.

| Age last Birthday. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| | Entered. | Lived. | Died. | Discontinued. | Existing. | Total of Cols. 3, 4, 5. | Half of Total of Cols. 1, 4, 5. | Exposed to Risk. | Died. |
| 14 | 1 | 1 | ... | ... | ... | ... | ·5 | ·5 | |
| 15 | 19 | 20 | ... | ... | ... | ... | 9·5 | 10·5 | |
| 16 | 34 | 54 | 1 | ... | ... | 1 | 17· | 37· | 1 |
| 17 | 67 | 120 | 1 | 3 | ... | 4 | 35· | 85· | 1 |
| 18 | 77 | 193 | 2 | 8 | 4 | 14 | 44·5 | 148·5 | 2 |
| 19 | 95 | 274 | ... | 6 | 2 | 8 | 51·5 | 222·5 | |
| 20 | 113 | 379 | 4 | 11 | 7 | 22 | 65·5 | 313·5 | 4 |
| 21 | 109 | 466 | 13 | 18 | 18 | 49 | 72·5 | 393·5 | 13 |
| 22 | 106 | 523 | 9 | 15 | 20 | 44 | 70·5 | 452·5 | 9 |
| 23 | 92 | 571 | 11 | 13 | 24 | 48 | 64·5 | 506·5 | 11 |
| 24 | 123 | 646 | 6 | 16 | 30 | 52 | 84·5 | 561·5 | 6 |
| 25 | 89 | 683 | 10 | 16 | 32 | 58 | 68·5 | 614·5 | 10 |
| 26 | 115 | 740 | 7 | 21 | 43 | 71 | 89·5 | 650·5 | 7 |
| 27 | 86 | 755 | 10 | 25 | 36 | 71 | 73·5 | 681·5 | 10 |
| 28 | 60 | 744 | 11 | 15 | 46 | 72 | 60·5 | 683·5 | 11 |
| 29 | 61 | 733 | 22 | 22 | 43 | 87 | 63· | 670· | 22 |
| 30 | 50 | 696 | 11 | 12 | 36 | 59 | 49· | 647· | 11 |
| 31 | 52 | 689 | 17 | 10 | 42 | 69 | 52· | 637· | 17 |
| 32 | 49 | 669 | 13 | 14 | 30 | 57 | 46·5 | 622·5 | 13 |
| 33 | 41 | 653 | 11 | 14 | 31 | 56 | 43· | 610· | 11 |
| 34 | 48 | 645 | 10 | 14 | 40 | 64 | 51· | 594· | 10 |
| 35 | 26 | 607 | 11 | 10 | 35 | 56 | 35·5 | 571·5 | 11 |
| 36 | 28 | 579 | 12 | 18 | 31 | 61 | 38·5 | 540·5 | 12 |
| 37 | 23 | 541 | 12 | 10 | 34 | 56 | 33·5 | 507·5 | 12 |
| 38 | 28 | 513 | 6 | 10 | 32 | 48 | 35· | 478· | 6 |
| 39 | 13 | 478 | 5 | 5 | 31 | 41 | 24·5 | 453·5 | 5 |
| 40 | 18 | 455 | 7 | 6 | 19 | 32 | 21·5 | 433·5 | 7 |
| 41 | 15 | 438 | 8 | 4 | 21 | 33 | 20· | 418· | 8 |
| 42 | 15 | 420 | 6 | 3 | 21 | 30 | 19·5 | 400·5 | 6 |
| 43 | 15 | 405 | 13 | 6 | 30 | 49 | 25·5 | 379·5 | 13 |
| 44 | 9 | 365 | 9 | 4 | 31 | 44 | 22· | 343· | 9 |
| 45 | 10 | 331 | 7 | 8 | 18 | 33 | 18· | 313· | 7 |
| 46 | 8 | 306 | 6 | 5 | 16 | 27 | 14·5 | 291·5 | 6 |
| 47 | 7 | 286 | 5 | 4 | 22 | 31 | 16·5 | 269·5 | 5 |
| 48 | 1 | 256 | 1 | 2 | 9 | 12 | 6· | 250· | 1 |
| 49 | 1 | 245 | 6 | 3 | 19 | 28 | 11·5 | 233·5 | 6 |
| 50 | 4 | 221 | 5 | 3 | 12 | 20 | 9·5 | 211·5 | 5 |
| 51 | ... | 201 | 1 | 1 | 11 | 13 | 6· | 195· | 1 |
| 52 | 2 | 190 | 5 | 2 | 13 | 20 | 8·5 | 181·5 | 5 |
| 53 | 1 | 171 | 1 | ... | 12 | 13 | 6·5 | 164·5 | 1 |
| 54 | 1 | 159 | 2 | ... | 14 | 16 | 7·5 | 151·5 | 2 |
| 55 | 2 | 145 | 8 | ... | 12 | 20 | 7· | 138· | 8 |
| 56 | ... | 125 | 1 | ... | 10 | 11 | 5· | 120· | 1 |
| 57 | 1 | 115 | 1 | ... | 7 | 8 | 4· | 111· | 1 |
| 58 | ... | 107 | ... | 1 | 9 | 10 | 5· | 102· | |
| 59 | ... | 97 | 2 | ... | 5 | 7 | 2·5 | 94·5 | 2 |
| 60 | ... | 90 | 2 | 2 | 13 | 17 | 7·5 | 82·5 | 2 |
| 61 | ... | 73 | 1 | ... | 15 | 16 | 7·5 | 65·5 | 1 |
| 62 | ... | 57 | 2 | ... | 7 | 9 | 3·5 | 53·5 | 2 |
| 63 | ... | 48 | 3 | ... | 6 | 9 | 3· | 45· | 3 |
| 64 | ... | 39 | 2 | ... | 6 | 8 | 3· | 36· | 2 |
| 65 | ... | 31 | 1 | ... | 4 | 5 | 2· | 29· | 1 |

TABLE C—(*continued*).

| Age last Birthday. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| | Entered. | Lived. | Died. | Discontinued. | Existing. | Total of Cols. 3, 4, 5. | Half of Total of Cols. 1, 4, 5. | Exposed to Risk. | Died. |
| 66 | ... | 26 | 2 | ... | ... | 2 | ... | 26· | 2 |
| 67 | ... | 24 | 1 | ... | 1 | 2 | ·5 | 23·5 | 1 |
| 68 | ... | 22 | 2 | ... | 3 | 5 | 1·5 | 20·5 | 2 |
| 69 | ... | 17 | ... | ... | 4 | 4 | 2· | 15· | |
| 70 | ... | 13 | ... | ... | 2 | 2 | 1· | 12· | |
| 71 | ... | 11 | ... | ... | 2 | 2 | 1· | 10· | |
| 72 | ... | 9 | 2 | ... | 1 | 3 | ·5 | 8·5 | 2 |
| 73 | ... | 6 | 2 | ... | 1 | 3 | ·5 | 5·5 | 2 |
| 74 | ... | 3 | ... | ... | ... | ... | ... | 3· | |
| 75 | ... | 3 | ... | ... | 1 | 1 | ·5 | 2·5 | |
| 76 | ... | 2 | 1 | ... | ... | 1 | ... | 2· | 1 |
| 77 | ... | 1 | ... | ... | ... | ... | ... | 1· | |
| 78 | ... | 1 | ... | ... | 1 | 1 | ·5 | ·5 | |
| | 1,715 | 18,486 | 330 | 360 | 1,025 | 1,715 | 1550· | 16936· | 330 |

TABLE D.—FEMALE MORTALITY IN INDIA.—*The Experience of the Unmarried Daughters of the Subscribers to the Uncovenanted Service Family Pension Fund, between the Years* 1837–1872.

| Age last Birthday. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| | Entered. | Lived. | Died. | Married. | Discontinued. | Existing. | Total of Cols. 3, 4, 5, 6. | Half of Total of Cols. 1, 4, 5, 6. | Exposed to Risk. | Died. |
| 0 | 307 | 307 | 12 | ... | ... | ... | 12 | 153·5 | 153·5 | 12 |
| 1 | 265 | 560 | 33 | ... | 1 | 8 | 42 | 137· | 423· | 33 |
| 2 | 255 | 773 | 18 | ... | 6 | 20 | 44 | 140·5 | 632·5 | 18 |
| 3 | 196 | 925 | 14 | ... | 15 | 36 | 65 | 123·5 | 801·5 | 14 |
| 4 | 174 | 1,034 | 9 | ... | 18 | 39 | 66 | 115·5 | 918·5 | 9 |
| 5 | 121 | 1,089 | 7 | ... | 21 | 47 | 75 | 94·5 | 994·5 | 7 |
| 6 | 130 | 1,144 | 10 | ... | 19 | 51 | 80 | 100· | 1044· | 10 |
| 7 | 117 | 1,181 | 9 | ... | 18 | 64 | 91 | 99·5 | 1081·5 | 9 |
| 8 | 74 | 1,164 | 8 | ... | 21 | 67 | 96 | 81· | 1083· | 8 |
| 9 | 79 | 1,147 | ... | ... | 16 | 58 | 74 | 76·5 | 1070·5 | |
| 10 | 58 | 1,131 | 7 | ... | 14 | 65 | 86 | 68·5 | 1062·5 | 7 |
| 11 | 51 | 1,096 | 7 | ... | 12 | 59 | 78 | 61· | 1033· | 7 |
| 12 | 37 | 1,055 | 4 | 1 | 13 | 79 | 97 | 65· | 990· | 4 |
| 13 | 42 | 1,000 | 2 | 3 | 16 | 50 | 71 | 55·5 | 944·5 | 2 |
| 14 | 25 | 954 | 2 | 11 | 12 | 55 | 80 | 51·5 | 902·5 | 2 |
| 15 | 17 | 891 | 6 | 27 | 8 | 58 | 99 | 55· | 836· | 6 |
| 16 | 17 | 809 | 6 | 30 | 14 | 49 | 99 | 55· | 754· | 6 |
| 17 | 12 | 722 | 8 | 40 | 12 | 51 | 111 | 57·5 | 664·5 | 8 |
| 18 | 9 | 620 | 1 | 41 | 8 | 41 | 91 | 49·5 | 570·5 | 1 |
| 19 | 9 | 538 | 4 | 44 | 8 | 37 | 93 | 49· | 489· | 4 |
| 20 | 11 | 456 | 3 | 34 | 9 | 28 | 74 | 41· | 415· | 3 |
| 21 | 9 | 391 | 2 | 49 | 4 | 21 | 76 | 41·5 | 349·5 | 2 |
| 22 | ... | 315 | 2 | 29 | 4 | 33 | 68 | 33· | 282· | 2 |

TABLE D—(*continued*).

| Age last Birth-day. | 1<br>Entered. | 2<br>Lived. | 3<br>Died. | 4<br>Married. | 5<br>Discon-tinued. | 6<br>Exist-ing. | 7<br>Total of Cols. 3, 4, 5, 6. | 8<br>Half of Total of Cols. 1, 4, 5, 6. | 9<br>Exposed to Risk. | 10<br>Died. |
|---|---|---|---|---|---|---|---|---|---|---|
| 23 | 3 | 250 | 1 | 22 | 1 | 22 | 46 | 24· | 226· | 1 |
| 24 | 3 | 207 | 2 | 14 | 2 | 19 | 37 | 19· | 188· | 2 |
| 25 | 1 | 171 | ... | 18 | 2 | 16 | 36 | 18·5 | 152·5 | |
| 26 | 3 | 138 | 2 | 8 | 2 | 9 | 21 | 11· | 127· | 2 |
| 27 | ... | 117 | ... | 10 | ... | 13 | 23 | 11·5 | 105·5 | |
| 28 | 1 | 95 | 2 | 1 | ... | 5 | 8 | 3·5 | 91·5 | 2 |
| 29 | 2 | 89 | 2 | 4 | 1 | 9 | 16 | 8· | 81· | 2 |
| 30 | ... | 73 | 1 | 2 | 1 | 6 | 10 | 4·5 | 68·5 | 1 |
| 31 | 1 | 64 | ... | ... | 1 | 4 | 5 | 3· | 61· | |
| 32 | ... | 59 | ... | 1 | 1 | 4 | 6 | 3· | 56· | |
| 33 | 1 | 54 | 1 | 5 | ... | 4 | 10 | 5· | 49· | 1 |
| 34 | 1 | 45 | ... | 2 | ... | 5 | 7 | 4· | 41· | |
| 35 | ... | 38 | 1 | 1 | ... | 6 | 8 | 3·5 | 34·5 | 1 |
| 36 | ... | 30 | ... | 1 | ... | 4 | 5 | 2·5 | 27·5 | |
| 37 | ... | 25 | ... | ... | ... | 6 | 6 | 3· | 22· | |
| 38 | ... | 19 | ... | ... | ... | 2 | 2 | 1· | 18· | |
| 39 | ... | 17 | ... | ... | ... | 6 | 6 | 3· | 14· | |
| 40 | ... | 11 | 1 | 1 | ... | 1 | 3 | 1· | 10· | 1 |
| 41 | ... | 8 | ... | ... | ... | ... | ... | ... | 8· | |
| 42 | ... | 8 | ... | ... | ... | ... | ... | ... | 8· | |
| 43 | ... | 8 | ... | ... | ... | ... | ... | ... | 8· | |
| 44 | ... | 8 | ... | ... | ... | 1 | 1 | ·5 | 7·5 | |
| 45 | ... | 7 | ... | ... | ... | ... | ... | ... | 7· | |
| 46 | ... | 7 | ... | ... | ... | 1 | 1 | ·5 | 6·5 | |
| 47 | ... | 6 | ... | ... | ... | ... | ... | ... | 6· | |
| 48 | ... | 6 | ... | 1 | ... | ... | 1 | ·5 | 5·5 | |
| 49 | ... | 5 | ... | ... | ... | 1 | 1 | ·5 | 4·5 | |
| 50 | ... | 4 | ... | ... | ... | ... | ... | ... | 4· | |
| 51 | ... | 4 | ... | ... | ... | ... | ... | ... | 4· | |
| 52 | ... | 4 | ... | ... | ... | ... | ... | ... | 4· | |
| 53 | ... | 4 | ... | ... | ... | 1 | 1 | ·5 | 3·5 | |
| 54 | ... | 3 | ... | ... | ... | ... | ... | ... | 3· | |
| 55 | ... | 3 | ... | ... | ... | 1 | 1 | ·5 | 2·5 | |
| 56 | ... | 2 | ... | ... | ... | ... | ... | ... | 2· | |
| 57 | ... | 2 | ... | ... | ... | ... | ... | ... | 2· | |
| 58 | ... | 2 | ... | ... | ... | ... | ... | ... | 2· | |
| 59 | ... | 2 | ... | ... | ... | 1 | 1 | ·5 | 1·5 | |
| 60 | ... | 1 | ... | ... | ... | 1 | 1 | ·5 | ·5 | |
| | 2,031 | 20,898 | 187 | 400 | 280 | 1,164 | 2,031 | 1937·5 | 18960·5 | 187 |

Table E gives the facts contained in the foregoing tables collected into groups according to years of age, as well as the average annual mortality per cent experienced during the periods; followed by short comparative statements contrasting the experience with rates for similar ages deduced from well known tables of mortality.

## TABLE E.—COMPARISON OF ANNUAL MORTALITY PER CENT.

| Age. | SUBSCRIBERS—MALES. | | | WIVES AND WIDOWS. | | | DAUGHTERS—UNMARRIED. | | | SONS. | | | Age. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Exposed to Risk. | Died. | Annual Mortality per Cent. | Exposed to Risk. | Died. | Annual Mortality per Cent. | Exposed to Risk. | Died. | Annual Mortality per Cent. | Exposed to Risk. | Died. | Annual Mortality per Cent. | |
| 1 | ... | ... | ... | ... | ... | ... | 423· | 33 | 7·80 | 394· | 26 | 6·60 | 1 |
| 2 | ... | ... | ... | ... | ... | ... | 632·5 | 18 | 2·85 | 543· | 15 | 2·76 | 2 |
| 3 | ... | ... | ... | ... | ... | ... | 801·5 | 14 | 1·75 | 678·5 | 13 | 1·92 | 3 |
| 4 | ... | ... | ... | ... | ... | ... | 918·5 | 9 | ·98 | 772·5 | 5 | ·65 | 4 |
| 5– 9 | ... | ... | ... | ... | ... | ... | 5273·5 | 34 | ·64 | 4278· | 25 | ·58 | 5– 9 |
| 10–19 | ... | ... | ... | ... | ... | ... | 8248·5 | 47 | ·57 | 5284· | 27 | ·51 | 10–19 |
| 20–29 | 2002· | 20 | 1·00 | 5527·5 | 103 | 1·86 | 2018· | 16 | ·79 | ... | ... | ... | 20–29 |
| 30–39 | 6501· | 110 | 1·69 | 5661·5 | 108 | 1·91 | 391·5 | 3 | ·77 | ... | ... | ... | 30–39 |
| 40–49 | 5605· | 141 | 2·52 | 3332· | 68 | 2·04 | | | ... | | | | 40–49 |
| 50–59 | 2467·5 | 100 | 4·05 | 1469·5 | 26 | 1·77 | | | ... | | | | 50–59 |
| 60–69 | 637· | 54 | 8·48 | 396·5 | 16 | 4·04 | | | ... | | | | 60–69 |
| 70–79 | 71· | 10 | 14·08 | | | | | | ... | | | | 70–79 |

## Males—*Annual Mortality per Cent.*

| Age. | INDIA. | | ENGLAND. | | Age. |
|---|---|---|---|---|---|
| | Subscribers to the Uncovenanted Service Fund. | Covenanted Civil Servants (S. Brown). | Experience of Life Assurance Companies (HM). | Peerage Families (Bailey and Day). | |
| 20–29 | 1·00 | 1·78 | ·69 | 1·05 | 20–29 |
| 30–39 | 1·69 | 1·72 | ·89 | ·87 | 30–39 |
| 40–49 | 2·52 | 1·99 | 1·20 | 1·23 | 40–49 |
| 50–59 | 4·05 | 2·52 | 2·02 | 1·76 | 50–59 |
| 60–69 | 8·48 | 4·28 | 4·06 | 3·76 | 60–69 |
| 70–79 | 14·08 | 7·46 | 8·44 | 8·71 | 70–79 |

## Males—*Annual Mortality per Cent.*

| Age. | INDIA. | | ENGLAND. | Age. |
|---|---|---|---|---|
| | Sons of Subscribers to the Uncovenanted Service Fund. | Children of European Parents (S. Brown). | Peerage Families (Bailey and Day). | |
| 1 | 6·60 | 7·24 | 1·63 | 1 |
| 2 | 2·76 | 3·17 | ·87 | 2 |
| 3 | 1·92 | 1·60 | ·39 | 3 |
| 4 | ·65 | ·79 | ·44 | 4 |
| 5– 9 | ·58 | ·51 | ·35 | 5– 9 |
| 10–19 | ·51 | ·42 | ·54 | 10–19 |

TABLE E—(*continued*).—Females—*Annual Mortality per Cent.*

| Age. | INDIA. Wives and Widows of Subscribers to the Uncovenanted Service Fund. | INDIA. Wives and Widows of European Military Officers and Civil Servants (S. Brown). | ENGLAND. Government Annuitants (A. G. Finlaison). | ENGLAND. Peerage Families (Bailey and Day). | Age. |
|---|---|---|---|---|---|
| 20–29 | 1·86 | 1·43 | ·85 | ·83 | 20–29 |
| 30–39 | 1·91 | 1·31 | 1·00 | ·92 | 30–39 |
| 40–49 | 2·04 | 1·11 | 1·15 | 1·18 | 40–49 |
| 50–59 | 1·77 | 1·41 | 1·63 | 1·71 | 50–59 |
| 60–69 | 4·04 | 3·01 | 3·04 | 3·51 | 60–69 |

Females—*Annual Mortality per Cent.*

| Age. | INDIA. Unmarried Daughters of Subscribers to the Uncovenanted Service Fund. | INDIA. Children of European Parents (S. Brown). | ENGLAND. Government Annuitants (A. G. Finlaison). | ENGLAND. Peerage Families (Bailey and Day). | Age. |
|---|---|---|---|---|---|
| 1 | 7·80 | 6·76 | ... | 1·64 | 1 |
| 2 | 2·85 | 3·68 | 1·89 | ·71 | 2 |
| 3 | 1·75 | 1·87 | 1·51 | ·56 | 3 |
| 4 | ·98 | 1·14 | 1·21 | ·56 | 4 |
| 5– 9 | ·64 | ·58 | ·64 | ·44 | 5– 9 |
| 10–19 | ·57 | ·43 | ·68 | ·72 | 10–19 |
| 20–29 | ·79 | ... | ·85 | ·83 | 20–29 |
| 30–39 | ·77 | ... | 1·00 | ·92 | 30–39 |

The abstract made for the purpose of comparing the tables with each other shows the numbers computed to be exposed to risk at the 1st, 2nd, 3rd, and 4th birthdays, and the numbers that died in each of the succeeding years. The year from birth to one year of age is omitted, as it is considered that the correction which is required to show the number exposed to risk at the commencement of the year of age could not be properly applied in this particular instance. The sum of the numbers exposed to risk at the commencement of each year of the quinquennium, from 5 to 9 years of age, is then next given, and is compared with the deaths that occurred during the same period. From 10 years of age upwards the facts have been grouped in decades, and have been continued until the numbers present became too small to return a trustworthy result.

If the mortality experienced by the subscribers and that which occurred among their wives and widows be compared, it will be seen that the deaths of the females were in excess of those of the males in the decades 20-29 and 30-39. This may be the result partly of the medical selection of the males, no choice being exercised in the case of the female lives; and it may also be in some degree occasioned by the complications which I am informed frequently supervene in childbirth among European women in India. After passing the age of 40, the course of things is reversed, and the mortality of the male sex becomes much greater than that of the female. At these older ages some of the females are widows in the receipt of pension, and resident in England, while, on the other hand, a large majority of the male subscribers are still resident in India. At corresponding ages in England the superior vitality of the female sex over the male sex is well known. It is, moreover, conceivable that in all Funds for the provision of benefit to widows and children, the natural tendency of the pecuniary interests involved in the contracts would impel those who had become the most feeble in health among the male members to remain to the last in the Fund, while the more robust the health of the female nominee, the less likely would it be that her interests would be allowed to lapse.

In the two decades 20-29 and 30-39, the facts are sufficiently numerous to allow a comparison to be made between the mortality of the unmarried daughters and that prevailing among their mothers. It will be seen that at these ages the deaths of the married women are more than twice as numerous as those which occurred among the spinsters. I do not know how far this can be taken as an indication of a general law, or how much the dangers peculiar to childbirth, alluded to above, may affect the rate of mortality at these particular ages; but at any rate the difference is of remarkable extent.

On comparing the rate of mortality prevailing among the children of the two sexes, it will be seen that the disparity is not very great. Among the girls, however, the deaths are more numerous than among the boys. This result is, I believe, somewhat at variance with the generally received ideas on the subject of the rate of mortality during the nonage of the sexes. For there has been an opinion that the excess of births of male children, discovered wherever the facts have been carefully observed, is balanced by a subsequent greater mortality among males during their minority.

In comparing the results at which I have arrived with other tables of mortality, I have confined myself to a few well known tables, and have in each instance presented the original observations before any process of graduation had been applied to them.

The papers concerning the rate of mortality prevailing among Europeans resident in India, contributed to the *Journal of the Institute of Actuaries* by Mr. Brown, one of our past Presidents—who has given us in them not only the many valuable results collected by himself, but also a recapitulation of the labours of other eminent persons—may be placed in the first rank of the information hitherto published on the subject.

I have selected as a standard of comparison for male mortality in India the results deduced from the experience of the Covenanted Civil Servants, and published in vol. xvi of the *Journal of the Institute of Actuaries.* "These gentlemen," it is justly remarked, "from the positions which they occupy, the considerable incomes "which they enjoy, furloughs in the case of illness, and other "social advantages, may be compared with the best classes in this "country in order to see the effect of climate on health."

The incomes of the Uncovenanted Civil Servants are less considerable than those of the members of the Covenanted Service, and their more moderate circumstances probably keep them longer in those districts of India where the harder work of life has to be performed.

Results deduced from the $\mathbf{H}^{\mathbf{M}}$ Table of the Mortality Experience of Life Assurance Companies, collected by the Institute, and from the Peerage Table of Messrs. Bailey and Day, have been made use of for the purpose of comparison, as exponents of rates of mortality which have prevailed among males in Great Britain.

In the case of the females, I have compared the rate of mortality experienced by the wives and widows of the Uncovenanted Civil Servants, with the observation of the mortality among the wives and widows of European Military Officers and Civil Servants on service in India, given by Mr. Brown, on page 207 of vol. xvi of the *Journal of the Institute of Actuaries.*

For female mortality which has prevailed in Great Britain, I have referred to the experience of the Government Life Annuitants by Mr. A. G. Finlaison; and also to the deductions made by Messrs. Bailey and Day, from the experience of the Peerage Families.

It will be noticed in the table of comparison of Annual Mortality per cent for Males, that in the decade 20–29 the mortality of the Uncovenanted Servants is under that shown in the table for the Covenanted Civil Service. This possibly may arise from the strict medical examination to which the members of the Uncovenanted Service Fund have been subjected in India; while I believe I am right in stating that, with the exception of the medical examination in England on entering the service, the whole of the members of the Covenanted Service become also members of their Fund, without reference being made to their state of health on arrival in India, and that all single as well as married men subscribe for its benefits.

In the decade of 30-39 the rates of mortality shown in the two tables are virtually identical. After the age of 40 is passed, however, the mortality of the Uncovenanted Servants begins to be in excess of that experienced by the members of the Covenanted Service; and after the age of 50 it is very greatly so.

It may here be observed that very probably more than two thirds of the subscribers to the Uncovenanted Service Fund are permanent residents in India; while, as Mr. Brown observes, page 191, vol. xvi of the *Journal of the Institute of Actuaries*, of the Covenanted Servants: "It must be noticed that these Tables " include the retired members; and as the ages of 50 to 55 may " be roughly taken as about the average ages of retirement, the " greater part of the observations after those ages may be considered as relating to residents in this country (England), who " have survived the effects of exposure to service in India."

On comparing the annual rate of mortality per cent experienced by the Uncovenanted Civil Servants with that shown in the $H^M$ Table of the mortality experience of British Life Assurance Companies, it is worthy of remark that, from the age of 30 to the age of 70, the average annual rate is in each decade of the Indian Table about the double of that shown in the British Table. The rate of progression from age to age of this function in the two tables may therefore be said to be generally similar.

The marked amount of difference between the rates of mortality shown in the two tables is a clear indication that the rate of mortality experienced by male Europeans in India is greatly in excess of that experienced by them in England. Some recent assertions to the contrary are certainly by no means borne out by the death rate that has occurred, during the last 35 years, among the subscribers to the Uncovenanted Service Family Pension Fund.

An inspection of the summaries of the mortality of the female sex, will show that the death rate of the wives and widows of the Uncovenanted Civil Servants is throughout higher than that prevailing among the wives and widows of the European Military

Officers and Covenanted Civil Servants on service in India. It is probable, however, that a greater number of the wives of the Uncovenanted Civil Servants are permanently resident in the plains of India than is the case with the wives of the Military Officers and Covenanted Civil Servants. If this conjecture be correct, it no doubt will have had an influence on the death rate. After the age of 40 is passed, it may be surmised that nearly all the wives and widows of the Military Officers and Covenanted Civil Servants are resident in Europe; and although a similar privilege may possibly be enjoyed by a considerable proportion of the widows of the Uncovenanted Civil Servants, it is not probable that the advantage is nearly so universal in their case.

At these ages, namely from 40 up to 70, moreover, it is to be noted that the rate of mortality shown in the table for the wives and widows of the Military Officers and Covenanted Civil Servants is lower than that shown in either of the purely English tables of female mortality already mentioned. The majority of the older lives in the Indian table are widows in receipt of pension; and it is perhaps needless to recall to recollection that the fact of being an annuitant has apparently been always coincident with an extended longevity. It is, however, remarkable to find that even Government Life Annuitants do not enjoy quite so protracted an existence as that which seems to be the lot of the pensioners of these Indian funds.

The rate of mortality found to prevail among the younger children, both male and female, of the subscribers to the Uncovenanted Fund, is much greater than that which is shown in the English tables. This result is corroborated by the experience of the children of European parents on service in India, given by Mr. Brown, in vol. xvi of the *Journal of the Institute of Actuaries.*

In the decade from 10–19, however, in both the Indian tables the rate of mortality is shown, in the case of both sexes, to be less than that given in the English tables. This result may perhaps in part arise from the nature of the pecuniary interests involved. The amount of the recorded mortality may be less than it would otherwise be by reason of the selection which, possibly, is made in the case of the children who, it will be seen, are withdrawn in considerable numbers at these ages. For although a large number of the lapses may be due in great measure to improvidence, yet it seems to be far from improbable that the rate of mortality among those children who are withdrawn at these ages is really subsequently greater than that experienced among those who are kept on the books by the continued payments of their parents.

ABLE F.—MALE MORTALITY IN INDIA.—*The Experience of the Subscribers to the Uncovenanted Service Family Pension Fund.*

| ge. | Probability of Dying in One Year. Unadjusted. | Number Living. Unadjusted. | Number Dying. Unadjusted. | Number Living. Adjusted. | Number Dying. Adjusted. | Log Number Living. Adjusted. | Log Probability of Surviving One Year. Adjusted. | Probability of Surviving One Year. Adjusted. | Probability of Dying in One Year. Adjusted. | Age. |
|---|---|---|---|---|---|---|---|---|---|---|
| $x$ | $q'_x$ | $l'_x$ | $d'_x$ | $l_x$ | $d_x$ | $\lambda l_x$ | $\lambda p_x$ | $p_x$ | $q_x$ | $x$ |
| 21 | ·00000 | 10000 | 0 | 10000 | 61 | ·000 0000 | ·997 3602 | ·99394 | ·00606 | 21 |
| 22 | ·01961 | 10000 | 196 | 9939 | 66 | ·997 3602 | ·997 0892 | ·99332 | ·00668 | 22 |
| 23 | ·00000 | 9804 | 0 | 9873 | 72 | ·994 4494 | ·996 8399 | ·99275 | ·00725 | 23 |
| 24 | ·01481 | 9804 | 145 | 9801 | 76 | ·991 2893 | ·996 6124 | ·99223 | ·00777 | 24 |
| 25 | ·00000 | 9659 | 0 | 9725 | 80 | ·987 9017 | ·996 4022 | ·99175 | ·00825 | 25 |
| 26 | ·01093 | 9659 | 106 | 9645 | 85 | ·984 3039 | ·996 1613 | ·99120 | ·00880 | 26 |
| 27 | ·00583 | 9553 | 56 | 9560 | 89 | ·980 4652 | ·995 9071 | ·99062 | ·00938 | 27 |
| 28 | ·01212 | 9497 | 115 | 9471 | 96 | ·976 3723 | ·995 6176 | ·98996 | ·01004 | 28 |
| 29 | ·01477 | 9382 | 138 | 9375 | 101 | ·971 9899 | ·995 2709 | ·98917 | ·01083 | 29 |
| 30 | ·00759 | 9244 | 70 | 9274 | 109 | ·967 2608 | ·994 8492 | ·98821 | ·01179 | 30 |
| 31 | ·01037 | 9174 | 95 | 9165 | 118 | ·962 1100 | ·994 3919 | ·98717 | ·01283 | 31 |
| 32 | ·00799 | 9079 | 73 | 9047 | 126 | ·956 5019 | ·993 8989 | ·98605 | ·01395 | 32 |
| 33 | ·02268 | 9006 | 204 | 8921 | 135 | ·950 4008 | ·993 3833 | ·98488 | ·01512 | 33 |
| 34 | ·01949 | 8802 | 172 | 8786 | 143 | ·943 7841 | ·992 8892 | ·98376 | ·01624 | 34 |
| 35 | ·01616 | 8630 | 139 | 8643 | 149 | ·936 6733 | ·992 4519 | ·98277 | ·01723 | 35 |
| 36 | ·01447 | 8491 | 123 | 8494 | 153 | ·929 1252 | ·992 0894 | ·98195 | ·01805 | 36 |
| 37 | ·01880 | 8368 | 157 | 8341 | 156 | ·921 2146 | ·991 7974 | ·98129 | ·01871 | 37 |
| 38 | ·02030 | 8211 | 167 | 8185 | 157 | ·913 0120 | ·991 5760 | ·98079 | ·01921 | 38 |
| 39 | ·02760 | 8044 | 222 | 8028 | 159 | ·904 5880 | ·991 3501 | ·98028 | ·01972 | 39 |
| 40 | ·01468 | 7822 | 115 | 7869 | 159 | ·895 9381 | ·991 0931 | ·97970 | ·02030 | 40 |
| 41 | ·02107 | 7707 | 162 | 7710 | 163 | ·887 0312 | ·990 7161 | ·97885 | ·02115 | 41 |
| 42 | ·01438 | 7545 | 109 | 7547 | 168 | ·877 7473 | ·990 2234 | ·97774 | ·02226 | 42 |
| 43 | ·02168 | 7436 | 161 | 7379 | 175 | ·867 9707 | ·989 5744 | ·97628 | ·02372 | 43 |
| 44 | ·02929 | 7275 | 213 | 7204 | 183 | ·857 5451 | ·988 8487 | ·97465 | ·02535 | 44 |
| 45 | ·03217 | 7062 | 227 | 7021 | 191 | ·846 3938 | ·988 0236 | ·97280 | ·02720 | 45 |
| 46 | ·03053 | 6835 | 209 | 6830 | 199 | ·834 4174 | ·987 1834 | ·97092 | ·02908 | 46 |
| 47 | ·03093 | 6626 | 205 | 6631 | 207 | ·821 6008 | ·986 2476 | ·96883 | ·03117 | 47 |
| 48 | ·01972 | 6421 | 127 | 6424 | 214 | ·807 8484 | ·985 2827 | ·96668 | ·03332 | 48 |
| 49 | ·04667 | 6294 | 293 | 6210 | 221 | ·793 1311 | ·984 2257 | ·96433 | ·03567 | 49 |
| 50 | ·02601 | 6001 | 156 | 5989 | 227 | ·777 3568 | ·983 2022 | ·96206 | ·03794 | 50 |
| 51 | ·05698 | 5845 | 333 | 5762 | 231 | ·760 5590 | ·982 2441 | ·95994 | ·04006 | 51 |
| 52 | ·02306 | 5512 | 127 | 5531 | 229 | ·742 8031 | ·981 6057 | ·95853 | ·04147 | 52 |
| 53 | ·06115 | 5385 | 330 | 5302 | 224 | ·724 4088 | ·981 2703 | ·95779 | ·04221 | 53 |
| 54 | ·04829 | 5055 | 244 | 5078 | 215 | ·705 6791 | ·981 2250 | ·95769 | ·04231 | 54 |
| 55 | ·04545 | 4811 | 219 | 4863 | 206 | ·686 9041 | ·981 2159 | ·95767 | ·04233 | 55 |
| 56 | ·05102 | 4592 | 234 | 4657 | 200 | ·668 1200 | ·980 9573 | ·95710 | ·04290 | 56 |
| 57 | ·01130 | 4358 | 49 | 4457 | 201 | ·649 0773 | ·979 9579 | ·95490 | ·04510 | 57 |
| 58 | ·02462 | 4309 | 106 | 4256 | 209 | ·629 0352 | ·978 0709 | ·95076 | ·04924 | 58 |
| 59 | ·05461 | 4203 | 230 | 4047 | 224 | ·607 1061 | ·975 3077 | ·94473 | ·05527 | 59 |
| 60 | ·06226 | 3973 | 247 | 3823 | 239 | ·582 4138 | ·971 9852 | ·93753 | ·06247 | 60 |
| 61 | ·09259 | 3726 | 345 | 3584 | 251 | ·554 3990 | ·968 4082 | ·92984 | ·07016 | 61 |
| 62 | ·09249 | 3381 | 313 | 3333 | 256 | ·522 8072 | ·965 2488 | ·92310 | ·07690 | 62 |
| 63 | ·09722 | 3068 | 298 | 3077 | 253 | ·488 0560 | ·962 8190 | ·91795 | ·08205 | 63 |
| 64 | ·09600 | 2770 | 266 | 2824 | 241 | ·450 8750 | ·961 1647 | ·91446 | ·08554 | 64 |
| 65 | ·03922 | 2504 | 98 | 2583 | 228 | ·412 0397 | ·960 0139 | ·91204 | ·08796 | 65 |
| 66 | ·07059 | 2406 | 170 | 2355 | 211 | ·372 0536 | ·959 1511 | ·91023 | ·08977 | 66 |
| 67 | ·16667 | 2236 | 373 | 2144 | 199 | ·331 2047 | ·957 7796 | ·90736 | ·09264 | 67 |
| 68 | ·03704 | 1863 | 69 | 1945 | 190 | ·288 9843 | ·955 2433 | ·90207 | ·09793 | 68 |
| 69 | ·13043 | 1794 | 234 | 1755 | 182 | ·244 2276 | ·952 4183 | ·89623 | ·10377 | 69 |
| 70 | ·05405 | 1560 | 84 | 1573 | 174 | ·196 6459 | ·949 2029 | ·88962 | ·11038 | 70 |
| 71 | ·18750 | 1476 | 277 | 1399 | 165 | ·145 8488 | ·945 6071 | ·88228 | ·11772 | 71 |

TABLE F—(*continued*).

| Age. | Probability of Dying in One Year. | Number Living. | Number Dying. | Number Living. | Number Dying. | Log Number Living. | Log Probability of Surviving One Year. | Probability of Surviving One Year. | Probability of Dying in One Year. |
|---|---|---|---|---|---|---|---|---|---|
| | Unadjusted. | Unadjusted. | Unadjusted. | Adjusted. | Adjusted. | Adjusted. | Adjusted. | Adjusted. | Adjusted. |
| $x$ | $q'_x$ | $l'_x$ | $d'_x$ | $l_x$ | $d_x$ | $\lambda l_x$ | $\lambda p_x$ | $p_x$ | $q_x$ |
| 72 | ·09524 | 1199 | 114 | 1234 | 155 | ·091 4559 | ·941 4850 | ·87395 | ·12605 |
| 73 | ·13333 | 1085 | 145 | 1079 | 146 | ·032 9409 | ·936 7545 | ·86448 | ·13552 |
| 74 | ·36364 | 940 | 342 | 933 | 137 | ·969 6954 | ·931 2177 | ·85353 | ·14647 |
| 75 | ... | 598 | ... | 796 | 127 | ·900 9131 | ·924 7077 | ·84083 | ·15917 |
| 76 | ... | ... | ... | 669 | 116 | ·825 6208 | ·916 8687 | ·82579 | ·17421 |
| 77 | ... | ... | ... | 553 | 107 | ·742 4895 | ·907 2347 | ·80767 | ·19233 |
| 78 | ... | ... | ... | 446 | 95 | ·649 7242 | ·895 3354 | ·78584 | ·21416 |
| 79 | ... | ... | ... | 351 | 85 | ·545 0596 | ·879 8220 | ·75827 | ·24173 |
| 80 | ... | ... | ... | 266 | 74 | ·424 8816 | ·859 3235 | ·72331 | ·27669 |
| 81 | ... | ... | ... | 192 | 62 | ·284 2051 | ·830 4059 | ·67672 | ·32328 |
| 82 | ... | ... | ... | 130 | 50 | ·114 6110 | ·785 7561 | ·61060 | ·38940 |
| 83 | ... | ... | ... | 80 | 39 | ·900 3671 | ·710 2931 | ·51321 | ·48679 |
| 84 | ... | ... | ... | 41 | 27 | ·610 6602 | ·541 6281 | ·34804 | ·65196 |
| 85 | ... | ... | ... | 14 | 14 | ·152 2883 | $-\infty$ | ·00000 | 1·00000 |
| 86 | ... | ... | ... | 0 | ... | $-\infty$ | ... | ... | ... |

An adjustment of the rate of mortality experienced by the subscribers has been made by me; and the results may perhaps be accepted as offering no unsafe bases for certain classes of pecuniary contracts made to depend upon the lives of males who are exposed to the hazard of residence in the climate of India.

The series of the numbers living at each age, as derived directly from the original facts, was first adjusted by the method used by Mr. Woolhouse for the tables of the Mortality Experience of Insurance Companies collected by the Institute of Actuaries. The radix of the table being assumed to be 10,000 living at the age of 21, the original facts were made use of up to the age of 74; and by the direct operation of this process an adjusted column of the number living was obtained up to the age of 68.

Although the general effect of this admirable method was highly satisfactory, yet the series resulting, of the probabilities of dying in one year, retrograded at two points, and it was therefore felt necessary to apply a further adjustment to fit the table for practical use, if it should be required. Accordingly, the series of the probabilities of dying in one year was readjusted, by the method which originated with the late Mr. John Finlaison, and which is given in page 10 of his well known Report of 1829 to the Lords of the Treasury, on the mortality of the Government Life Annuitants.

It is to the following effect:—

$$q_x=\frac{5q'_x+4({}^{1}q'_x+{}^{-1}q'_x)+3({}^{2}q'_x+{}^{-2}q'_x)+2({}^{3}q'_x+{}^{-3}q'_x)+({}^{4}q'_x+{}^{-4}q'_x)}{25}.$$

The use of this formula had the effect desired; while it did not materially alter the rates except in the cases to which allusion has been made.

It will be seen, on looking down the column containing the adjusted number living, that it exhibits substantially the same rate of mortality as that shown in the similar unadjusted series obtained directly from the facts. For it will be found, at many of the ages, that the numbers of the survivors, out of the 10,000 living at age 21, in the two columns correspond with a considerable degree of closeness.

The paucity of the facts at the older ages necessitated an artificial termination to the table, if it were to be used for any practical purpose.

Now it is generally admitted that, in India, man attains physical maturity at an earlier age than is the case in the colder regions of the north; and on this ground I think that it may be fairly assumed that the limit of his life is also reached at an earlier period than it is in those countries where his physical development proceeds at a more gradual rate.

It may be perhaps regarded as a confirmation of this assumption that, in the table of comparison of the Indian with the English rates of mortality, the rates experienced in the two decades, 50-59 and 60-69, by the male subscribers to the Uncovenanted Service Fund coincide with the rates shown respectively for the succeeding decades 60-69 and 70-79, in the $H^M$ Table of the Mortality Experience of British Life Insurance Companies.

For practical purposes it may be allowed that the limit of life in England should be fixed at about 96 years of age. Accordingly I have ventured to assume that the limit of life, for similar purposes, may be fixed in India at 86 years of age in respect of males who have experienced the rate of mortality of the Uncovenanted Civil Servants, up to the age of 70.

The adjusted number living was therefore completed from the age of 68 to the age of 86 by assuming that none would be left alive at the age last mentioned; and by obtaining the intermediate numbers by the help of the method of finite differences, proceeding on the further assumption that the third order of differences obtained from $l_{66}$, $l_{67}$, $l_{68}$ and $l_{86}$ was constant.

Through these steps the graduation of the table of mortality for males was arrived at up to its ultimate point.

The materials at my disposal in respect of female mortality and marriage in India afford the means of further investigation in other interesting directions; and to these subjects, on which I am engaged, I hope to return on a future day.

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The following abstract of the discussion which followed the reading of the paper is abridged from the *Insurance Record.*

The President welcomed a contribution from anyone of the name of Finlaison. The writer was a grandson of the first President of this Institute, and he hoped the name of Finlaison would always be found amongst its members. He quite concurred in Mr. Finlaison's opinion that the inquiry which he had prosecuted is of the highest importance to the actuary, and may even be considered of national importance. He supposed there was no actuary who had had so much experience of Indian mortality as Mr. Brown, who, he hoped, would favour the meeting with some observations.

Mr. Samuel Brown said that the paper was a very valuable one, and came at a very opportune time with reference to the recent controversy with regard to mortality in India. Dr. Mouat had unfortunately chosen to conduct an enquiry into rates of mortality, which depends upon large aggregations of facts, in precisely the same manner as any medical gentleman would give the results of his own individual practice. In these instances, isolated cases on one side of the question form the greater part of the observations that a medical man has to do deal with. The actuary, on the contrary, is compelled to look at all these isolated cases after they have been collected into considerable masses and compared together; and he does not reason except upon the averages of large numbers. The question of Indian mortality is very important, because every year the numbers going to India are increasing, and probably within a very short time there will be a still wider field opening up for assurance business in that country. We ought, therefore, to take all the opportunities we can of adding in every way to the facts upon which we base our reasoning. He should mention that, in his last paper to which Mr. Finlaison had alluded, he had put together all the authentic observations already in existence up to that date. The Government have closed the old Indian military pension funds, and therefore, as there will be no more entrants, there will be no more collection of facts relating to mortality from these sources. With regard to the civil provident pension funds, with which we are more immediately concerned, as showing the mortality of non-military lives, they are not transferred to the Government, and he did not know that it was intended to do so. But the combination of the facts relating to the covenanted service is very much of the same class, with the exceptions which Mr. Finlaison has pointed out, as those with which we should have to deal for assured lives. The largest numbers of facts as to the mortality of Indian civilians which he knew of up to the present time are those which he had put together in the paper which had been referred to, amounting to more than 40,000 years of life observed, with 900 deaths. To these Mr. Finlaison had made a considerable addition; and he thought that he should himself be able to assist this enquiry still further, as he had just received a large bundle of papers relating to the Widows and Orphan Fund in Madras, which will probably add some thousands more years of observed lives. He had also received the continuation of the experience of the Bengal Civil Fund up to the present time. The former extended back for thirteen years, from 1850 to 1863, and this will give ten years more. When these two bodies of facts are added to Mr. Finlaison's, and to those given in his (Mr. Brown's) last publication, he fancied we should have more than 120,000 years of life observed, both amongst adults and children in India, to deal with. There was one point upon which Mr. Finlaison had not touched that he should like to have heard something about. It is a point which has been very much in debate, and on which information is much needed; and that is as to the deaths of children in the first year of life in these funds. That would not affect an assurance company, but with regard to these pension funds, of which the solvency and position and the valuations to ascertain their condition, are likely to continue to press their claims upon the actuary's attention, the mortality in the first year of life is an important element. One of the contingencies to be provided against is the chance of future children being born in the family, and surviving to obtain the benefits; and the question whether the contribution is sufficient to cover this risk, is of consequence. The point is whether the small amount of mortality shown in the first year of life in the Indian pension funds is true or not. It is generally less than one-half of what is represented by the registers in England; and he believed that Mr. Griffith Davies came to the conclusion that there was something in the climate of India which was beneficial to children in the first year of life. It was a singular fact that that rate is confirmed by Messrs. Bailey and Day's observations of the Peerage for the first year of life, but their second year's mortality is still smaller as compared with the first year. The mortality of the first year of life is about one-half, and that of the second year only one-fourth of the ordinary rate. Mr. Finlaison had passed this question over, and begun the observations from the end of one year after birth. Another important question as affecting pension funds is as to the mortality of married women in India. The observations which he had been led to make are that the mortality follows the law of male mortality, in being greatest at the younger ages, and gradually diminishing in the excess over the English rate till about the middle period of life, when the worst lives are most likely to have succumbed to the climate; and these having been removed, there remains a stronger body behind, and by the time they return to England they are nearly equal in point of longevity to the women of this country of the same age. [Mr. Bailey—That is, women generally, not married women alone.] He spoke of married women, as no observations can be obtained in the middle period of life of any other class. What he meant to imply was, that, as they are married, they are likely to be residing in India with their husbands, and are obliged to remain exposed to the same climatic influences. The daughters of subscribers are not necessarily confined to the climate of India. They may come to England for education, and remain and marry here. It does not necessarily follow that they continue to live in India exposed to the risks of its climate. If Mr. Finlaison, when reviewing this subject, could examine these two or three suggestions, and see whether he could collect additional observations, especially if he could go further into the interesting question of the proportion of marriages amongst the widows, and amongst the unmarried women respectively, which is a matter of great consequence in connection with these widows and orphan funds, it would greatly add to the worth of this very valuable contribution.

Mr. WOODROW said that having been for some time the chairman of the Uncovenanted Pension Fund, the details of which had been laid before the meeting, he had given considerable time to the subject, and deduced results which, to his great satisfaction, were the same as those Mr. Finlaison had brought out. The uncovenanted servants in India take their pensions generally between the ages of fifty-four and fifty-eight. Now, in watching the death-rates about the ages from fifty to sixty, there is a remarkable change in the curve. The feeling of satisfaction which the members experience at the time they have their pensions, seems to be conducive to longevity; for at the age of fifty-five there is a most remarkable turn in the rate of mortality, and the lives at fifty-five, fifty-six, fifty-seven, and fifty-eight, are much better than in the four preceding years. That fact rather confirms a suggestion of Mr. Finlaison's, that the annuitants do not die. When he went out to India, an old lady, called "the mother of Calcutta", Mrs. Ellerton, mother-in-law of Bishop Corry, a lady respected by every one in Calcutta for more than half a century—he believed she had been there for seventy years—said to him, "Now, Mr. Woodrow, be content. Discontent has killed more people in India than cholera, fever, and dysentery together." He was somewhat surprised at the results with reference to unmarried daughters. They had an impression in the fund that that is a very losing part of the business, for the young ladies in India do not marry now so fast as they used to do. The journey to England, which used to occupy four or five months, is now only about twenty-three days, and the consequence is the men come to England, and go back married. The ladies in India have not therefore such a good chance of marrying as they had twenty years ago, and the number of spinsters on the fund is increasing at a dreadful rate. Not only that, but one or two other things perplexed them exceedingly. One is, that in this age of ritualism, some enter convents, and never marry at all. That was not bargained for. In that case the pensions are paid to the convent, and we do not know how long they may live. Another thing, and one which is much sadder, is that ladies sometimes have families without being married. The benefit is forfeited on marriage, but it is not forfeited when the members have families; and the consequence is, the fund is paying unmarried people with families of children. A father insures for the daughter: the daughter goes wrong; does not marry, and has children: she does not go off the fund, and they could not reckon her as married. They had not provided for that, and, not wishing to have a law-suit, go on paying and say nothing about it. If, however, that continued to increase, they would have to protect themselves by some clause. He was never so astonished as when he read Dr. Mouat's statement, that the mortality in India is the same as that in England. He had spoken about it to several medical men in India, and they entirely disagreed with Dr. Mouat.

General HANNYNGTON said that in the report of the Uncovenanted Pension Fund Society it is remarked that the law of mortality deduced from the uncovenanted servants differs widely from that formerly employed by Mr. Finlaison. [Mr. WOODROW—The clerk made a little error there. It was corrected afterwards, and what was published was correct.] It remained to be noticed that the Uncovenanted Service Family Pension Fund is a very mixed community, in which may be traced the names of Arminians, British, French, Germans, Hindoos, Portuguese, and Spanish; so that the mortality prevailing in such a community must be of a very special and local character. [Mr. WOODROW—The Hindoos must be Christians.] He used the word as showing that they are natives of Hindustan. He was not putting it on the ground of religion, but of nationality.

Mr. H. W. PORTER observed a statement in Mr. Finlaison's paper in reference to the excessive mortality of females at the child-bearing ages as compared with males in the decades 20-29 and 30-39, to the following effect, that this excess "may be in some degree occasioned by the complications which I am informed frequently supervene in childbirth among European women in India." Now he had had some experience of India, having been there himself, and also intimate relations with a great many Indian people; and so far from there being any complications connected with childbirth in India, he had always understood that it is a simpler process than in our own country; and the only circumstance at all likely to affect the matter is a slight tendency to puerperal fever. But, granting that these complications do exist, it would account for the increased mortality of women in India over those of corresponding ages in England, but not for the extraordinary difference which he saw in one of these tables between the mortality at those ages in India, as shown by Mr. Finlaison's table and those of Mr. Samuel Brown, Mr. Finlaison's being the uncovenanted civil servants and Mr. Brown's the covenanted civil servants. He knew what the nature of the covenanted service was, but not much about that of the uncovenanted service; and he could only conceive that the difference arises from the respective positions of the families, the wives of the uncovenanted servants being of an inferior class, and probably not having the same comforts and medical attendance as those of the officers of the army. Mr. Brown's mortality at ages 20-29 is 1·43, and Mr. Finlaison's 1·86; and the mortality is greater all through in Mr. Finlaison's table than it is in Mr. Brown's.

Mr. BADEN referred to the low rate of mortality amongst unmarried daughters, say from five to twenty, from ten to twenty, and possibly from seven to twenty. It occurred to him that this might probably be owing, not simply to the fact which Mr. Finlaison had mentioned, and which might have some influence, that there is some selection exercised on account of the purpose for which the lives are nominated, but also from the fact that the children are sent to this country for their education. He thought it was extremely likely that that might be one cause influencing the low rate of mortality at those ages.

Mr. WOODROW—The uncovenanted service contains the working bees of India, and they are engaged in carrying on all the great offices with the civilians receiving the better paid appointments. The consequence is that only a few of the uncovenanted servants can go to the hills. He had been twenty-five years in India, and for twenty-five Mays he stayed in Calcutta, having no opportunity of going to the hills. He was never sick enough to come home, and on only two occasions did he have six months' leave, and those were in the autumn. The uncovenanted service consists of several great divisions. One is the education department, which is very large. There are men in that receiving only five rupees (which is a pound) a month, and the director receives 3,000 rupees a month, or £3,600 a year. The upper portion in Bengal only contains about thirty members, who are sent out from this country. The highest in the education, police, and public works departments generally, insure for their widows £240 a year, which is the highest they can insure; and the lower members insure for smaller sums. Undoubtedly the uncovenanted servants have not the same means of going to the hills or coming to England as the covenanted servants; and their being thus tied to the plains will probably suggest a reason why the females of the uncovenanted servants have a higher rate of mortality than those of the covenanted servants. Change of air is absolutely necessary for a woman after her confinement; and unless this can be secured, exhaustion takes place, and carries her off. That is one great reason for the difference in the rate of mortality which has been pointed out.

Mr. LAWSON states that he came from Madras, where the question of the rates of mortality in India has attracted a great deal of attention. The paper which he had heard applied to rates of mortality as ascertained in thirty-seven years of observation. During that thirty-seven years, everybody who has been in India has noticed a very great change for the better. The change of life in India during the last twelve years has been so marked that these tables, which embrace a period of thirty-seven years, do not give us sufficient information regarding the actual present condition of human longevity in India. We all know there is an enormous increase in the European population of India; and that increase is likely to continue as the resources of the country are developed. But the present rates of premiums for insurance are so prohibitive, that a large portion of the community do not avail themselves of the facilities which are offered by the insurance companies, and the proportion of the insured to the uninsured does not increase in anything like the same ratio as the population. In order to ascertain accurately whether or not the office rates of premium are justified, more information should be obtained with regard to the rates of mortality during the last few years, and more reference should be made to the improved conditions of life generally in India. [The PRESIDENT—Would you limit that to 12 years?] To 12 or 15 years, but not anything like so far back as 37 years, or even 25 years. Everyone who has been in India for a long time, will tell you that in recent years the European population do not drink so much and do not eat so much—which is perhaps of still greater importance,—as they formerly did, and moreover they came home at shorter intervals. They marry and live more decent lives. There is also a greater knowledge of sanitary science, and a greater appreciation of the duty of maintaining one's health than formerly existed. It has been asserted by some persons that the mortality in India is not greater than it is at home. That is an assertion which cannot be admitted for a moment. It stands to reason that the mortality in India must be greater than it is here; but from a policyholder's point of view, taking the general experience of people who go out to India at a tolerably youthful age, and having regard to the way in which they live and the effect of the climate, he thought there was not a sufficient recognition of the fact that the change in the mode of living has been very marked in late years, and that this change justifies more liberality on the part of the insurance offices than has yet been manifested.

Mr. A. H. BAILEY said that a theory has been lately propounded that the rate of mortality in India is no worse than that which prevails in England, and that the rates for insurance in India should therefore be the same as the rates in England. That theory has been reduced to practice, and is the subject of controversy at this moment. In support of it, Dr. Mouat, the honorary secretary of the Statistical Society of London, has put forward one or two reports of a somewhat elaborate character, which, coming from a gentleman of large Indian experience and well deserved reputation as a medical man, and holding the position which he does of honorary secretary of the Statistical Society, have had some weight. Now, what do these reports contain? No trustworthy facts such as Mr. Finlaison has given to-night; but several assumptions. Dr. Mouat has assumed the mortality in India to be the same as the mortality of England, and he has given certain reasons why that should be the case; but he has ignored facts. The value of Mr. Finlaison's paper, which contains facts of as recent a character as could be obtained, embracing the years between 1837 and 1872, is beyond question. If you want the mortality at the later periods of life, you must go back for several years to those who entered the services at about 18 or 20 years of age. You cannot get the mortality of the years of age of 40 or 50, unless you go back twenty or thirty years or more; and it is difficult to conceive that we can get any more recent facts than those which Mr. Finlaison's paper contains. And what do they prove? They prove, as conclusively as possible, that the trustworthy observations of Mr. Brown, which embraced an unusually large number, are correct; that the mortality in India is most materially in excess of that in England; and that therefore it is madness to think of charging the same rates of premium for residents in India as for residents in this country. He agreed to some considerable extent with the last speaker. He thought sufficient justice had not been done by the insurance companies, not so much as regards the matter of premiums, as in respect to bonus and the distribution of profits to residents in India. But he was bound to say that much greater improvements than the gentleman who has just spoken supposed have been effected. Since he first entered the profession the rates of premium for India had been reduced by something like one-half; and that surely is a very substantial improvement. Speaking very roughly, he should say that about thirty years ago the rates in India were about double those prevailing in England; speaking again very roughly he should say they are now about fifty per-cent above the English rates. Therefore, if the average rate in England for an insurance was £3, in India it would have been £6 thirty years ago, and now it is about £4. 10*s*. Something considerable, therefore, has been done in the way of improvement, but he did not think the grievance has so much to do with rates of premium as with the way the Indian policyholders are treated in the division of the profits. In this latter respect he did not think the Indian constituents of the assurance offices are quite fairly treated.

Mr. WALFORD pointed out two sets of figures which seemed to him somewhat remarkable. In the table of male lives, sons of subscribers to the Uncovenanted Service Fund, the mortality shown in the first year is 6·60; in the case of unmarried daughters, it is 7·80 in the first year. Then in Mr. Brown's table, children of European parents, the mortality of males is 7·24, being considerably more than the males in the uncovenanted service; and in the table of females the figures are almost entirely reversed, the rate for year one being 6·76. Could Mr. Finlaison give any explanation?

Mr. JAMES BOGIE stated that he went to Bombay in 1856, and had been backwards and forwards four times. It occurred to him that the Europeans, who have more moderate salaries than the favoured covenanted civilians, do not get so much to the hills, or have so many chances of returning to England, as they do. At the time of the share mania especially there was a scarcity of house room, and the sickness which formerly existed must have been intensified by the bad house accommodation and the unhealthiness of the locality as compared with other parts of the island of Bombay. Since that time, owing to commercial disaster, and houses being built when they were not wanted, when he was last out there, a year and a half ago, there were too many houses in some places; but the sanitary condition of many parts of India, and notably of the island of Bombay, had greatly improved in the last seven years. From the census of two years ago, it appears that in round numbers the mortality of Bombay, Europeans and natives together, in a population of 650,000, was a shade under 30 per thousand all round. A Scotch clergyman told him a couple of years ago, when making up his quarterly returns of funerals, that he never had any funerals, and he accounted for it by the number of steamers which had commenced running two years before, and the number of men who went home for change of air, who formerly would have been unable to get away, through not being able to afford the passage by the P. and O. Company, and would have remained behind and died. [Mr. BADEN—If these gentlemen did not linger in Bombay to die, then where did they go to die?] Possibly not one in four of those who go away from the unhealthy districts do die—perhaps not one in fourteen die immediately. Their health is so improved that they have another lease of life for ten or twenty years.

Mr. FINLAISON, in reply, said that, with reference to Mr. Brown's remarks on the rate of mortality in the first year of age, that is from the date of birth to the first birthday, he believed, in the covenanted service fund, the whole of the children are entered at their births, and therefore Mr. Brown's observation on their rate of mortality in the first year of life is most valuable, inasmuch as it includes without exception the whole of the children born to the members. In the uncovenanted service fund it is optional on the father whether he subscribes for his children or not. Having had the advantage of having the age stated in months and days, he had examined the ages of those children who were entered in the first year of life, and ascertained that, on the average, they were entered rather after they were six months old than before. Therefore he inferred that weakly infants were possibly not subscribed for. He remarked in the course of his paper that this year of life is omitted in the abstract made for the purposes of comparison, as he considered that the usual correction, required to show the number exposed to the risk of mortality at the commencement of the year, could not be properly applied for this reason in this particular instance. The facts are given, however, in the detailed table. General Hannyngton had remarked on the mixed class of lives that are included in the uncovenanted civil service. That is no doubt the case, but the members of this fund are the upper classes, the select portion of that service; and so far as he could judge from the names of the members, there are a dozen pure Hindoos amongst them. He had no other means of telling the nationality of the subscribers. There are a good many Portuguese names, but the great majority are English. The complications which supervene on childbirth, to which he alluded, and to which Mr. Porter had referred, are, that unless European women are enabled either to go to the hills or come to England, or at any rate to escape from the great heat of Calcutta, prolonged weakness arises after their confinements Mr. Woodrow had to-night confirmed what he had ascertained with regard to this point. Mr. Lawson spoke of the improved mortality which has taken place in India in very recent years. An observation of the mortality of these same subscribers to the uncovenanted fund had been made by his father in 1857; and he had for his own satisfaction separated the mortality which occurred between 1837 and 1857, and that which occurred between 1857 and 1872, and these are the results. The average annual mortality per-cent of the males between twenty and twenty-nine years of age in the former period was 1·19, and in the latter ·88, showing a diminution; for the decade 30-39 in the former period 1·49, and in the latter 1·83, being an increase; for 40 to 49, in the first period 3·19, and in the second 2·18, being an improvement of 1 per-cent; for ages 50 to 59, in the first period 4·11, and in the second 4·02; for ages 60 to 69 in the first period it was 9·11, and in the second 8·19 per-cent, but there are very few at these older ages. There are in all 163 deaths in the first period, and 273 in the second. They are all uncovenanted servants. With regard to Mr. Walford's remark, he certainly had noticed the difference as to the way in which the mortality is reversed in the year one to two, between the sexes of the children. He had given the facts exactly as they came from the returns. There were not a great number of deaths at those ages, and he did not think any theoretical deductions could be made with regard to those ages from so small a number of facts.

# *On the Practice of the Eagle Company with regard to the Assurance of Lives classed as Unsound, and on the Rates of Mortality prevailing amongst the Lives so classed, assured during the Sixty-three Years ending* 30 *June* 1871. *By* GEORGE HUMPHREYS, M.A., *Actuary to the Company.*

[Read before the Institute, 30 March 1874.]

MANY years having elapsed since the Eagle Insurance Company first undertook to issue policies of assurance upon unsound lives, and some doubts having arisen as to the way in which the funds of the Society had been affected by such transactions, the Directors decided that a thorough investigation should be made into the business connected therewith, with the view of obtaining not only a record of the past, but also, if possible, of establishing a guide for the future, and I now have the pleasure, with their permission, of making public the results of that investigation.

Assurances upon unsound lives, in the full acceptation of the term, do not appear to have been effected in the early years of the Society's existence, only eight cases, and those of persons liable to Gout, having been accepted during the period from 1808 to 1813.

In the last-named year a policy was effected on the life of a person afflicted with Hernia, and from that time to the year 1824, when the subject had attracted more serious attention, but few assurances on unsound lives, other than those suffering from Gout and Hernia, were carried out. For some years past, however, the principle, as is well known, of assuring unsound lives at an increased rate of premium, has been more and more recognized and acted upon, and risks of that class now form a not unimportant section of the business of many Life Assurance Companies.

The total number of assurances upon unhealthy lives from the year 1813 up to the end of June 1871, effected in the Eagle and in the several companies which have been amalgamated with it, amounted, exclusive of lapsed, surrendered and term-expired policies, to 2,094, of which number I may remark *en passant* that only 723 had become claims.

Having had a full account of all these cases extracted from the records of the Office, and having also, with the kind assistance of one of our medical advisers (Dr. Ogle), arranged the different *Causes of addition* under the following seven heads, viz.,

GOUT,
HERNIA,
AFFECTIONS OF THE ORGANS OF CIRCULATION,
AFFECTIONS OF THE ORGANS OF RESPIRATION,
OBESITY,
INTEMPERATE HABITS,
FAMILY HISTORY, OR GENERAL WANT OF ROBUSTNESS,

I proceeded to classify the cases accordingly for every decade of life, with regard to each cause of addition as well as for the same periods combining all the causes.

It will be observed that I have also added an eighth section, entitled *Miscellaneous Affections.* This arose in consequence of the causes of addition which could not be included under any of the above heads, being rather numerous, and the number of cases appertaining to each being far too few to allow of their being grouped under separate heads with any real advantage, and also from the reasons of the addition having been, in very many of the earlier instances—particularly in the Amalgamated Companies—omitted from the records. These last were accordingly classed under the head of "*No Information*"; and they, together with the other cases referred to, were collected under the above title of *Miscellaneous Affections;* and, although they could not be of immediate service with regard to any special class of disease, yet as their number would affect the general result, it was quite necessary that they should be taken into account in the inquiry.

Having made a financial investigation with respect to the 2,094 cases alone, I found that, on the whole, the ordinary surplus had been realized, and this result might, perhaps, of itself have been considered sufficient to warrant, without further research, a continuance of the practice hitherto pursued by the Company, supported as it is by the length of time over which that practice has extended; but as the safety of such a course could not upon that investigation alone be properly asserted, I have thought it advisable to pursue the subject still further, and by the construction of the usual mortality table, to ascertain with technical precision to what extent the results already obtained might be trusted as a guide for future reference.

In order accurately to show the past working of the Office, I also had extracts made of the particulars of all the policies on these lives which either had been allowed to lapse, or had been surrendered, or the term of which had expired, and then combining these results with those previously obtained I drew up Table A, which represents the conditions as regards provision against extra risk upon which unsound lives are at present assured in the Eagle. I have given the number of cases under each head, in order that the value of the additions may be rightly appreciated, the total number of the cases being 3,147.

In the construction of the mortality table marked B, in which, as it will be seen, account has been taken of all lapsed, surrendered, and term-expired policies, I have availed myself of the formulæ employed by Mr. Jellicoe in obtaining the rates of mortality prevailing amongst the lives assured in the Eagle Company during the forty-four years ending December 1851, the manner of using which is fully explained by Mr. Jellicoe in his paper on the subject, published in the year 1854, in the fourth volume of the "*Journal of the Institute of Actuaries*".

Having thus deduced the rate of mortality, as shown in Table B, I proceeded to the construction of Table C, in which is shown the expectation or average duration of after-life at each age.

It will be observed that I have made use of the Office ages in this investigation. I did this, considering that by such a course it would be more readily shown whether the several additions which had been made to the true ages were sufficient for the purpose, seeing that, if that were the case, the "expectations" deduced therefrom would in all material respects fairly bear comparison with those obtained from other tables of mortality.

These "expectations" are, as it will be seen, unadjusted, but they are so nearly equal to the "Experience" expectations, as shown in Table D, that I have not deemed it necessary to enter upon the process of adjustment.

It is a matter of considerable satisfaction that the result obtained from this more precise and exhaustive investigation has quite corroborated the financial conclusions previously deduced; and considering the great uncertainty which has prevailed up to a late period as to the effect of specific diseases on the duration of human life, it is not a little remarkable that the purely empirical practice resorted to by the Company should have so closely met the exigencies of the case. It will be seen by Table D how nearly the additions made have placed the sound and unsound lives on the like footing, and an examination of the curves shown in Table E will serve still further to illustrate this very proximate identity.

I have thus endeavoured briefly to describe the practice of the Eagle, and of the other companies connected with it, in regard to the assurances which have been effected on unsound lives, and I can only now conclude with the expression of a hope that humble as the effort has been, the report now put forth may form a fitting, and I trust a useful, addendum to the paper by Mr. Jellicoe to which I have before referred.

TABLE A.—*Average additions which have been made to the true ages in cases of assurances upon lives classed as unsound. From* 1808 *to* 30*th June* 1871.

| Age at entry. | CAUSES OF ADDITION. | | | | | | | | | | | | | | | | Total Number of Cases. | Average Addition. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Gout. | | Hernia. | | Affections of the Organs of Respiration. | | Affections of the Organs of Circulation. | | Obesity. | | Intemperate Habits. | | Family History, or General want of Robustness. | | Miscellaneous Affections. | | | |
| | Number of cases. | Average addition, years. | Number of cases. | Average addition, years. | Number of cases. | Average addition, years. | Number of cases. | Average addition, years. | Number of cases. | Average addition, years. | Number of cases. | Average addition, years. | Number of cases. | Average addition, years. | Number of cases. | Average addition, years. | | |
| Under 20 ... | ... | ... | 3 | 4 | ... | ... | ... | ... | 1 | 10 | ... | ... | 14 | 10 | 26 | 10 | 44 | 9 |
| 20 to 29 ... | 16 | 6 | 64 | 4 | 60 | 10 | 16 | 8 | ... | ... | 15 | 12 | 187 | 8 | 242 | 8 | 600 | 8 |
| 30 „ 39 ... | 71 | 5 | 157 | 3 | 118 | 7 | 33 | 7 | 12 | 6 | 42 | 9 | 192 | 6 | 393 | 7 | 1,018 | 6 |
| 40 „ 49 ... | 170 | 4 | 133 | 3 | 69 | 6 | 27 | 7 | 17 | 7 | 41 | 6 | 132 | 6 | 293 | 6 | 882 | 5 |
| 50 „ 59 ... | 123 | 3 | 88 | 3 | 31 | 5 | 19 | 6 | 12 | 4 | 17 | 5 | 43 | 5 | 120 | 5 | 453 | 4 |
| 60 „ 69 ... | 38 | 2 | 29 | 2 | 3 | 5 | 6 | 4 | 3 | 4 | 3 | 4 | 11 | 4 | 45 | 4 | 138 | 3 |
| 70 and above | ... | ... | 7 | 2 | ... | ... | 3 | 6 | ... | ... | 1 | 1 | ... | ... | 1 | 3 | 12 | 3 |
| | 418 | 4 | 481 | 3 | 281 | 7 | 104 | 7 | 45 | 6 | 119 | 7 | 579 | 6 | 1,120 | 6 | 3,147 | 6 |

### TABLE B.—*Mortality of Unsound Lives from* 1808 *to* 30*th June* 1871.

| Office age ($x$). | Completed the age $x-\frac{1}{2}$. | Existing 30th June, 1871, between the ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. | Discontinued between the ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. | Died between the ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. | Number exposed to the risk of death between the ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. | Number exposed to the risk of death between the ages $x$ and $x+1$. | Number who died between the ages $x$ and $x+1$. | Mortality per cent per annum. | Office age ($x$). |
|---|---|---|---|---|---|---|---|---|---|
| 15 | 11 | ... | ... | ... | 11·0 | 22·0 | 0·0 | 0·0000 | 15 |
| 16 | 11 | ... | ... | ... | 11·0 | 22·0 | 0·0 | 0·0000 | 16 |
| 17 | 11 | ... | ... | ... | 11·0 | 22·5 | ... | 0·0000 | 17 |
| 18 | 12 | ... | 1 | ... | 11·5 | 21·5 | ... | 0·0000 | 18 |
| 19 | 11 | ... | 2 | ... | 10·0 | 17·5 | ... | 0·0000 | 19 |
| 20 | 10 | ... | 5 | ... | 7·5 | 21·5 | ... | 0·0000 | 20 |
| 21 | 14 | ... | ... | ... | 14·0 | 29·0 | ... | 0·0000 | 21 |
| 22 | 15 | ... | ... | ... | 15·0 | 35·5 | ... | 0·0000 | 22 |
| 23 | 21 | ... | 1 | ... | 20·5 | 44·5 | 1·0 | 2·2472 | 23 |
| 24 | 25 | ... | 2 | 1 | 24·0 | 60·0 | 1·0 | 1·6667 | 24 |
| 25 | 37 | ... | 2 | ... | 36·0 | 75·5 | ... | 0·0000 | 25 |
| 26 | 43 | 2 | 5 | ... | 39·5 | 92·5 | ... | 0·0000 | 26 |
| 27 | 55 | ... | 4 | ... | 53·0 | 120·0 | ... | 0·0000 | 27 |
| 28 | 69 | 1 | 3 | ... | 67·0 | 163·0 | ... | 0·0000 | 28 |
| 29 | 101 | 1 | 9 | ... | 96·0 | 259·0 | 1 | 0·3861 | 29 |
| 30 | 169 | 4 | 8 | 1 | 163·0 | 345·0 | 2 | 0·5797 | 30 |
| 31 | 193 | 8 | 14 | 1 | 182·0 | 437·5 | 3 | 0·6857 | 31 |
| 32 | 259 | 2 | 5 | 2 | 255·5 | 550·5 | 4 | 0·7266 | 32 |
| 33 | 304 | 4 | 14 | 2 | 295·0 | 640·0 | 3 | 0·4688 | 33 |
| 34 | 355 | 5 | 15 | 1 | 345·0 | 804·0 | 3 | 0·3729 | 34 |
| 35 | 469 | 4 | 16 | 2 | 459·0 | 965·0 | 8 | 0·8290 | 35 |
| 36 | 525 | 8 | 30 | 6 | 506·0 | 1081·0 | 10 | 0·9251 | 36 |
| 37 | 592 | 8 | 26 | 4 | 575·0 | 1200·5 | 7 | 0·5831 | 37 |
| 38 | 646 | 14 | 27 | 3 | 625·5 | 1296·5 | 7 | 0·5399 | 38 |
| 39 | 689 | 10 | 26 | 4 | 671·0 | 1489·0 | 11 | 0·7387 | 39 |
| 40 | 834 | 7 | 25 | 7 | 818·0 | 1646·0 | 19 | 1·1543 | 40 |
| 41 | 854 | 21 | 31 | 12 | 828·0 | 1700·0 | 27 | 1·5880 | 41 |
| 42 | 899 | 20 | 34 | 15 | 872·0 | 1760·0 | 23 | 1·3068 | 42 |
| 43 | 922 | 23 | 45 | 8 | 888·0 | 1806·0 | 19 | 1·0520 | 43 |
| 44 | 951 | 24 | 42 | 11 | 918·0 | 1935·0 | 21 | 1·0852 | 44 |
| 45 | 1042 | 17 | 33 | 10 | 1017·0 | 2054·5 | 23 | 1·1195 | 45 |
| 46 | 1067 | 31 | 28 | 13 | 1037·5 | 2056·5 | 32 | 1·5560 | 46 |
| 47 | 1058 | 37 | 41 | 19 | 1019·0 | 2030·0 | 33 | 1·6256 | 47 |
| 48 | 1053 | 47 | 37 | 14 | 1011·0 | 2014·5 | 28 | 1·3899 | 48 |
| 49 | 1045 | 48 | 35 | 14 | 1003·5 | 2042·0 | 30 | 1·4692 | 49 |
| 50 | 1081 | 43 | 42 | 16 | 1038·5 | 2028·0 | 29 | 1·4300 | 50 |
| 51 | 1036 | 54 | 39 | 13 | 989·5 | 1994·5 | 26 | 1·3036 | 51 |
| 52 | 1040 | 42 | 28 | 13 | 1005·0 | 2003·0 | 34 | 1·6976 | 52 |
| 53 | 1032 | 44 | 24 | 21 | 998·0 | 1943·0 | 42 | 2·1617 | 53 |
| 54 | 991 | 65 | 27 | 21 | 945·0 | 1877·0 | 49 | 2·6105 | 54 |
| 55 | 971 | 52 | 26 | 28 | 932·0 | 1804·0 | 51 | 2·8269 | 55 |
| 56 | 915 | 50 | 36 | 23 | 872·0 | 1708·5 | 46 | 2·6924 | 56 |
| 57 | 875 | 54 | 23 | 23 | 836·5 | 1629·0 | 40 | 2·4555 | 57 |
| 58 | 832 | 43 | 36 | 17 | 792·5 | 1539·5 | 40 | 2·5984 | 58 |
| 59 | 781 | 38 | 30 | 23 | 747·0 | 1477·0 | 45 | 3·0465 | 59 |
| 60 | 767 | 44 | 30 | 22 | 730·0 | 1402·0 | 41 | 2·9244 | 60 |
| 61 | 707 | 50 | 20 | 19 | 672·0 | 1317·0 | 32 | 2·4298 | 61 |
| 62 | 673 | 40 | 16 | 13 | 645·0 | 1265·0 | 26 | 2·0553 | 62 |
| 63 | 644 | 35 | 13 | 13 | 620·0 | 1199·0 | 28 | 2·3352 | 63 |
| 64 | 607 | 34 | 22 | 15 | 579·0 | 1111·5 | 34 | 3·0589 | 64 |
| 65 | 562 | 40 | 19 | 19 | 532·5 | 1021·0 | 46 | 4·5054 | 65 |

TABLE B.—*Mortality of Unsound Lives*—(continued).

| Office age ($x$). | Completed the age $x-\frac{1}{2}$. | Existing 30th June, 1871, between the ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. | Discontinued between the ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. | Died between the ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. | Number exposed to the risk of death between the ages $x-\frac{1}{2}$ and $x+\frac{1}{2}$. | Number exposed to the risk of death between the ages $x$ and $x+1$. | Number who died between the ages $x$ and $x+1$. | Mortality per cent per annum. | Office age ($x$). |
|---|---|---|---|---|---|---|---|---|---|
| 66 | 508 | 27 | 12 | 27 | 488·5 | 916·0 | 43 | 4·6943 | 66 |
| 67 | 454 | 34 | 19 | 16 | 427·5 | 823·0 | 33 | 4·0098 | 67 |
| 68 | 412 | 28 | 5 | 17 | 395·5 | 747·5 | 46 | 6·0539 | 68 |
| 69 | 366 | 25 | 3 | 29 | 352·0 | 659·5 | 54 | 8·1880 | 69 |
| 70 | 323 | 26 | 5 | 25 | 307·5 | 578·5 | 38 | 6·5687 | 70 |
| 71 | 287 | 23 | 9 | 13 | 271·0 | 514·5 | 24 | 4·6647 | 71 |
| 72 | 256 | 19 | 6 | 11 | 243·5 | 458·0 | 26 | 5·6769 | 72 |
| 73 | 224 | 14 | 5 | 15 | 214·5 | 400·5 | 27 | 6·7416 | 73 |
| 74 | 197 | 14 | 8 | 12 | 186·0 | 337·5 | 31 | 9·1853 | 74 |
| 75 | 161 | 17 | 2 | 19 | 151·5 | 282·5 | 29 | 10·2658 | 75 |
| 76 | 138 | 13 | 1 | 10 | 131·0 | 241·0 | 19 | 7·8838 | 76 |
| 77 | 117 | 10 | 4 | 9 | 110·0 | 195·5 | 22 | 11·2532 | 77 |
| 78 | 92 | 11 | 2 | 13 | 85·5 | 151·5 | 24 | 15·8416 | 78 |
| 79 | 71 | 8 | 2 | 11 | 66·0 | 116·5 | 19 | 16·3090 | 79 |
| 80 | 55 | 8 | 1 | 8 | 50·5 | 92·0 | 14 | 15·2174 | 80 |
| 81 | 44 | 5 | ... | 6 | 41·5 | 71·0 | 13 | 18·3099 | 81 |
| 82 | 32 | 5 | ... | 7 | 29·5 | 53·5 | 9 | 16·8224 | 82 |
| 83 | 26 | 4 | ... | 2 | 24·0 | 42·0 | 7 | 16·6667 | 83 |
| 84 | 19 | 2 | ... | 5 | 18·0 | 31·5 | 9 | 28·5714 | 84 |
| 85 | 14 | 1 | ... | 4 | 13·5 | 22·0 | 8 | 36·3640 | 85 |
| 86 | 9 | 1 | ... | 4 | 8·5 | 17·5 | 4 | 22·8571 | 86 |
| 87 | 9 | ... | ... | ... | 9·0 | 15·0 | 3 | 20·0000 | 87 |
| 88 | 6 | ... | ... | 3 | 6·0 | 12·0 | 3 | 25·0000 | 88 |
| 89 | 6 | ... | ... | ... | 6·0 | 12·0 | ... | 0·0000 | 89 |
| 90 | 6 | ... | ... | ... | 6·0 | 9·5 | 1 | 10·5263 | 90 |
| 91 | 4 | 1 | ... | 1 | 3·5 | 6·5 | 2 | 30·7692 | 91 |
| 92 | 3 | ... | ... | 1 | 3·0 | 6·0 | 1 | 16·6667 | 92 |
| 93 | 3 | ... | ... | ... | 3·0 | 6·0 | ... | 0·0000 | 93 |
| 94 | 3 | ... | ... | ... | 3·0 | 6·0 | ... | 0·0000 | 94 |
| 95 | 3 | ... | ... | ... | 3·0 | 3·0 | ... | ... | 95 |
| 96 | 1 | 1 | 1 | ... | 0·0 | 1·0 | ... | ... | 96 |
| 97 | 1 | ... | ... | ... | 1·0 | 2·0 | ... | ... | 97 |
| 98 | 1 | ... | ... | ... | 1·0 | 2·0 | ... | ... | 98 |
| 99 | 1 | ... | ... | ... | 1·0 | 2·0 | ... | ... | 99 |
| 100 | 1 | ... | ... | ... | 1·0 | 1·0 | 1 | ... | 100 |
| 101 | ... | ... | ... | 1 | ... | ... | 1 | ... | 101 |

## Table C.

| Office age. | Number who complete the age opposite. | Mortality per cent per annum. | Number who die in their next year. | Average duration of after life. | Office age. |
|---|---|---|---|---|---|
| 23 | 10000 | 2·247 | 225 | 40·52 | 23 |
| 24 | 9775 | 1·667 | 163 | 40·44 | 24 |
| 25 | 9612 | ... | ... | 40·12 | 25 |
| 26 | 9612 | ... | ... | 39·12 | 26 |
| 27 | 9612 | ... | ... | 38·12 | 27 |
| 28 | 9612 | ... | ... | 37·12 | 28 |
| 29 | 9612 | 0·386 | 37 | 36·12 | 29 |
| 30 | 9575 | 0·580 | 56 | 35·26 | 30 |
| 31 | 9519 | 0·686 | 65 | 34·46 | 31 |
| 32 | 9454 | 0·727 | 69 | 33·70 | 32 |
| 33 | 9385 | 0·469 | 44 | 32·93 | 33 |
| 34 | 9341 | 0·373 | 35 | 32·09 | 34 |
| 35 | 9306 | 0·829 | 77 | 31·21 | 35 |
| 36 | 9229 | 0·925 | 85 | 30·47 | 36 |
| 37 | 9144 | 0·583 | 53 | 29·75 | 37 |
| 38 | 9091 | 0·540 | 49 | 28·92 | 38 |
| 39 | 9042 | 0·739 | 67 | 28·07 | 39 |
| 40 | 8975 | 1·154 | 103 | 27·28 | 40 |
| 41 | 8872 | 1·588 | 141 | 26·51 | 41 |
| 42 | 8731 | 1·307 | 114 | 26·01 | 42 |
| 43 | 8617 | 1·052 | 91 | 25·35 | 43 |
| 44 | 8526 | 1·085 | 93 | 24·61 | 44 |
| 45 | 8433 | 1·120 | 94 | 23·88 | 45 |
| 46 | 8339 | 1·556 | 128 | 23·14 | 46 |
| 47 | 8211 | 1·626 | 134 | 22·51 | 47 |
| 48 | 8077 | 1·390 | 112 | 21·86 | 48 |
| 49 | 7965 | 1·469 | 117 | 21·16 | 49 |
| 50 | 7848 | 1·430 | 112 | 20·47 | 50 |
| 51 | 7736 | 1·304 | 101 | 19·75 | 51 |
| 52 | 7635 | 1·698 | 130 | 19·01 | 52 |
| 53 | 7505 | 2·162 | 162 | 18·33 | 53 |
| 54 | 7343 | 2·611 | 191 | 17·73 | 54 |
| 55 | 7152 | 2·827 | 202 | 17·18 | 55 |
| 56 | 6950 | 2·692 | 171 | 16·67 | 56 |
| 57 | 6779 | 2·456 | 167 | 16·08 | 57 |
| 58 | 6612 | 2·598 | 172 | 15·47 | 58 |
| 59 | 6440 | 3·047 | 196 | 14·87 | 59 |
| 60 | 6244 | 2·924 | 183 | 14·32 | 60 |
| 61 | 6061 | 2·430 | 147 | 13·74 | 61 |
| 62 | 5914 | 2·055 | 122 | 13·07 | 62 |
| 63 | 5792 | 2·335 | 135 | 12·33 | 63 |
| 64 | 5657 | 3·060 | 173 | 11·61 | 64 |
| 65 | 5484 | 4·505 | 247 | 10·96 | 65 |
| 66 | 5237 | 4·694 | 246 | 10·46 | 66 |
| 67 | 4991 | 4·010 | 200 | 9·95 | 67 |
| 68 | 4791 | 6·154 | 295 | 9·34 | 68 |
| 69 | 4496 | 8·188 | 368 | 8·92 | 69 |
| 70 | 4128 | 6·569 | 271 | 8·67 | 70 |
| 71 | 3857 | 4·665 | 180 | 8·27 | 71 |
| 72 | 3677 | 5·677 | 209 | 7·63 | 72 |
| 73 | 3468 | 6·742 | 234 | 7·06 | 73 |
| 74 | 3234 | 9·185 | 297 | 6·55 | 74 |
| 75 | 2937 | 10·266 | 308 | 6·15 | 75 |
| 76 | 2629 | 7·884 | 207 | 5·80 | 76 |
| 77 | 2422 | 11·253 | 273 | 5·26 | 77 |
| 78 | 2149 | 15·842 | 340 | 4·86 | 78 |
| 79 | 1809 | 16·309 | 295 | 4·68 | 79 |
| 80 | 1514 | 15·217 | 230 | 4·50 | 80 |
| 81 | 1284 | 18·310 | 235 | 4·21 | 81 |

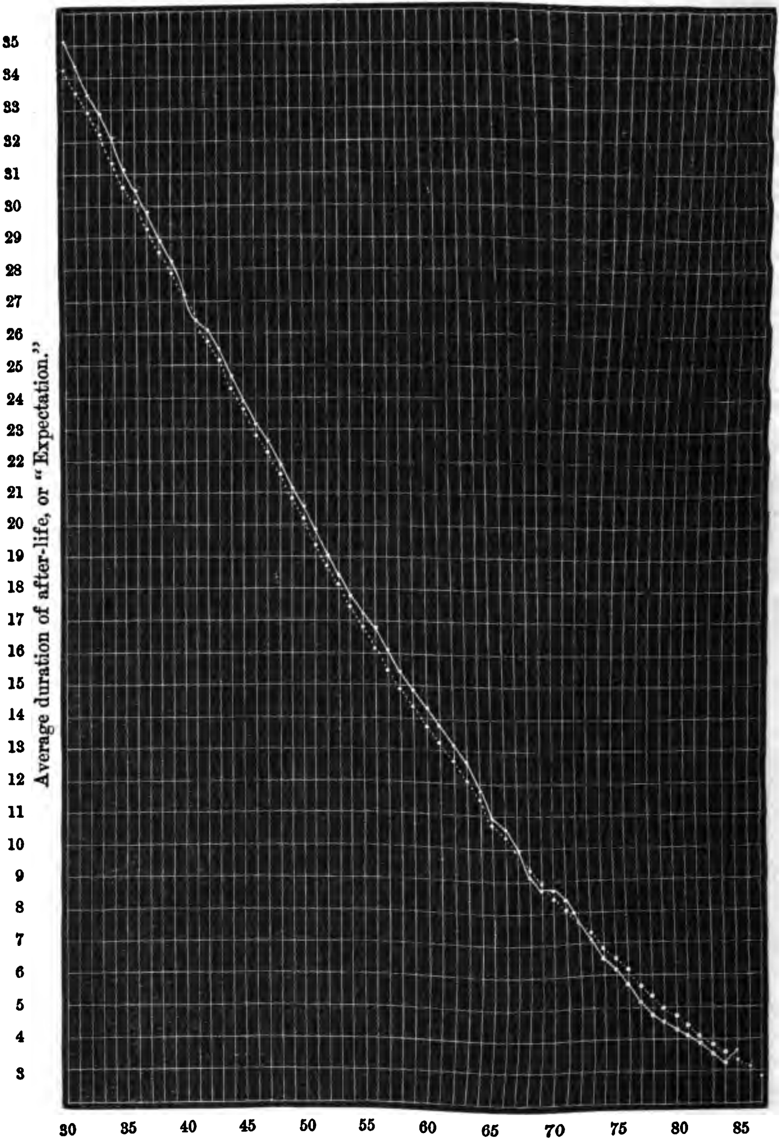
TABLE C—(continued).

| Office age. | Number who complete the age opposite. | Mortality per cent per Annum. | Number who die in their next year. | Average duration of after life. | Office age. |
|---|---|---|---|---|---|
| 82 | 1049 | 16·822 | 176 | 4·04 | 82 |
| 83 | 873 | 16·667 | 145 | 3·75 | 83 |
| 84 | 728 | 28·571 | 208 | 3·40 | 84 |
| 85 | 520 | 36·364 | 189 | 3·56 | 85 |
| 86 | 331 | 22·857 | 76 | 4·31 | 86 |
| 87 | 255 | 20·000 | 51 | 4·44 | 87 |
| 88 | 204 | 25·000 | 51 | 4·43 | 88 |
| 89 | 155 | ... | ... | 4·67 | 89 |
| 90 | 155 | 10·526 | 16 | 3·67 | 90 |
| 91 | 139 | 30·769 | 43 | 3·03 | 91 |
| 92 | 96 | 16·667 | 16 | 3·17 | 92 |
| 93 | 80 | ... | 18 | 2·70 | 93 |
| 94 | 62 | ... | 16 | 2·34 | 94 |
| 95 | 46 | ... | 14 | 1·98 | 95 |
| 96 | 32 | ... | 12 | 1·63 | 96 |
| 97 | 20 | ... | 9 | 1·30 | 97 |
| 98 | 11 | ... | 7 | ·92 | 98 |
| 99 | 4 | ... | 3 | ·75 | 99 |
| 100 | 1 | 100·000 | 1 | ·50 | 100 |

TABLE D.—*Comparative Table of Expectations.*

| Office Age. | Carlisle. | Experience. | Eagle, 1873. Unadjusted. | Office Age. |
|---|---|---|---|---|
| 30 | 34·34 | 34·43 | 35·26 | 30 |
| 32 | 33·03 | 33·01 | 33·70 | 32 |
| 34 | 31·68 | 31·58 | 32·09 | 34 |
| 36 | 30·32 | 30·15 | 30·47 | 36 |
| 38 | 28·96 | 28·72 | 28·92 | 38 |
| 40 | 27·61 | 27·28 | 27·28 | 40 |
| 42 | 26·34 | 25·84 | 26·01 | 42 |
| 44 | 25·09 | 24·40 | 24·61 | 44 |
| 46 | 23·82 | 22·97 | 23·14 | 46 |
| 48 | 22·50 | 21·56 | 21·86 | 48 |
| 50 | 21·11 | 20·18 | 20·47 | 50 |
| 52 | 19·68 | 18·82 | 19·01 | 52 |
| 54 | 18·28 | 17·50 | 17·73 | 54 |
| 56 | 16·89 | 16·22 | 16·67 | 56 |
| 58 | 15·55 | 14·97 | 15·47 | 58 |
| 60 | 14·34 | 13·77 | 14·32 | 60 |
| 62 | 13·31 | 12·61 | 13·07 | 62 |
| 64 | 12·30 | 11·51 | 11·61 | 64 |
| 66 | 11·27 | 10·46 | 10·46 | 66 |
| 68 | 10·23 | 9·47 | 9·34 | 68 |
| 70 | 9·18 | 8·54 | 8·67 | 70 |
| 72 | 8·16 | 7·67 | 7·63 | 72 |
| 74 | 7·33 | 6·86 | 6·55 | 74 |
| 76 | 6·69 | 6·11 | 5·80 | 76 |
| 78 | 6·12 | 5·42 | 4·86 | 78 |
| 80 | 5·51 | 4·78 | 4·50 | 80 |
| 82 | 4·93 | 4·18 | 4·04 | 82 |
| 84 | 4·39 | 3·63 | 3·40 | 84 |
| 86 | 3·90 | 3·10 | 4·31 | 86 |
| 88 | 3·59 | 2·59 | 4·43 | 88 |
| 90 | 3·28 | 2·11 | 3·67 | 90 |
| 92 | 3·37 | 1·67 | 3·17 | 92 |
| 94 | 3·53 | 1·28 | 2·34 | 94 |
| 96 | 3·46 | ·99 | 1·63 | 96 |
| 98 | 3·07 | ·75 | ·92 | 98 |
| 100 | 2·28 | ... | ·50 | 100 |

TABLE E.—*Curves denoting the Expectation or Average Duration of After-life.*



The dotted curve denotes the "Expectation" according to the 17 Offices' Experience, the other that of the Eagle Unsound lives.

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Since communicating to the Institute of Actuaries the above article, I have traced the causes of death in 505 of the cases, thinking that it might be of advantage to know how far the diseases and affections which rendered it necessary in the first instance to make the extra charge, were connected with those complaints which eventually caused the death of the life assured.

This information I have collected and arranged in the following table.

| Cause of Addition. | Cause of Death. | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Gout. | Hernia. | Intemperance. | Natural Decay. | Cancer. | Dropsy. | Erysipelas. | Fever. | Affections of the Organs of Respiration. | Affections of the Organs of Circulation. | Affections of the Nervous System. | Affections of the Liver. | Affections of the Digestive Organs. | Affections of the Genito-Urinary Organs. | Number of Cases. |
| Gout ... ... ... ... | 16 | ... | ... | 17 | 5 | 4 | 2 | 1 | 17 | 14 | 37 | 6 | 14 | 15 | 148 |
| Hernia ... ... ... | ... | 1 | 1 | 9 | 3 | ... | 1 | 2 | 26 | 15 | 20 | 3 | 10 | 6 | 97 |
| Organs of Respiration ... | ... | ... | ... | 2 | ... | ... | 1 | ... | 52 | 1 | 8 | 6 | 2 | 9 | 81 |
| Organs of Circulation ... | ... | ... | 1 | ... | ... | 2 | ... | 1 | 7 | 8 | 4 | 3 | ... | 2 | 28 |
| Obesity ... ... ... | ... | ... | ... | ... | ... | 1 | ... | ... | 6 | 3 | 3 | 1 | 1 | 1 | 16 |
| Intemperance ... ... | ... | ... | ... | ... | 3 | ... | ... | 2 | 11 | 4 | 5 | 7 | 2 | 3 | 37 |
| Family History, or General want of Robustness | ... | ... | 3 | 5 | 1 | ... | ... | ... | 39 | 10 | 17 | 5 | 8 | 10 | 98 |
| Totals ... ... ... | 16 | 1 | 5 | 33 | 12 | 7 | 4 | 6 | 158 | 55 | 94 | 31 | 37 | 46 | 505 |

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The following abstract of the discussion which followed the reading of the paper is taken from the *Insurance Record*.

The President having invited discussion,

Mr. Bunyon asked Mr. Humphreys whether he could trace the number of diseased lives, classed under the heads of organs of respiration and intemperance, that had died; the average addition in each case being only seven years.

Mr. Humphreys said he had traced whether there had been a loss or gain on those lives, and he found that on the whole they had not paid sufficiently, and therefore a heavier addition would be required. He felt that it was no use to make a table of premiums for each disease, because people naturally expected the state of their disease to be assessed. It might be a fair average case, or a very bad one; and the same premium would not do for both cases. Supposing, for instance, it were an average case of gout, the company would add a certain number of years in consequence, but people would not pay an average rate in other cases.

Mr. Bunyon asked whether that was not specially the case with intemperate lives.

Mr. Porter enquired what was meant by intemperate lives?

Mr. **HUMPHREYS** said the Mutiny Act defined as an intemperate man anyone who got drunk three times in a month, or something of that sort.

The **PRESIDENT** recollected a celebrated case in which the Eagle was concerned, where the late Lord Chief Baron laid it down that no amount of after-dinner conviviality would constitute intemperance, but that the man who drank brandy in the morning was the really intemperate.

Mr. **BROWN** asked Mr. Humphreys whether the causes of death were traced, and whether the connection between those causes and the reason for putting up the life, had been investigated.

Mr. **HUMPHREYS** said he had got the data in some cases, but not in all. The early records were faulty. So far as he could ascertain, a great many of the lives had died from causes quite irrespective of the disease for which the addition was originally made.

Mr. **WALFORD** said that he had had occasion recently to go through a great deal of the history of diseased life insurance. Since the advent of the Clerical and Medical in 1824, no less than 50 different companies had started, more or less with the view of insuring diseased lives. A considerable percentage of these companies had had tables specifically prepared for their business, but not one of them furnished any clue to the materials from which they were deduced. He had taken a great deal of trouble to obtain publications of parliamentary committees, reports on Friendly Societies, and other official documents, from which people expected to obtain information, in order to ascertain whether there were any sources of information open to the companies; but he could not find anything clear or definite on the subject. There was no doubt that the officials of the Asylum Company did take the trouble to go through some army agents' books and trace the lives, more particularly with reference to the influence of foreign residence on health; and thus got a large collection of facts, which were no doubt the most trustworthy data which could at that time be obtained. But how the facts thus collected were applied to the tables, did not appear clear. The impression on his mind after careful observation was that hardly any of the companies had made the business pay. That might arise from several causes; probably from the fact that they did not get enough of it to make it pay. There was, he thought, one obvious reason, and that was that in cases of bad colds,— as also for intemperance and other common forms of disease, —the companies had put on a small extra charge; but for gout, hernia, and palpable diseases of that sort, they had put on a large addition sufficient to drive the business away. Now, he believed that the specific charges for what would be regarded as serious diseases had paid very well, but that small additions for that class of ailments which were considered trivial had not paid. It might be taken for granted that cases of consumption, for instance, which had been slightly surcharged, generally resulted in a loss to the companies. It was quite clear that no tables could be constructed which would be applicable to all cases, the only method was to regard each case on its own merits, submitting it to the best medical scrutiny they could obtain, and quoting a rate in accordance with the medical officer's report. There had been several attempts in America to found diseased life insurance companies, and he had had the honour of being consulted upon the subject. He had looked into the facts and made investigations, but the conclusion was inevitable that nothing could guide the office but the experience and skill of their medical staff. With regard to the diseased life experience published by the Institute, they did not take the rated-up age, but the true age; and that would no doubt account for the difference between the Institute table and the table of the Eagle, the expectation under which was higher all the way through, up to the extreme ages, than the Experience table. That would naturally be so, and it harmonized very well with the rating-up applied to most of the cases which Mr. Humphreys had submitted to them. The question, which was one of great interest, had hitherto been enveloped in the greatest obscurity, and anything which tended to elucidate it would always be welcomed by every member of that Institute.

Mr. A. H. Bailey thought they would all agree with Mr. Walford that any attempt to elucidate this obscure subject was welcomed by all of them. The result of his experience was that during the fifty years in which the insurance of diseased lives had been studied and practised, very little progress had been made, and no real information obtained as to the proper premiums to charge. In 1824, the date of the establishment of the Asylum and the Clerical and Medical Offices, it was thought possible to fix the proper premium to be charged for a man, say, in an advanced stage of phthisis, or for a man both of whose parents had died of consumption, or for a man suffering from organic disease of the heart. His own notion was that if a life was below the ordinary standard, it was much below it; and therefore that small additions were useless, whilst large additions the public would not pay. He thought therefore they were reduced very much to the state of things existing fifty years ago, with this slight difference, that the lives which are admitted now as diseased lives, and for which an extra premium is charged, would have been taken fifty years ago at the ordinary rates. Statistics are readily obtainable, by which comparisons can be made of the numbers living with the numbers dying, and ratios of mortality deduced; but how is it possible to determine the mortality amongst persons exposed to any particular disease? In the case of gout, for example,—a common disease enough,—they could not determine what was the increased mortality due to that disease; nor, indeed, to any other. Mr. Walford had alluded to the Asylum, and in the latest number of his *Insurance Cyclopædia*, which had come out quite opportunely for this discussion, he had with his usual industry and research brought together almost all the available information on the subject. He (Mr. Bailey) had in his possession one of the early prospectuses of the Asylum Company, wherein it was stated that "the Directors had by great care and research ascertained the true law of mortality for various diseases." No traces of that discovery were to be found in the books and papers of the Asylum that came into the possession of the London Assurance Corporation, and no law beyond mere guesswork seemed to have been acted upon in fixing the premiums. Some results of the Asylum business might be of interest. The London Assurance contributed with several other companies to the mortality experience collected by the Institute of Actuaries;

and before he parted with the cards, he thought it well to get out the particular mortality of the London Assurance, which of course included the Asylum with its considerable number of diseased lives. He found there were no less than 558 deaths amongst diseased lives up to 31st December 1863; and in comparing those with the healthy lives, he found that, taken throughout, the annual mortality per-cent was 3·55 against 1·99 for healthy lives, which was not far short of double. At the ages from 25 to 45 it was as nearly as possible double. Omitting the first ten years of assurance, in order to get rid of the effect of selection, it was found that, instead of what was hoped and might have been expected, the mortality amongst the diseased lives diminishing, it steadily increased, and he had no doubt whatever that the additions could not possibly have compensated for that increased mortality in any shape or way. There were about 8,000 policies issued by the Asylum, of which there were between 500 and 600 remaining; and if he lived a few years longer, he did not despair of seeing the greater part of this experience worked out; and then, if his health permitted, he hoped to be able to give to the Institute the whole of the results connected with the mortality experience of the Asylum. But, so far as the assurance of diseased lives was concerned, he had no doubt whatever that the scheme had been a failure. As he had said, he believed that no progress whatever had been made to solve this difficult question, and it is now just as difficult for a man with an undoubted mortal disease upon him to get his life assured as ever it was. [Mr. WALFORD—More difficult.] He believed it was. The doctors have acquired more experience and knowledge, and there is a greater attempt at discrimination now than formerly, but he firmly believed that the solution of the problem to find a rate for every case that offered is as far off as ever.

Mr. MACFADYEN thought that there was no question in which insurance practice was more chaotic than that of which Mr. Humphreys's paper treated; and its frequent discussion, though it might not lead to scientific accuracy of treatment, would at least help to bring out clearly the true difficulties of the matter, and perhaps in time might lead to less diversity of practice. He did not quite see how Mr. Humphreys arrived at the conclusion that on the whole the ordinary surplus had been realized on the lives taken at extra by the Eagle office. It might be so; but he did not see the proof of it in the paper. It did not follow from the interesting fact arrived at by Mr. Humphreys, namely, that the expectations of life in the invalid cases were as good as those of healthy lives of corresponding premiums. Expectations of life might be equal, but premium rates might be very different. Neither, in such a conglomerate mortality, would it do to conclude that the profit had been as great as on normal risks, because the extra premiums deduced from the under-average mortality table, showed something like the average extra received. And it would not do to accumulate from year to year the premiums received after deducting the corresponding claims, because the future mortality among the current policies was unknown. It seemed to him, then, not at all an easy matter to ascertain that the premiums charged had been such as to provide the ordinary surplus. Even admitting that in the past this had been so, they were not much nearer the end of the difficulties.

In under-average risks it was to him rather doubtful whether past experience could altogether be relied on for future guidance. For example, if he remembered aright, the recent investigation by the Scottish Faculty of Actuaries showed that persons afflicted with hernia are positively better lives than the normal risks. Is hernia, then, calculated to lengthen life, and is it in future to be an inducement for an office to take an assurance? Again, family history had only lately among medical men received much attention, and many a person that would now be charged extra for consumptive taint, would have escaped notice in under-average statistics. Altogether, he saw no hope of ever getting beyond a very rough classification of extra premiums. Still, things might be better than now. If, instead of as at present trying to predict the length of individual life,—for that was really what it came to,—they would be content to have two or three great classes for such cases as could be taken at all, there might be some chance, with care in selection and the publication of such experience as Mr. Humphreys had brought before them, of arriving in time at safe working tables for these two or three classes. No doubt, the rougher the classification, the less likely was the individual case to have its mortality closely defined by it. But so long as the deviation was not great enough to affect the class mortality severely, this was not absolutely necessary. In fact, risks are taken every day in ordinary assurance business that the premium does not exactly represent; for it can not be supposed that all lives taken at the ordinary rates are of the same vitality.

Mr. HODGE said that, although the observations of Mr. Humphreys were valuable in many respects, they were not so valuable as those observations which applied to ordinary purposes, inasmuch as they did not enable them to judge what premiums ought really to be charged upon diseased lives. From the observations deduced from healthy lives it was possible to estimate with scientific accuracy the value of a pecuniary interest depending upon a life at a certain age; but when they came to deal with the mortality of persons suffering from some disease, they immediately had in question the intensity of the disease, each person suffering from it in a different degree of intensity. Unless they could make a great many classifications of diseases, he did not see how they could do anything in the way of anticipating the future. As Mr. Humphreys had pointed out, the method of dealing with the question is purely empirical; and the only point is whether there is any faith to be placed in the judgment of medical men on the subject. Now, the paper before them was valuable, as showing that what they hardly believed could be done, had been done in the Eagle with considerable success. That company had, as the result of a great deal of experience and judgment exercised upon such subjects, succeeded in estimating the increased risk of disease. There was another point in which he thought the paper was a valuable one. All persons connected with insurance offices were, probably, not aware that there is a general impression among a large portion of the public that when a man has anything the matter with him,—any taint in the character of his health,—the offices are apt to take advantage of it to extort from him a heavier premium than the circumstances demand, so as to make a large profit out of him. Now, it is quite clear from the experience of the Eagle in regard to these cases that that has not been done; and he thought it might be satisfactory to those who, being subject to disease, are prevented from obtaining insurances at the ordinary rates, to know that they are fairly and justly dealt by, and that they can insure without paying a heavier premium than the circumstances of the case render necessary. So far that is satisfactory. There is no doubt the process has been a tentative one, and they knew very well that most of the offices which have undertaken this kind of business have at first been unsuccessful. The Asylum failed entirely; the Clerical and Medical, established about the same time as the Asylum, was, according to the evidence of Mr. Pinckard, in the early part of its career a loser by this kind of business; and the Medical Invalid was obliged to transfer its business, but whether it was because they found the insurance of diseased lives unprofitable or not, he did not know. It was reported at the time that they would insure persons suffering from an attack of cholera. With the exception of the transactions of that Institute, he did not believe there had ever been any information upon this subject at all to be relied upon. He remembered very well that the directors of the Asylum did publish a statement to the effect that they had satisfactory information with regard to various diseases and climates. He happened to know a great deal about the getting up of the Asylum, and he believed that the data about which so much was made were mere moonshine. He recollected a gentleman who was intimately connected with the promotion of the company being strongly advised not to give up the data which he possessed upon the subject, and he as strenuously asserted his intention not to do so, for the simple reason that he had none to communicate. And as to the promoter having a knowledge of diseases in foreign countries, he (Mr. Hodge) happened to know that the man was quite ignorant as to the part of the globe in which the different countries were. No other information of the kind is accessible, and therefore they were greatly indebted to Mr. Humphreys for the accumulation of facts which he had brought before them. He (Mr. Hodge) did not think they would ever get beyond an empirical method of treating these cases. They must be dependent upon the judgment of the medical officers. It was satisfactory to find that the result of the experience in the case of the Eagle justified their judgment, and that they could proceed upon the same principles as had guided them in their decisions hitherto.

Mr. R. P. HARDY said that, considering the unsatisfactory mode in which medical assessments are usually made, he thought the results shown by the tables most encouraging. The medical men never consider, in assessing a rate for a diseased life, what the effect will be upon the premium. They put on an addition of five, seven, or ten years, but never six, eight, or nine years; and if they are asked for a reason why they add seven instead of six, they cannot give a reply. It was not twelve months ago that a joint life insurance was proposed upon a gentleman in a high position and his wife. The wife was a delicate woman, and it was doubtful whether the change of climate she was about to get, would not develop rather than retard the phthisis which was hereditary in her family. The medical man proposed to put seven years on her life, which made a most insignificant difference in the premium, which he (Mr. Hardy) did not consider was at all an equivalent for running a risk of that nature. It was usual for medical men to have by them a table of the expectation of life; and if they thought a man would not live the period there set down, they took him at an age corresponding to the diminished expectancy. But surely if that were the measure of the greatest possible duration of his life, he ought to have been rated at an age where by the tables he could not live that term. What they should look to, was to see what effect it would have upon the premium; because, unless they had a large premium, an early death would hit the company very hard. One point the medical men attach great importance to is the early deaths of the parents; but as a large number of deaths occur in the prime of life, it is evident there are a large number of persons in existence whose parents could not have attained a very old age. With regard to some of the additions made in the case of the Medical Invalid, he had had the schedules of that company under his notice, and he found that some of the rated-up cases had reached to the extreme of life—one case at the commencement of the company, where the addition was 30 years.

Mr. HODGE asked Mr. Humphreys whether he knew what was the highest rate of addition—not the average rate—in each class.

Mr. HUMPHREYS replied that he had got the whole of the data. He knew there was one case in which an addition of 34 years was made.

Mr. PORTER asked Mr. Humphreys whether it had been the custom in the company he represented, in dealing with cases in which there was a slight deviation from the normal standard of health, to charge the extra premium for a number of years, say seven or fifteen, and then if the disease had not developed itself to revert to the ordinary premium. With regard to a remark made by Mr. Hodge as to the varying intensity of disease, and the consequent impossibility of comparing tables of diseased lives, he had heard of a curious instance of that. A child under the care of Dr. West was suffering from mesenteric disease; and it was said that if there was consumption in either of the parents, the child must have died.

Mr. BUNYON enquired of Mr. Humphreys whether he had investigated the particular mortality which occurred amongst the gout and hernia cases, because although, as had been more than once observed that evening, the individual cases were all marked in the ordinary practice according to the supposed intensity of the disease that existed, yet they knew that the common practice with insurance companies had been to charge so much for gout and so much for hernia. The old offices used to make an addition of 10 per-cent, and sometimes 11½ per-cent, to the ordinary premium for gout. He should like to know, as a matter of curiosity, what was the additional rate of mortality in the particular cases which had been assumed to have something like a uniform intensity of disease. In cases of consumption and so on, it is impossible to fix any uniform premium; because, in one instance, a small addition will suffice, and in another, an addition of 35 years will not be enough to meet the risk.

Mr. HUMPHREYS said he had worked out each disease financially, but not as a separate mortality table.

Mr. HODGE enquired how the cases of hernia turned out?

Mr. HUMPHREYS said they turned out very successfully,—rather better in fact than the others. The average duration of the policies with regard to hernia was 12 years, and of each policy on the whole of the diseased lives, 10 years. The average of the policies on lives afflicted with gout was 10 years. The financial result followed somewhat the same law.

The PRESIDENT remarked that he believed the old extra charge of 10 per-cent was fixed because the Equitable charged 11 per-cent.

Mr. MANLY wished to know whether in the different columns family history was included with personal history. He noticed in the family history column there were only 579 cases out of a total of 3,147, which appeared a very small number indeed.

Mr. HUMPHREYS said that in the family history column, which included cases of general want of robustness, a very small addition was required. There was often no actual disease, but the lives were not what could be called first-class.

Mr. MANLY said he did not know whether Mr. Humphreys had examined Mr. Meikle's investigations into the diseased lives amongst the ten Scottish offices, but he there divided the doubtful lives into two classes—personal history and family history; and if he (Mr. Manly) remembered rightly, Mr. Meikle found that amongst those who were rated-up for family history, there being no disease found in the individual, the mortality was very nearly the ordinary average; but that the mortality amongst the lives which were rated-up for some defect in the personal constitution, was so bad that the business was hardly worth having. [Mr. BAILEY remarked that there were not 500 deaths in those observations.] Mr. MANLY said he was aware of that, but still the fact was very striking, and it was most important that it should be followed out. It was very difficult to reject a man because his family might have a tendency to disease, he himself being in apparently good health. This was a subject which would, he thought, be worthy of consideration, and if Mr. Humphreys would take it in hand on a future occasion, it would be an acceptable addition to their knowledge of the subject, which is one of real practical importance.

Mr. BAILEY remarked that with regard to family history, when he insured in the Eagle many years ago, questions on that subject were never asked. It was quite a modern practice to enquire into family history.

The PRESIDENT, in proposing a vote of thanks to Mr. Humphreys for his valuable contribution to their transactions, expressed a hope that they might again have the pleasure of listening to him on this difficult but important subject.

Mr. HUMPHREYS said he was very much obliged to them for the attention they had given him on that occasion. With reference to Mr. Porter's question, he could not call to mind how many cases had been met by a limited number of payments. In a recent case it was arranged that the larger premium should extend over twelve payments, so that the whole of the premiums at the increased age should be paid in eleven years,—the insurance not to be paid till the death of the person insured. He was fully aware of the difficulties environing this matter which Mr. Macfadyen had brought to their notice; but he had placed before them the best materials he had at command, and he hoped they would be found of some practical value.

## *On the method of comparing the Expected with the Actual Experience of a Life Insurance Company, as regards the number of Deaths and amount of Claims. By* GEORGE M. LOW, F.F.A., *of the English and Scottish Law Life Association, Edinburgh.*

THE business of Life Assurance is founded on the principle that the number of deaths which will occur among a large number of persons in a given time is not a matter dependent entirely on what is called chance; but is subject to a law of average so uniform in its operations, and so trustworthy as to its results, as to be capable of forming the basis of calculations on which the shareholder may stake his capital, and the assured the welfare of those for whom it is his duty to provide. Proceeding on this principle, tables of mortality have been constructed,—from data collected at different places and under a variety of circumstances, and tabulated and adjusted with various degrees of accuracy,—which show, out of a given number of persons born, the number who survive each year of age, and the number who die at each age from year to year. From such tables, companies transacting the business of assurance calculate the rates of contribution to be required from their clients; and on the sufficiency of the rate of mortality founded on in any instance depends, of course, the sufficiency of the premiums which the company is to receive in consideration of its risks. It is therefore of great consequence that the company should ascertain from time to time how the rate of mortality actually experienced compares with that indicated by the table on which the calculations are founded, in order that there may be proper grounds for being satisfied that the basis is a correct one. For it is not sufficient that the mortality table founded on should be known to represent faithfully the death-rate prevailing among the general population, or even among assured lives generally. Every office has, in the peculiar nature of the benefits which it offers—the localities from which its business is drawn—the character and social position of its agents—the skill, and, it may be, the peculiar opinions of its medical officers and its executive — and, indeed, in an endless variety of circumstances—influences at work which tend to modify in a greater or less degree the quality of the lives which come upon its books, and consequently the rate of mortality which it is likely to experience. Nor do these remarks apply to offices only in the earlier stages of their existence. It is well known that the value of some of the most popular tables is not equal at all ages; and it is not until a company has been in existence for a very considerable period that it is able to test, from its own experience, the trustworthiness of the table on which its calculations have been founded, as regards the later periods of life. The Carlisle Table, for instance, once almost universally popular, is now gradually being abandoned, mainly because the experience of the older offices has shown its untrustworthiness at the higher ages of assured life.

Another important use of the calculated mortality occurs in connection with the periodical investigations which are made into the affairs of companies with the view of ascertaining the amount of profit arising from their transactions. It is very desirable to know what portions of the total profit have been derived from interest, mortality, margins, or other sources. Now the only way in which the actuary can ascertain what portion of his surplus has arisen from the actual mortality experienced being lighter than that calculated upon—or to what extent the surplus has been encroached upon by mortality in excess of that provided for,—is by calculating the amount of the expected claims for the period over which the investigation extends, upon the basis of the mortality table from which the premiums were charged.

Having thus endeavoured to show the importance of the calculations to which this paper refers, I will now proceed to consider some of the methods which are adopted by offices for ascertaining the number and amount of their expected claims, and to inquire to what extent these methods appear to diverge from strict accuracy. But, first, it may be necessary to say that the word "expected" is here used, not in its literal sense, as implying something which is likely to happen, but in a sense resembling in some degree that in which the phrase "expectation of life" is used:—the "claims to be expected" being, not those which it is anticipated will actually emerge, but those which are provided for by the table of mortality employed. Of course, no one would "*expect*" this number actually to arise, any more than a man of, say, 30 would "*expect*" to survive exactly the number of years enjoyed on the average by persons of his age.

In most offices such estimates as we are now to consider are made and recorded from year to year, so that the materials are always at hand for instituting a comparison between the expected mortality and that which has been developing itself among the lives assured during the various stages of the company's progress; while in some offices it is only thought necessary to institute a strict inquiry into the results of mortality experience at the investigation periods; any intermediate estimates being merely approximate, and not relied upon as furnishing the materials for comparison with any great degree of accuracy. In the observations which follow, I shall have in view primarily the first of these two systems, namely, that of estimating mortality from year to year—but the same general principles are applicable to both cases; the quantities on which the calculations are founded being always the number of lives and the amount of sums assured which are exposed to the risk of mortality for one year at each age.

Of all the various circumstances which affect the chances of life and death in individual instances, that only of age is taken into account by assurance companies in their calculations; it being assumed that selected lives of the same age are equally exposed to the chances of mortality, and any deviation from the general standard of eligibility in particular cases being generally met by referring the life in question to a class in which the chances of death are greater—that is, by making an addition to the real age in calculating the premium to be charged for the risk. Hence it is, that in dealing with the whole number of lives upon the books of a company, it is only necessary to treat separately the numbers existing at the different ages; and as has been said, that in calculating the number of expected deaths, the quantities founded on are the numbers exposed to risk at each age. The first step in the calculation, therefore, is to collect together the numbers existing at the several ages, which is accomplished by recording each assurance under the year in which the policyholder was born; care being taken, of course, to eliminate duplicate policies, so that each life may only be reckoned once, no matter how many different assurances may depend upon its existence.

Having thus ascertained the numbers alive at the commencement of any particular year, nothing would remain to be done in order to arrive at the expected mortality for that year but to multiply these numbers by the respective rates of mortality at the various ages, were these numbers not subject to increase, and to diminution from other causes than the death of the parties. But some of the policies are likely to be allowed to drop, others to be surrendered, and the period for which others have been effected may expire before the close of the year; while, at most ages, new assurances are likely to come upon the books in each of the twelve months. In all these instances, it is evident that if the office were to treat each policy as exposed to a whole year's risk of mortality, the liability in respect of such assurances would be over-estimated. Hence it is found necessary to defer calculations of this nature until the end of the year, when the extent of the withdrawals which may have taken place, and the number of new assurances effected, may be ascertained, and proper allowance made for them. One consideration suffices to indicate the nature of this allowance, and the manner in which such cases fall to be treated, namely, that as all the lives of a particular age are assumed to be exposed to the same risk of mortality, any which are at risk only for portions of a year may be treated as so many fractions of a life exposed to risk during the whole year; and thus the sum of the terms (say, in months) during which the lives of those who have dropped their policies, and those who have effected new assurances, have been at risk during the year, being added to the number of lives on which a whole year's risk has been incurred, will represent the total number at risk for that year. The following example will serve to illustrate this. It is not put forward in the shape in which the calculation would occur in practice, but is merely intended to show the application of the principle.

AGE 55.

| | |
|---|---|
| Number exposed to risk for the whole year . . . | 50 |
| Lapsed 31 January . . . . . . . | $\frac{1}{12}$ |
| Surrendered 30 June . . . . . . | $\frac{6}{12}$ |
| Expired 31 August . . . . . . . | $\frac{8}{12}$ |
| Effected 28 February . . . . . . | $\frac{10}{12}$ |
| „ 30 June . . . . . . . | $\frac{6}{12}$ |
| „ 31 July . . . . . . . | $\frac{5}{12}$ |
| „ 30 September . . . . . . | $\frac{3}{12}$ |
| „ 30 November . . . . . . | $\frac{1}{12}$ |
| No. of "lives at risk" for the year . . . . | $53\frac{4}{12}$ |
| Rate of mortality at age 55, 2 percent, or per life | ·02 |
| Number of deaths to be expected at this age= $53\cdot\dot{3}\times\cdot02=$ . . . . . . . | 1·067 |

The method here indicated appears to be the most thoroughly accurate one; but others have been adopted involving assumptions more or less approximately true, but which are thought by those who employ them to be sufficiently correct for practical purposes.

One of these methods is that of adding to the number alive at each age at the commencement of a year the whole number of new entrants at that age, and deducting the whole number of withdrawals,—the nett result thus brought out being simply the number of lives remaining on the books at each age at the close of the year. It is obvious that the number thus found is not in any true sense the number of lives which have been exposed to risk during the year; and that if the number of admissions has exceeded that of the withdrawals (the average duration of the policies being the same), the number of lives exposed to risk will be stated too high; while if the withdrawals have exceeded the admissions, the number stated will be too low. Such a method, therefore, could give a correct result only in the event of the withdrawals being exactly balanced by the admissions at each age, with reference both to their number and to their average duration within the year. This, however, is never likely to happen in practice, except, perhaps, at a few of the middle ages of life. At the older and younger ages, where the admissions and withdrawals respectively are of comparative insignificance, the assumption here involved will generally be very far from true; and on the whole this method will be found to be utterly fallacious as applied to the business of an office whose membership is in any other than a stationary condition; for so long as the new admissions are considerably in excess of the withdrawals (which is, of course, generally the case), the system under consideration will inevitably show an expectation of mortality exceeding that warranted by the table of observations employed; while if the number of withdrawals should ever come to exceed the admissions, the result would obviously be that the number of expected deaths brought out by the method would fall short of the number which a more accurate system of calculation would have indicated. Taking the example already given, in which it was found that the correct number of "lives at risk" was $53\frac{1}{3}$, it will be seen that (the admissions having exceeded the withdrawals, and the average duration of the policies having been the same) this method would have stated the number at risk too high; thus—

| | |
|---|---|
| Number at risk at commencement of year . . | 53 |
| Add new entrants . . . . . . . | 5 |
| | 58 |
| Deduct withdrawals . . . . . . . | 3 |
| "Lives at risk" . . . . . . . . | 55; |

being $1\frac{2}{3}$ (or rather more than 3 percent) in excess of the true number.

| | |
|---|---|
| Multiplying the number then found by the rate of mortality (·02) gives the number of expected deaths . . . . . . . . | 1·100, |

instead of 1·067 as already found.

A better system, and one which seems to be more generally adopted than any other, is that of adding to the number alive at the beginning of the year one-half of the number admitted during the year, and deducting one-half of the withdrawals which have taken place otherwise than by death. This method is a decided improvement on the other, and in a series of extensive calculations will give results which may be regarded as pretty nearly accurate, since the assumption which it involves seems not unreasonable, namely, that on each of the lives entering and withdrawing the office will, on the average, have run during the year six months' risk. If, for greater accuracy, allowance be made in the proportion taken of the new entrants, for the acceleration of business which usually takes place towards the close of the financial year—which may be done by adding, say, only *one-third* of the new lives—the deviations of the results from strict accuracy will probably fall within very narrow limits. But since even when such a method as this is employed, each admission and each withdrawal must be separately dealt with in one way or another, and as the exact term of risk run upon each life is known, it may be questioned whether the additional trouble involved in finding the sum of these terms would not be compensated by the greater accuracy of the result. In all actuarial work of this kind, *some* assumptions and approximations have necessarily to be made; and for this reason, if for no other, it is desirable that strict accuracy should be adhered to as far as practicable.

In the example which has already been given, the calculation by this method would stand as follows:—

| | |
|---|---|
| Number of lives at risk at commencement of the year . . . . . . . . . . | 53 |
| Add one half new entrants . . . . . | 2½ |
| | 55½ |
| Deduct one half withdrawals . . . . . | 1½ |
| "Lives at risk" for the year . . . . . | 54 |
| Multiplying by ·02, the rate of mortality, gives the number of expected deaths . . . . . | 1·080; |

while the true number was found to be 1·067.

It will be observed that the number of lives at risk brought out by this method is 54, as against $53\frac{1}{8}$, the number found by summing the exact terms of risk. Had the average duration of the new and dropped policies been exactly six months, the numbers found by both methods would, of course, have been alike; but this is a condition which does not always prevail in practice.

Another method which is sometimes adopted, and which differs somewhat in principle from that just referred to, seems to call for more partieular remark: it is that of including the claims which have actually arisen in the deduction made on account of withdrawals—the number of "lives at risk" being obtained either by adding to the number existing throughout the whole year the fractions corresponding to the respective terms of risk run under the new assurances and the withdrawals, including claims; or by adding to the number existing at the commencement of the year one half of the new admissions (or such other proportion as may be thought more nearly accurate), and deducting one half of the withdrawals, including those which have taken place by death. In this case the rate of mortality employed is not $\frac{d_x}{l_x}$ (the tabular number of deaths taking place between ages $x$ and $x+1$ divided by the tabular number alive at age $x$), but $\frac{d_x}{l_x-\frac{1}{2}d_x}$, or $\frac{d_x}{\frac{1}{2}(l_x+l_{x+1})}$ —the tabular number of deaths divided by the tabular number alive in the middle of the year, assuming the deaths to be equally distributed over the period in which they occur. In accordance with this latter hypothesis, it will be convenient, in investigating this method, to assume that the deaths which actually occur will take place after the lives have been on an average six months at risk during the year.

The formula employed in this case $\left(\frac{d_x}{l_x-\frac{1}{2}d_x}\right)$ is of itself objectionable, inasmuch as its arithmetical values occasionally prove somewhat anomalous. Mr. Meikle has remarked upon this in his "Observations on the Mortality Experience of the Scotch Offices," where he shows, for example, that if at a certain age two lives have been at risk, and have both failed within a year, this formula will give a rate of mortality for that year of 2 per life. For

$$d_x=2, \quad l_x-\tfrac{1}{2}d_x=1,$$

and

$$\frac{d_x}{l_x-\frac{1}{2}d_x}=\frac{2}{1}=2.$$

Such anomalies, however, will occur only when the number of deaths bears a very considerable proportion to the number of lives exposed to risk; so that at the younger and middle ages of life, where the numbers dying are very small as compared with the numbers surviving, the formula will represent nearly enough the average rate of mortality during each year of age. The values of this formula are, of course, greater than those of the other throughout the whole period of life.

But the method of calculation in which the formula referred to is employed, is objectionable on other grounds. Those who adopt it proceed on the principle that allowance should be made for the fact, that lives which have failed during the year to which the calculation of expected mortality refers have not been exposed to risk during the whole of that period; and if this principle be admitted, then the plan adopted, or some modification of it, would seem to be the only practicable one. But it should be borne in mind that these calculations, although necessarily deferred until a period when the actual facts of the year's experience have become known, are in their nature *forecasts*—their object being to ascertain, among the whole number of lives on which the office incurs risk during a year, the number which, according to the mortality table, may be expected to fail *during the same period;* and it would seem to be somewhat inconsistent to take account, in a calculation made with this view, of the mortality which has actually prevailed, for the result will be that in that case the actual number of deaths will be allowed to some extent to modify the theoretical number. This modification, moreover, will take effect in such a way that (unless the ordinary method of calculation be fallacious) the theoretical mortality will be slightly *overstated* when the actual mortality has been *less* than that which was to have been expected, and *understated* when the actual mortality has been *more* than the theoretical: so that the effect of applying this method is to make actual results appear more divergent from theoretical expectation than they really are, and consequently to exaggerate any profit or loss which may appear to have arisen from the mortality of a particular year.

This may be demonstrated as follows:—If we take $\lambda_x$ to represent the number of lives aged $x$ on the books of an office at the beginning of a particular year, and, for simplicity, assume that there are to be no new admissions during that year, and no withdrawals otherwise than by death (such admissions and withdrawals having in reality nothing to do with the merits of the question before us), then by the ordinary method of calculation the number of deaths to be expected during the year will be

$$\lambda_x \times \frac{d_x}{l_x};$$

while by the method under consideration the number would be (putting $\delta_x$ for the deaths which will actually have taken place by the end of the year)

$$(\lambda_x - \tfrac{1}{2}\delta_x) \times \frac{d_x}{l_x - \frac{1}{2}d_x}.$$

The number of expected deaths, therefore, which this method will bring out, will be greater than, equal to, or less than, the number which would be brought out by the ordinary method of calculation, according as

$$(\lambda_x - \tfrac{1}{2}\delta_x) \times \frac{d_x}{l_x - \frac{1}{2}d_x} > = < \lambda_x \times \frac{d_x}{l_x}$$

As $$\frac{\lambda_x - \frac{1}{2}\delta_x}{l_x - \frac{1}{2}d_x} > = < \frac{\lambda_x}{l_x}$$

$$\frac{\lambda_x - \frac{1}{2}\delta_x}{\lambda_x} > = < \frac{l_x - \frac{1}{2}d_x}{l_x}$$

$$1 - \frac{\frac{1}{2}\delta_x}{\lambda_x} > = < 1 - \frac{\frac{1}{2}d_x}{l_x}$$

As $$\frac{d_x}{l_x} > = < \frac{\delta_x}{\lambda_x}$$

or, conversely, as $$\frac{\delta_x}{\lambda_x} < = > \frac{d_x}{l_x};$$

that is, according as the actual rate of mortality is less than, equal to, or greater than, the theoretical. So that the results brought out by this method will always differ from those of the ordinary method, except only in the event of the office's mortality experience coinciding, at all points, with the tabular expectation—an event which, of course, is never likely to happen.

Now, in the case we have been considering—that of a certain number of lives exposed to the risk of mortality for a year, and not subject to increase, nor to diminution from any other cause than death, it will scarcely be disputed that the method of multiplying the number of lives of each age on the books at the commencement of the year by the rate of mortality $\frac{d_x}{l_x}$, will give correctly the number of deaths to be expected at that age. For since, by the ordinary rules of proportion,

$$l_x : d_x :: \lambda_x : \lambda_x \times \frac{d_x}{l_x},$$

it follows that $\lambda_x \times \frac{d_x}{l_x}$ will represent the number of deaths which will take place among $\lambda_x$ persons alive at the commencement of the

$(x+1)$th year of age, so long as $d_x$ represents the number which will occur among $l_x$ such persons. This being so, it will follow that the results of any other method of calculation are correct only in so far as they coincide with those brought out in this way. But it has been shown that the results of the method we have been considering will always differ from those thus obtained. It would appear, therefore, that that method may fairly be characterized from a theoretical point of view as both anomalous and fallacious, —anomalous, because it allows actual experience to modify theoretical expectation; and fallacious, because its results do not agree with those of a more proper and consistent mode of calculation.

It is right to observe, however, that the deviations of the results of this method from those of the ordinary one are not very considerable, since (except at extreme ages) they may be said to consist generally of the mortality expectation in respect of but a small proportion of the lives which fail from year to year. An illustration of this will be seen in the table on next page, which exhibits the number of deaths (calculated by both methods) to have been expected, according to the English Life Table, No. 1, among the lives comprised in the statistics of the Scotch Offices' Experience to 1863.

In order to make the comparison more complete, there has been set down, beside the calculated number of deaths, the respective rates of mortality deduced from the English Life Table and from the actual experience of the offices. Where the actual rate of mortality is *greater* than the theoretical, the former is printed in antique type; and, in accordance with what has already been said, it will be observed that wherever this is the case, the theoretical number of deaths calculated from the formula $\frac{d_x}{l_x - \frac{1}{2}d_x}$ is *less* than the number calculated from the ordinary formula $\frac{d_x}{l_x}$, while the former number is greater at all other ages. The difference in the total results, however, is not very important—the ordinary method showing 13,799 deaths, while the other brings out 13,818, being a difference of only 19 deaths, or about 1·4 per 1000.

| | |
|---|---|
| The actual number of deaths which occurred was | 11,947 |
| And the theoretical number, as already stated . | 13,799 |
| The actual number being thus less than the computed number by . . . . . . | 1,852 |

SCOTCH OFFICES' EXPERIENCE.—*Deaths to have been expected according to the English Life Table No.* 1 (*Males*)*; calculated by two different methods.*

| Age. | MORTALITY PER CENT. | | EXPECTED DEATHS. | | Age. | MORTALITY PER CENT. | | EXPECTED DEATHS. | |
|---|---|---|---|---|---|---|---|---|---|
| | English Life Table. | Offices' Experience (Unadjusted). | $\lambda_x \times \frac{d_x}{l_x}$ | $(\lambda_x - \frac{1}{2}\delta_x) \times \frac{d_x}{l_x - \frac{1}{2}d_x}$ | | English Life Table. | Offices' Experience (Unadjusted). | $\lambda_x \times \frac{d_x}{l_x}$ | $(\lambda_x - \frac{1}{2}\delta_x) \times \frac{d_x}{l_x - \frac{1}{2}d_x}$ |
| 0 | 15·93 | ·00 | 6·06 | 6·58 | 50 | 1·70 | 1·68 | 307·39 | 307·42 |
| 1 | 6·30 | ·00 | 5·20 | 5·37 | 51 | 1·74 | 1·61 | 297·67 | 297·86 |
| 2 | 3·39 | ·00 | 3·49 | 3·55 | 52 | 1·79 | 1·71 | 287·57 | 288·34 |
| 3 | 2·45 | 1·67 | 2·92 | 2·93 | 53 | 1·83 | 1·63 | 275·72 | 275·99 |
| 4 | 1·78 | ·00 | 2·27 | 2·30 | 54 | 1·88 | 1·74 | 264·11 | 264·29 |
| 5 | 1·45 | ·00 | 2·04 | 2·05 | 55 | 2·11 | 2·22 | 277·79 | 277·63 |
| 6 | 1·17 | ·65 | 1·81 | 1·81 | 56 | 2·33 | 2·31 | 285·47 | 285·51 |
| 7 | ·95 | ·00 | 1·66 | 1·67 | 57 | 2·51 | 2·22 | 283·15 | 283·56 |
| 8 | ·77 | ·00 | 1·61 | 1·61 | 58 | 2·70 | 2·44 | 281·99 | 282·35 |
| 9 | ·62 | ·00 | 1·52 | 1·53 | 59 | 2·92 | 2·26 | 280·86 | 281·78 |
| 10 | ·50 | ·70 | 1·44 | 1·43 | 60 | 3·14 | 3·05 | 277·89 | 278·01 |
| 11 | ·51 | ·00 | 1·65 | 1·66 | 61 | 3·39 | 2·98 | 269·98 | 270·54 |
| 12 | ·51 | ·54 | 1·86 | 1·86 | 62 | 3·67 | 3·27 | 265·94 | 266·47 |
| 13 | ·62 | ·23 | 2·66 | 2·67 | 63 | 3·95 | 3·75 | 259·26 | 259·53 |
| 14 | ·68 | ·38 | 3·57 | 3·58 | 64 | 4·26 | 3·49 | 249·12 | 250·11 |
| 15 | ·70 | ·31 | 4·53 | 4·54 | 65 | 4·60 | 4·57 | 239·65 | 239·70 |
| 16 | ·71 | ·12 | 5·87 | 5·88 | 66 | 4·96 | 4·42 | 227·49 | 228·12 |
| 17 | ·73 | ·48 | 7·72 | 7·73 | 67 | 5·35 | 4·52 | 216·41 | 217·33 |
| 18 | ·75 | ·59 | 10·26 | 10·27 | 68 | 5·77 | 5·41 | 204·66 | 205·03 |
| 19 | ·77 | ·82 | 14·14 | 14·13 | 69 | 6·22 | 5·41 | 190·90 | 191·70 |
| 20 | ·79 | ·72 | 19·71 | 19·71 | 70 | 6·70 | 5·80 | 178·90 | 179·74 |
| 21 | ·81 | ·45 | 27·32 | 27·37 | 71 | 7·22 | 6·04 | 167·21 | 168·24 |
| 22 | ·84 | ·58 | 38·76 | 38·81 | 72 | 7·79 | 8·32 | 158·14 | 157·70 |
| 23 | ·85 | ·86 | 52·33 | 52·32 | 73 | 8·38 | 7·47 | 143·59 | 144·28 |
| 24 | ·88 | ·89 | 68·96 | 68·95 | 74 | 9·02 | 10·42 | 131·54 | 130·58 |
| 25 | ·90 | ·66 | 87·84 | 87·95 | 75 | 9·72 | 8·07 | 116·82 | 117·83 |
| 26 | ·92 | ·75 | 107·42 | 107·52 | 76 | 10·45 | 10·67 | 105·71 | 105·59 |
| 27 | ·95 | ·67 | 129·18 | 129·35 | 77 | 11·26 | 9·27 | 92·25 | 93·21 |
| 28 | ·97 | ·85 | 150·82 | 150·92 | 78 | 12·10 | 12·47 | 81·49 | 81·33 |
| 29 | ·99 | ·74 | 173·23 | 173·45 | 79 | 13·02 | 13·79 | 67·96 | 67·68 |
| 30 | 1·02 | ·89 | 196·93 | 197·06 | 80 | 13·97 | 12·37 | 55·32 | 55·80 |
| 31 | 1·05 | ·69 | 216·70 | 217·10 | 81 | 15·00 | 16·91 | 46·13 | 45·65 |
| 32 | 1·08 | ·77 | 237·24 | 237·61 | 82 | 16·10 | 15·69 | 36·94 | 37·02 |
| 33 | 1·10 | ·89 | 256·90 | 257·11 | 83 | 17·26 | 24·63 | 29·43 | 28·24 |
| 34 | 1·13 | ·91 | 274·73 | 275·03 | 84 | 18·49 | 17·12 | 20·52 | 20·67 |
| 35 | 1·16 | ·88 | 292·06 | 292·47 | 85 | 19·83 | 22·89 | 16·46 | 16·17 |
| 36 | 1·19 | 1·01 | 306·54 | 306·82 | 86 | 21·16 | 25·00 | 11·00 | 10·77 |
| 37 | 1·22 | 1·03 | 316·50 | 316·82 | 87 | 22·68 | 25·00 | 7·26 | 7·16 |
| 38 | 1·25 | 1·00 | 326·20 | 326·61 | 88 | 24·04 | 26·32 | 4·57 | 4·51 |
| 39 | 1·29 | 1·05 | 336·21 | 336·61 | 89 | 25·81 | 16·67 | 3·10 | 3·26 |
| 40 | 1·32 | 1·06 | 343·38 | 343·84 | 90 | 27·43 | 20·00 | 2·74 | 2·86 |
| 41 | 1·36 | 1·15 | 347·22 | 347·58 | 91 | 29·11 | 28·57 | 2·04 | 2·05 |
| 42 | 1·39 | 1·06 | 347·89 | 348·48 | 92 | 30·81 | 20·00 | 1·54 | 1·64 |
| 43 | 1·42 | 1·08 | 347·22 | 347·83 | 93 | 33·18 | 66·67 | 1·49 | 1·19 |
| 44 | 1·46 | 1·12 | 345·42 | 346·01 | 94 | 34·07 | ·00 | ·68 | ·83 |
| 45 | 1·50 | 1·24 | 342·17 | 342·61 | 95 | 37·87 | ·00 | ·57 | ·70 |
| 46 | 1·54 | 1·33 | 336·10 | 336·46 | 96 | 37·18 | 100·00 | ·37 | ·23 |
| 47 | 1·58 | 1·34 | 329·30 | 329·69 | | | | | |
| 48 | 1·62 | 1·40 | 321·52 | 321·88 | | | | | |
| 49 | 1·66 | 1·45 | 314·81 | 315·14 | | Total | … … | 13,799·66 | 13,818·44 |

The other method of calculation, therefore, which was found to bring out 19 more deaths, exaggerates the gain to the offices, in respect of the number of deaths, to the extent of about 1 percent.

We have thus far dealt only with calculations having for their object the determination of the theoretical *number of deaths* to be expected among assured lives; and if assurance offices existed merely for the purpose of testing the merits of mortality tables with respect to the death-rates which they exhibit, there would be little further to remark. But the business of an assurance office is to guarantee the payment of certain stipulated sums of money on the occurrence of stated events; and its prosperity, therefore, depends mainly upon the relation which is maintained between the amount of the fund at its disposal to meet these eventualities, and the amount of the claims which are made upon it from time to time; so that any question as to the results of the office's experience is of ultimate importance only when viewed from a financial point of view. Hence it would seem to be a matter of much greater consequence to know whether the amount of the claims brought upon its funds by the death of policyholders were greater or less than the amount provided for by the premiums received, than simply to ascertain whether the actual number of deaths had been greater or less than the expected number. For it will not always follow that if in any year the number of deaths which have occurred has fallen short of the number to have been anticipated, the financial results of the year in respect of mortality will likewise have proved satisfactory. The claims may have fallen most heavily among lives assured for large amounts, and thus the total amount claimed may have actually exceeded the amount to have been expected. In a similar way the mortality of a particular year might be excessive, and yet the amount of the claims be only equal to, or even under, the expectation.

It is curious to note that some actuaries appear to attach but little importance to the mere number of the lives which have dropped; while others rest their comparisons of actual mortality with theoretical upon the number of deaths, to the exclusion of the sums claimed. But for any really useful comparison of actual results with theoretical expectation, it is extremely desirable that both the amount of expected claims and the number of expected deaths should be accurately ascertained.

A method sometimes adopted for ascertaining the amount of claims to be expected in a year, is to multiply the number of expected deaths by the average amount exposed to risk on a single life. This is done on the assumption that the assurances claimed are likely to be of the average amount at risk; it being apparently taken for granted that if this method will not always furnish correct results, yet that there exists no reason why its results should necessarily be wrong. But if regard be had to the following considerations, it will appear that this assumption is by no means a trustworthy one.

In the first place, it is quite evident that in any office which distributes surplus among its policyholders in the shape of reversionary bonus additions to the sums assured, the average amount at risk will be greatest on the older lives, among whom the rate of mortality is at its maximum; and consequently, the heavy mortality falling among the larger sums claimable, the average amount to be claimed must certainly be expected to exceed the mere average amount at risk at all ages. This consideration is so obvious, that probably no one would apply the method alluded to except with reference to original sums assured only; but, apart altogether from the question of bonus, there are other influences tending in the same direction, which are not quite so apparent, and which seem to be overlooked by those who employ the method referred to. For example, policies of small amount and on young lives are more readily dropped than those which are more valuable, either on account of the sums which they assure, or of the age of the persons on whose lives they depend. Further, persons who defer assuring until comparatively late in life, are generally more able to afford policies of large amount than those who assure at earlier ages; while again, this latter class frequently add to their assurances from time to time, as their means increase, and they become better able to provide in this way for those dependent on them.

The effect of such influences as these must obviously be to increase the average amount at risk on the older lives as compared with the younger; and that this effect is not inconsiderable will be seen on reference to the appended diagram, which exhibits the average amount assured at each age in the Scotch offices, whose experience has already been used in illustration of this paper. There is, of course, no influence of bonus in these averages, the amounts given in the table from which they are calculated being original sums assured. It will be observed that during the most important period of life—indeed, so long as the number of lives assured is sufficient to ensure a good average—the amount increases from age to age with almost perfect regularity. The averages for the quinquennial groups of age are as follows:—

Average amount at risk on each life at ages

| | | | |
|---|---|---|---|
| Under 10 | £414 | 50 to 54 | £961 |
| 10 to 14 | 668 | 55 „ 59 | 1035 |
| 15 „ 19 | 607 | 60 „ 64 | 1095 |
| 20 „ 24 | 619 | 65 „ 69 | 1130 |
| 25 „ 29 | 639 | 70 „ 74 | 1183 |
| 30 „ 34 | 702 | 75 „ 79 | 1117 |
| 35 „ 39 | 753 | 80 „ 84 | 1105 |
| 40 „ 44 | 832 | 85 „ 89 | 966 |
| 45 „ 49 | 901 | 90 & upwards | 703 |

while the average over the whole lives and sums at risk is £838.

It is obviously necessary then that, to make a correct estimate of the amount of the claims to be expected by a company, some better method must be adopted than that of merely multiplying the whole number of expected deaths by the general average of the sums at risk. The only possible way of performing the calculation with anything like precision, is to treat sums at risk separately, in the same way as lives at risk, namely, to ascertain the amount on which, at each age, a year's risk of mortality has been incurred, and to multiply that amount by the rate of mortality at the particular age.

If $s_x$ represent the sum exposed to risk for a year at age $x$, and, as before, $\lambda_x$ the number of lives at risk, the average amount of assurance on each life is $\frac{s_x}{\lambda_x}$. Now since the chances of death to each individual in this class are assumed to be equal, the average amount of the claims to be expected will coincide with that of the sums exposed to risk; and consequently, the number of expected deaths being $\lambda_x \times \frac{d_x}{l_x}$, the amount of expected claims will be

$$\frac{s_x}{\lambda_x} \times \lambda_x \times \frac{d_x}{l_x}$$

$$= s_x \times \frac{d_x}{l_x}.$$

For the purpose of contrasting the results of calculation by this method with those of the system last alluded to, I again take an illustration from the Scotch Offices' Experience. We have already seen that the sums at risk increased with the age of the lives assured, and that consequently claims were to have been expected of a greater average amount than the mere general average of sums at risk.

| | |
|---|---|
| The amount of the expected claims calculated upon the sums at risk at each age is . . | £12,732,695 |
| While the amount obtained by multiplying the total number of expected deaths by the average sum assured among the whole lives is . . | 11,569,181 |
| The result of this latter method therefore falls wide of the mark by no less than . . | £1,163,514 |

or fully 9 percent of the correct amount of expected claims—a discrepancy of very material importance. It is interesting to note that while the average amount exposed to risk was £838, and the average amount of the expected claims £923, the average of the claims which actually occurred was £926. The results of the experience of these offices were slightly more favourable in respect of the number of deaths which occurred than of the amount of the claims which emerged.

Throughout these observations we have been dealing only with the case of assurances on single lives. These are the only kind of risks which admit of being classified and dealt with in the mass in any calculation which has reference to the value of life at different ages. Assurances on joint lives, joint lives and survivor, survivorship assurances, and the like, generally occur in such comparatively small numbers that they can only be dealt with individually, and the mortality due in respect of each calculated from the formula applicable to the status on which it depends. It may be noted, however, that the formulæ for ascertaining the number of deaths and the amount of claims in such cases will not always be alike. For example, in the case of an assurance depending on the longest of two lives, the formula to be employed for estimating the addition which it implies to the amount of the expected claims will be that which expresses the chance of the sum assured becoming payable within the year, namely,

$$\frac{d_x}{l_x} \times \frac{d_y}{l_y};$$

while the quantum of mortality due in respect of the assurance must be obtained from the formula

$$\frac{d_x}{l_x} + \frac{d_y}{l_y},$$

since the failure of either of the lives within the year would have to be regarded as an addition to the year's deaths, although the actual falling in of the assurance will not happen until the termination of the other life involved.

One other point remains to be noticed. If assurances are classified with reference to year of birth only, the lives involved will be on an average six months older at the end of each calendar year than the age which they attained during the year. Now it is a common practice in making calculations of the nature referred to in this paper to employ the ratios applicable to the ages to be attained during the year for which the estimates are made, so that if the calculation be made for a calendar year, each life will be treated as half a year older than it really is. It is interesting to notice the extent to which this affects the results of the calculation; and for this purpose I will again take an illustration from the Offices' Experience.

| | |
|---|---|
| It has been already shown that the number of expected deaths, calculated at completed ages, as given in the published statistics, was according to the English Life Table . . . . . . . | 13,799 |
| Had the lives been taken as each six months older, the number of expected deaths would have been brought out as . . . . . . . . | 14,118 |
| There being a difference of . . . . . . | 319 |

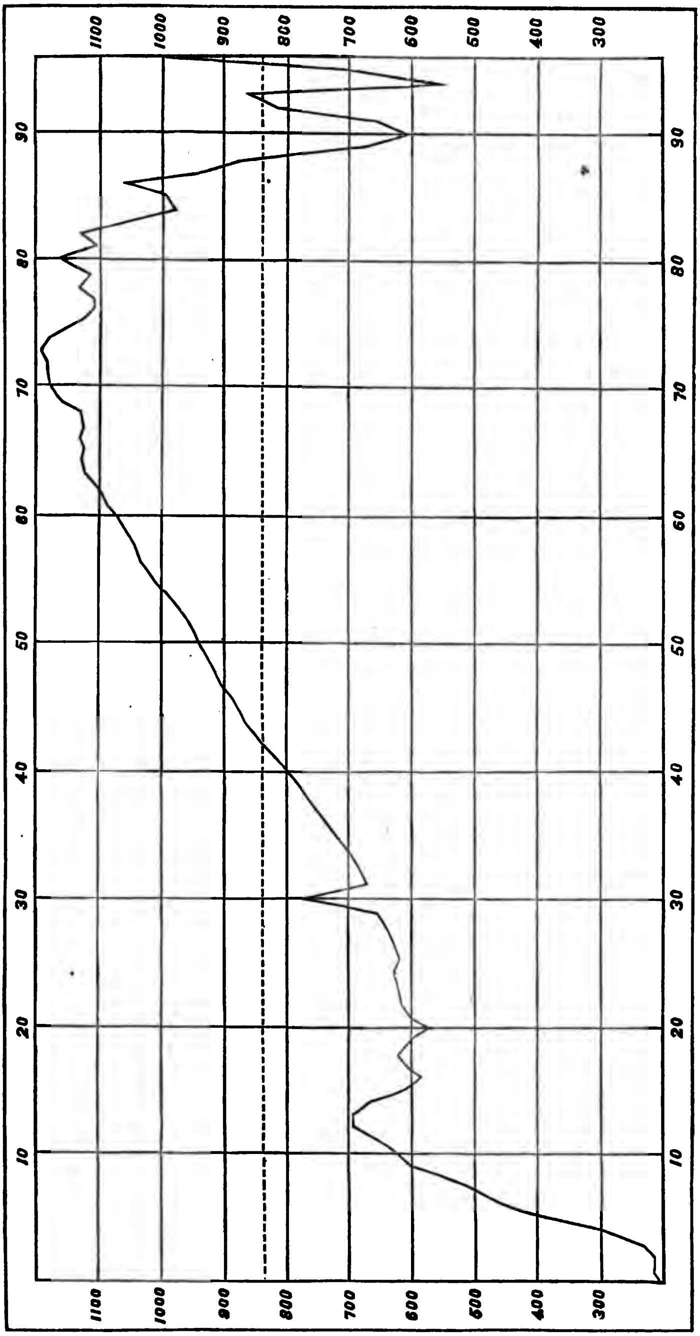
deaths, or fully $2\frac{1}{4}$ percent of the computed number.

| | |
|---|---|
| Again, the amount of expected claims, calculated at completed ages, was . . . . . | £12,732,695 |
| While at ages increased by six months the amount would have been . . . . . . | 13,055,710 |
| The difference being . . . . . . | £323,015 |

or fully $2\frac{1}{2}$ percent of the computed amount.

It would seem therefore that the over-estimate of mortality made by computing at ages next birthday is not quite insignificant. It is, no doubt, to some extent met by the premiums being charged at ages next birthday; but the gain to the office from this source does not amount to six months of age, since many persons purposely defer making their assurances until about to complete a year of their age; and besides, it is probably better that the theoretical claims should be accurately estimated at the real ages of the assured, in order that the deviations of experience from expectation may be made clearly apparent.

*Average Sums at Risk.—Scotch Offices' Experience.*



The continuous line shows the average amount at risk on a single life at each age. The dotted line shows the average per life of the sums at risk at all ages.

## *On Rates of Mortality and their Causes. By* W. Lazarus, *of Hamburg, Actuary of the General Insurance Company of Trieste. Translated by* T. B. Sprague, M.A., *Manager of the Scottish Equitable Life Assurance Society.*

(Continued from page 61.)

We are so accustomed, even when we do not aim at scientific exactness, to regard every phenomenon as produced by causes,—as the resultant of forces,—that we cannot help looking on what we call "life" as the effect of certain definite forces, and on death as the consequence of the extinction of those forces. But when we thus contrast life and death, we must always bear in mind not only that what we call "life" is a condition which presents itself under circumstances of unexhaustable variety, but that in all probability quite as many differences in its manifestation entirely elude our observation. Given the effect, we reason as to the cause; when the effects are different, we infer that either the causes are different, or the conditions on which the effect depends. But if the differences are so numerous that we can scarcely find two cases exactly alike, we can draw no conclusion either as to the causes or as to the conditions. Since in no single case can we recognize the various conditions under which the cause of life works, we cannot hope to decide how the effect is connected with the conditions and how with the causes. Death too is a phenomenon as to which we have not the least idea why it happens; whether because the vital force in the individual has sunk below a certain degree which is necessary for the maintenance of life, or whether as a result of wholly different forces. On this point we scarcely know more than that the possibility of organic life depends upon certain conditions, that it can exist so long as these conditions remain within limits more or less wide, and that a transgression of the limits causes death. We do not, however, know exactly what these conditions are, and still less do we know the limits within which human life can exist; and if we reflect how complicated are the circumstances to be taken into account, we can scarcely hope to see these relations settled for some time to come. It is further to be observed that life, being regarded as the effect of a force, and death as its extinction, there must be a gradual transition from the one to the other. But the contrast between them is so complete that we are quite unable to measure the diminution of life, for the purpose of inferring the magnitude of the force that supports it. Altho' it is in complete harmony with our ordinary conceptions to speak of the gradual diminution of the force which sustains life, and altho' in many cases we observe a slow transition from life to death, we must yet wholly abandon the idea of measuring the intensity of life, because it is a phenomenon of which the manifestations are so various.

It is only by means of observing large numbers that we can obtain another starting point. Combining into a group a sufficiently large number of new-born children, and considering this group as a unit, we can disregard all minor matters and take the number surviving at any time as a measure of the vital force then existing in the group. The deaths that occur from time to time will then show us how the vital force contained in the group gradually decreases; and we thus find that the gradual dying out of the group, or the extinction of its aggregate life, proceeds with perfect regularity. If, on the contrary, we resolve the group again into its single elements, we can no longer find in them the regularity that we observed in the group.

When the observation of large numbers thus exhibits clearly a regularity in certain respects which is not perceptible among smaller groups, we shall be justified in fixing our attention exclusively upon the cause of this regularity. Though, possibly it is one part of a force which produces at one and the same time altogether different effects, according to the conditions which it meets with, yet there is nothing to prevent our considering a cause as consisting of different parts, of which one or several operating alone produce the precise effect we have observed. If we consider the decrease of the vital force as a cause which occasions in individuals, sometimes sufferings of the most varied kinds, bodily or mental, sometimes sickness, sometimes death, so that these different effects, depending upon conditions unknown to us, appear to be wholly irregular, capricious, and incapable of being foreseen, yet we know that in observing large bodies of persons, each of these effects appears with perfect regularity; and we are even entitled to attribute each of these effects to a corresponding part of the general cause,—a part which we can imagine separated from the whole; and with reference to the effect in question we may fix our attention exclusively on this part of the cause.

This analysis of causes is, I believe, the chief means by which the observation of large numbers can be applied to the discovery of new truths. The nature of the causes and the forces is absolutely hidden from us; but in all cases in which an effect is recognized, we infer the existence of a cause producing it. The simpler the effect, and the more distinct from other phenomenons, the sooner and the more thoroughly do we succeed in analyzing the causes; but in cases where a thousand different effects of the same cause constantly cross each other, where not only are a large proportion of the effects such as we cannot measure, but a perhaps still larger proportion altogether elude our powers of observation, it becomes impossible to draw any conclusion as to the cause. In such cases we call to our assistance the method of observing large numbers, registering separately each of the different effects so far as they are visible; and whenever this method can be applied, instead of a confused mass of tangled threads, the general design becomes apparent, in which the separate lines run with great regularity. The observation of large numbers by thus resolving the irregular phenomenons into regular ones, at the same time decomposes the cause into parts corresponding to the various partial patterns which make up the whole design. Altho' probably all these different patterns, and many others which have not yet come within our range of vision, should be considered as the total effect of one and the same cause, yet we appropriate to each pattern its own special portion of the total cause, and treat this as if it had an independent existence. Since we know nothing about the nature and essence of the causes in general, it is quite immaterial in what way we picture to ourselves this decomposition of the cause. It is also immaterial whether we consider the cause itself as split up, or whether we imagine the coexisting conditions as grouped, for the effect only takes place if the cause meets with the necessary conditions, and consequently the latter really form part of the cause. Even in simpler subjects, we constantly have causes analyzed and separated; for instance, we consider the weight and impenetrability of matter as two separate causes of different phenomenons, altho' we know of no single instance in which the effects of the one appear without those of the other; so, again, we know that in every chemical combination heat is set free, and yet we consider chemical affinity apart from heat. It should certainly not be difficult to prove that in general we never see an effect towards the production of which more causes than one did not contribute; but this does not in the least prevent us from fixing attention exclusively upon a single cause and investigating its action independently of all others. We proceed in just the same way with regard to the phenomenons the regularity of which is made known to us by the observation of large numbers. We consider each as the effect of a definite cause, or what is the same thing, as the effect of a certain part of a cause that operates in different directions; and we disregard entirely the effect of its other parts when we refer each regular series of phenomenons to a separate cause.

The processes of meteorology may be cited as an example. In these, too, we have to do with phenomenons which are so many-sided and complex as not to admit of a conclusion as to the causes that produce them. Meteorology can scarcely be considered a statistical science; but the statistical method is applicable to it. This method distributes the phenomenons into distinct groups; and as soon as we see in these groups a steady regular progression, we infer that there is a cause producing this progression, and consider it as a partial cause, under certain conditions, of meteorological processes in general, and then perhaps proceed to examine what these conditions are. Thus by observing large numbers we succeed in recognizing particular parts of the causes, their effect being exhibited distinct from the effect of the other parts. But if, from the total pattern given by the statistical method, we would draw conclusions as to a smaller group or an individual, it must be borne in mind that we shall not in this case find the same pattern on a reduced scale. The total cause, which produced in the whole group completely regular phenomenons, appears to operate quite irregularly in the smaller groups and in the individual—a fact fully explained by the complexity of their method of operation.* This might perhaps be represented by saying that the same cause operating upon a large number of individuals produces an extensive series of *different* regular patterns by arranging those individuals in certain groups; but that this mode of operation cannot be recognized if, instead of observing the effect on the large number of persons, we observe only the effect on a few of them. Supposing, on the other hand, that the arrangement in each single group is characterized by a regularity of its own, the observation of one group would suffice to enable us to draw conclusions as to a partial effect of the cause, or what is the same thing, as to a part of the cause itself, disregarding altogether its other parts. Whenever then we would use the results obtained from the observation of large numbers, to draw conclusions as to smaller groups and individuals, we must bear in mind that the elements of the large group are not all precisely similar,—that there are, on the contrary, among them elements of deviation in different directions; but as these compensate each other, the whole body has the appearance of being composed of exactly similar individuals, which is very far from the case. If we imagine the properties which exist in the whole group to be uniformly divided among the individuals composing it, we shall get what is called the average man, and we may legitimately attribute to this imaginary individual the phenomenons brought to light by our observations of large numbers.

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* We remind the reader of a very closely allied method of investigation applied in the calculus of probabilities.

Death, which the observation of large numbers shows us to occur with complete regularity, may in the sense above explained, and in that sense only, be regarded as the effect of a separate cause, for the purpose of further investigating this cause. It may then be shown that the observed regularity in the mortality may be explained by means of very simple suppositions as to this cause.

Life, as opposed to death, may be conceived of as the result of a force, the diminution of which is manifested by the deaths that occur. The momentary diminution of this force (which we will denote by $dz$) is to be regarded as the cause of death; and the product of this into the number living, must be equal to the number dying in the next instant: consequently

$$dzl_x = -dl_x,$$

whence

$$dz = -d\log l_x;$$

that is, the cause of death is equal to the probability of dying in the next instant, or in other words, to the mortality of the next instant. Hence

$$-z = \log\frac{l_x}{c}, \text{ $c$ being the constant of integration;}$$

and

$$l_x = c\epsilon^{-z}.$$

We thus see the form that $l_x$ takes when different assumptions are made as to the form of $z$. In particular, we see that $l_x$ appears in the form of a product when $z$ is put equal to the sum of several functions. $dz$ being regarded in general as the cause that produces death in the next instant, the above conclusion holds good even if $dz$ expresses the variation, not merely of a single force, but that of the sum of different forces. It is also immaterial whether death is caused by the diminution of a force that supports life, or by the direct operation of another force. If $dz$ is the effective cause of death in the next instant, then $z$ is the sum total of the force which has caused death—a force which must be inversely proportional to the power of resisting death.

To begin with the simplest case, assume that death is simply a consequence of the diminution of the power to resist death, which we will denote by $y$; also that the diminution in any infinitely small interval of time is proportional to the power of resistance at the beginning of that interval, or, in other words, that the differential of the power to resist is proportional to that power.

Putting $$z=\frac{1}{y},$$

we have $$y=\frac{1}{z}, \text{ and } dz=-\frac{dy}{y^2}.$$

Let now $dy=\log\frac{1}{q}\cdot ydx$, where $\log\frac{1}{q}$ is a constant;

then $$\frac{dy}{y}=\log\frac{1}{q}\cdot dx$$

$$\therefore \quad \log y=x\log\frac{1}{q}+\text{const.}=\log\frac{1}{q^x\log\frac{1}{k}}$$

$$\therefore \quad y=1\div q^x\log\frac{1}{k}.$$

But $$z=\frac{1}{y}=q^x\log\frac{1}{k},$$

whence $$l_x=ce^{-z}=ck^{q^x}.$$

This is the well known formula first given by Gompertz. If the constants are suitably determined, it agrees tolerably well with the numbers contained in mortality tables,—not, however, throughout the table, but only for a greater or less part of it. If we wish to apply the formula to the whole table, we have to change the constants at least three times,—a circumstance which places the greatest difficulties in the way of using it for purposes of calculation or otherwise. This inconvenience is partly remedied by a change of Gompertz's hypothesis suggested by Makeham. We have supposed above that the diminution of the power of resistance is the sole cause of death, and is always proportional to that power, and therefore becomes extremely small at the higher ages, where the power of resistance has become very trifling; but if, starting from the middle of the table, we calculate for a series of years the values of $l_x$ according to this hypothesis, we very soon at the higher ages get values which are much larger than those contained in the mortality table; in other words, the actual mortality is considerably greater than that deduced from the formula. According to Makeham's hypothesis we are to regard the force whose differential is the cause of death, as consisting of two forces, independent of each other, which we will denote by $v$ and $w$.* One of these, $v$, acts according to Gompertz's original hypothesis, so that $v = q^x \log \frac{1}{k}$; while the other, $w$, is constant throughout life. If we put $dw = \log \frac{1}{h} \cdot dx$, where $\log \frac{1}{h}$ is a constant, we get

$$w = x \log \frac{1}{h}, \text{ the constant of integration being } 0.$$

Hence $\quad z = v + w = q^x \log \frac{1}{k} + x \log \frac{1}{h}.$

Also, since $\quad l_x = c\epsilon^{-z}$

we have $\quad l_x = c\epsilon^{q^x \log k + x \log h} = ck^{q^x} h^x.$

It is a numerical consequence of this formula, which was first given by Makeham, that the logarithms of the probability of living a definite period, at equidistant ages, being increased by a constant quantity, form a geometrical series.* If the constants are suitably chosen, the formula gives values for the mortality and vitality agreeing sufficiently well with those of different mortality tables, from about the age of 15 or 20 to the extremity of life; but the deaths at younger ages are not correctly represented by the function. The agreement of the values found from the formula with those obtained from observation, has been sufficiently proved by Makeham himself; and theoretically the hypothesis is justified by the simplicity and naturalness of the assumption that the causes of death consist of two parts, one of which acts quite generally and independently both of the age and of the power of resistance, while the other appears as a consequence of the wasting of the vital force with increasing age.

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* Since the above was written, I have become aware that my deduction is not quite correct. Different causes of death, although they may be quite independent of each other, constantly disturb one another; and their joint effect is not equal to the sum of effects of every cause taken separately. On the other hand, the analytical expression of a cause of death acting concurrently with other causes, differs from the expression of the cause acting alone. This ought to have been taken into consideration before I tried to explain the constants of the formula; $v$ and $w$ being the causes, each acting concurrently with the other.

*Nov.* 1873. W. L.

With reference to this, a correspondent writes:—

Does not the modification of his hypothesis, which Mr. L. here finds himself constrained to admit, strike at the root of his whole argument? In Vital Statistics, as in other branches of Social Science, in Physiology, and even in Chemistry, it is not true, but very far from being true, that the effects of causes acting in combination are equal to the sum of the effects of the same causes acting separately. But if this is so, must we not regard Vital Statistics as a strictly experimental or empirical science, the complex and often fugitive phenomena of which cannot be legitimately brought within the boundaries of formulæ which are applicable to dynamics or the strictly mathematical branches of physics?

Mr. L. takes as an illustration of the value of his hypothesis, drawn from another branch of science, the Epicycles of Ptolemy. But were the epicycles really a help or a hindrance to the progress of astronomical discovery? Did they not rather stand in the way of true investigation, until their increasing number and complexity showed clearly that they did not answer to anything in nature; but were mere arbitrary hypotheses *invented* to cover facts which they did not truly explain, and they gave way before the simple and natural theory of Copernicus? Without absolutely denying the value of a *temporary* hypothesis—*clearly recognized as such*—as a mode of arranging and classifying facts, it is scarcely necessary to say that any practical use of it in such a field as Vital Statistics might prove misleading and hazardous.

W. S.

If we wish to adapt the formula to the younger ages, we must again change it by the introduction of a third force. Makeham's formula gives a vitality gradually decreasing without a minimum. This can be got by assuming as a third cause of death, a force acting in opposition to "life" and gradually diminishing; and the constants must be so determined that the effect entirely vanishes at about the age of 15, after which Makeham's formula is applicable.

If we assume that the changes of this force also, that occur in infinitely small intervals of time, are proportional to the force, we get by suitable determination of the constants an approximation to the observed values. Let $u$ denote this force: then

$$du = \log a \, . \, dx$$

$$u = a^x \log \frac{1}{g}, \text{ where } \log \frac{1}{g} \text{ is the constant of integration.}$$

Hence $s = v + w + u$

$$= q^x \log \frac{1}{k} + x \log \frac{1}{h} + a^x \log \frac{1}{g};$$

and consequently

$$l_x = c k^{q^x} h^x g^{a^x}.$$

This formula accurately reflects the characteristics of human mortality from birth to death, so that by giving proper values to the constants, the figures of different mortality tables can be represented with sufficient accuracy. The determination of the constants for a given mortality table, although requiring a long and tedious computation, offers but little difficulty*; and the agreement of the functional with the tabular values is surprizingly close during the greater portion of the table. It is only at the very lowest and the very highest ages that we find deviations of any magnitude; but these are exactly the points at which the tables are least trustworthy, since the mortality at the highest ages is imperfectly known on account of the insufficient number

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* Let $r$ denote the period, then the probability of living thro' it is $\frac{l_{x+r}}{l_x}$. The logarithm of this, $\log l_{x+r} - \log l_x$, is easily seen to be equal to $\log c + q^{x+r} \log k + (x+r) \log h - \log c - q^x \log k - x \log h = q^x(q^r - 1) \log k + r \log h$; and if we add to this the constant quantity $r \log \frac{1}{h} = -r \log h$, we get $q^x(q^r - 1) \log k$, the values of which for equidistant values of $x$, form a geometrical series.

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* Disregarding the mortality of the first period, the constants in the formula $l_x = ck^{q^x}h^x$ may be approximately determined from the mortality table as follows:—

$$\begin{aligned} \log l_x &= \log c + q^x \log k + x \log h, \\ \log l_{x+n} &= \log c + q^{x+n} \log k + (x+n) \log h, \\ \log l_{x+2n} &= \log c + q^{x+2n} \log k + (x+2n) \log h; \end{aligned}$$

$$\begin{aligned} \therefore\ \log l_{x+n} - \log l_x &= q^x(q^n-1)\log k + n \log h, \\ \log l_{x+2n} - \log l_{x+n} &= q^{x+n}(q^n-1)\log k + n \log h. \end{aligned}$$

In the same manner we find

$$\log l_{x+3n} - \log l_{x+2n} = q^{x+2n}(q^n-1)\log k + n \log h.$$

Forming now the second differences, we get

$$\begin{aligned} \log l_{x+2n} - 2\log l_{x+n} + \log l_x &= q^x(q^n-1)^2 \log k, \\ \log l_{x+3n} - 2\log l_{x+2n} + \log l_{x+n} &= q^{x+n}(q^n-1)^2 \log k; \end{aligned}$$

and dividing the latter of these equations by the former,

$$q^n = \frac{\log l_{x+3n} - 2\log l_{x+2n} + \log l_{x+n}}{\log l_{x+2n} - 2\log l_{x+n} + \log l_x};$$

whence $q$ can be immediately determined, and with the greater accuracy the greater $n$ is taken. We require, for this purpose, to take four equidistant terms of the mortality table, $l_x$, $l_{x+n}$, $l_{x+2n}$, $l_{x+3n}$. If we put

$$\begin{aligned} \log l_{x+n} - \log l_x &= \Delta \log l_x, \\ \log l_{x+2n} - \log l_{x+n} &= \Delta \log l_{x+n}; \end{aligned}$$

also

$$\Delta \log l_{x+n} - \Delta \log l_x = \Delta^2 \log l_x,$$

then

$$q^n = \frac{\Delta^2 \log l_{x+n}}{\Delta^2 \log l_x}.$$

From the same four values of the mortality table $h$ is most simply determined as follows:—

$$\begin{aligned} \Delta \log l_x - n \log h &= q^x(q^n-1)\log k, \\ \Delta \log l_{x+n} - n \log h &= q^{x+n}(q^n-1)\log k, \\ \Delta \log l_{x+2n} - n \log h &= q^{x+2n}(q^n-1)\log k. \end{aligned}$$

Consequently

$$\frac{\Delta \log l_x - n \log h}{\Delta \log l_{x+n} - n \log h} = \frac{\Delta \log l_{x+n} - n \log h}{\Delta \log l_{x+2n} - n \log h},$$

and

$$n \log h = \frac{\Delta \log l_x . \Delta \log l_{x+2n} - (\Delta \log l_{x+n})^2}{\Delta \log l_x + \Delta \log l_{x+2n} - 2\Delta \log l_{x+n}}.$$

The determination of $k$ and $c$ presents no difficulty.

of the observed cases, and that of childhood on account of the difficulty of making exact observations. In general, too much importance ought not to be attached to the exact agreement of the numbers living, as calculated by the formula, with those of the mortality tables, because the latter themselves are only approximations to the truth, more or less close. The numbers entered in the tables have been very rarely indeed arrived at by actual observation, being more frequently deduced from the observed values by means of various corrections and adjustments; and the one circumstance that the numbers of the living and dying are always made integers, necessitates a deviation from the true value, which becomes greater as the numbers are smaller. A comparison of the observed with the tabular values shows further that in most mortality tables very considerable corrections have been introduced without hesitation for the purpose of getting rid of irregularities. The numbers given in a mortality table being thus only approximations, more or less close, and not the real ratios themselves, we need not scruple to replace them by others, agreeing closely with them, but obtained from a formula.

It still remains to submit to closer examination the constants that appear in the formula. In the first place, $c$ has evidently no reference to the mortality, but relates simply to the radix of the table, or the number of persons born. For putting $x=0$, we get

$$g^{a^x}=g,\ k^{q^x}=k,\ h^x=1.$$

$$\therefore\quad l_0=cgk,\ \text{and}\ c=\frac{l_0}{gk},$$

whence $$l_x=l_0.g^{a^x-1}.k^{q^x-1}.h^x.$$

In this formula there are five constants, $g$, $a$, $k$, $q$, $h$, the two first of which have to do only with the mortality of childhood. The meaning of $h$ is clear from what has been already said; for $\log\frac{1}{h}$ is the intensity of the general cause of death which is supposed to be independent of the age. In order that $\log\frac{1}{h}$ may be positive, $h$ must lie between 0 and 1; and the greater $\log\frac{1}{h}$ is, or the smaller $h$ is, the greater will be the mortality. The vitality, which $=\frac{l_{x+1}}{l_x}=\frac{g^{a^{x+1}}k^{q^{x+1}}}{g^{a^x}k^{q^x}}h$, becomes $\frac{g^{a^{x+1}}k^{q^{x+1}}}{g^{a^x}k^{q^x}}(h+\Delta)$, if $h$ becomes $h+\Delta$, and consequently increases with $h$. If the vitality of one mortality table bears always a constant ratio to that of another, if, for instance, the vitality in one country is at all ages five percent greater than that in another, this may be explained by the different intensity of a general cause of death which is supposed to be quite independent of the age. It is in this case immaterial whether we conceive that the cause itself directly produces death, or that the increased mortality is the consequence of a diminished protection against the attacks on life resulting from different circumstances.

In interpreting $k$ and $q$, we must not forget that $y$, the power to resist death, has been determined in terms of the age by the equation $dy = \log\frac{1}{q} \cdot y\,dx$. Since $y$, the power to resist death, must be considered as essentially positive, and since $dy$ is to be negative, $\log\frac{1}{q}$ must also be negative; therefore $q$ is greater than 1: but since $\log q$ must be $< 1$, therefore $q$ must be $< \epsilon$, and the fraction $\log q$ denotes what proportion of the power of resistance is lost in each infinitely small interval of time.

Since $y$, the power of resistance, $= 1 \div q^x \log\frac{1}{k}$, we get, by putting $x = 0$, the power of resistance at birth $= 1 \div \log\frac{1}{k}$; but in order that this may be positive, $\log\frac{1}{k}$ must be positive, whence $\frac{1}{k}$ must be $> 1$, and $k < 1$. The greater $k$ is, and the nearer $q$ is to 1, the greater is the vitality.

The increase, $du$, of our third cause of death in any infinitely small interval, $= \log a \,.\, u\,dx$. This must be negative if the force $u$ is considered as essentially positive, and positive if $u$ is negative; consequently $\log a$ must be negative, and $a < 1$. Also, $u$ denoting a cause of death, we must choose the negative sign for it, $u = \log\frac{1}{g}\,a^x$, consequently $\log\frac{1}{g}$ is negative, and $g > 1$. Putting $x = 0$, we have at birth $u = \log\frac{1}{g}$; and since $a$ is a proper fraction, the value of $u$ becomes smaller and smaller until in fact it vanishes.

The influence which the change of the constants produces on the vitality and mortality, can thus be not only numerically calculated, but also explained with reference to the causes of death; the formula therefore contains all the elements necessary for the comparison of different mortalities and vitalities; and if the constants are suitably chosen, the figures of any mortality table can be replaced by values of the function with sufficient accuracy for most practical purposes. One of the principal practical applications of mortality tables is made by life insurance companies. Extensive investigations have satisfied me that the differences caused by substituting for the 17 Offices table, a table calculated by means of Makeham's formula* from the age 15 upwards, are so trifling that they may be unhesitatingly neglected, far greater deviations being constantly allowed without scruple.

These circumstances are certainly in favour of the formula in question; but there can be no doubt whatever that it will eventually give place to a still better. It appears to me that in the assumption of one cause of death which is independent of the age, and a second cause which can be expressed as a function of the age, altho' it may be in a different way from that here adopted, lie the elements for deducing the law of mortality, not only in its general character, but also perhaps in its details. It would only depend upon finding the most suitable expression for this function; further observation of large numbers may likewise lead to discoveries that will have a decisive influence on the investigation of the causes of death.

At all events, we cannot regard investigations in any one direction as concluded; nor can we even venture to assert that in the question we are now discussing we have arrived at a point, which has the same relative importance as the introduction of epicycles had for the determination of the planetary orbits.

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* The constants in this case are, if we start with $l_{20}$, $l_{40}$, $l_{60}$, $l_{80}$, and use the uncorrected observed numbers, so that

$$\log l_{40} - \log l_{20} = -\cdot 07427$$
$$\log l_{60} - \log l_{40} = -\cdot 14707$$
$$\log l_{80} - \log l_{60} = -\cdot 63282$$
$$q = 1\cdot 098771788, \quad (\log_{10} q = \cdot 040914)$$
$$h = 0\cdot 9929949$$
$$k = 0\cdot 9991717$$

To complete the table at the younger ages, I put

$$g = 1\cdot 291219 \qquad a = \cdot 4$$

*Hamburg, August* 1867.

# CORRESPONDENCE.

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## ON THE FORMULA FOR THE MARKET VALUE OF A COMPLETE ANNUITY.

*To the Editor of the Journal of the Institute of Actuaries.*

Sir,—The error involved in the formula $\frac{1-p}{i+p}$, given by D. Jones, vol. i, page 189, of his work on Annuities, for the value of a complete annuity as an investment, has been so well pointed out and corrected in the letter of Mr. A. Baden, in vol. xvii, page 447, of the *Journal of the Institute*, that I do not hope to add to the completeness of the demonstration there given. The following considerations, however, on the subject have occurred to me, with which I beg to trouble you, in case you should think them worthy of insertion.

I will first show how the result arrived at by Mr. Baden can be deduced in another way. The annuity being 1, payable annually, the last payment, or that in respect of which such annuity is rendered complete, is on an average equal, or nearly so, to $\frac{1}{2}$ the annual payment, or simply to $\frac{1}{2}$; and this is assumed to be due, not at the instant of death, but at the end of a year from the preceding payment of the annuity. So that, in fact, if we suppose the annuity of 1 to be divided into two annuities of $\frac{1}{2}$ each, both payable at the same periods, one of these annuities will be payable during the life, and its value is given by the well known formula $\frac{1}{2}\left(\frac{1}{d+\varpi_x}-1\right)$, (using $\varpi_x$ instead of $p$, the symbol adopted by Jones); but in the case of the other, one further payment is made, viz.: of $\frac{1}{2}$ at the end of the year in which the life drops.

Now a little consideration will show that if each payment of an annuity of $\frac{1}{2}$ be made one year later than is assumed in the formula $\frac{1}{2}\left(\frac{1}{d+\varpi_x}-1\right)$, that is, at the end of each year, provided the life was in existence at the *beginning* of the year, we have an annuity of $\frac{1}{2}$, payment of which is to commence at the end of the second year, to continue during life, and also be made at the end of the year of death. But since the present value of 1 payable a year hence is $v$, the value of the latter annuity is $\frac{v}{2}\left(\frac{1}{d+\varpi_x}-1\right)$; and as the payment of $\frac{1}{2}$ in respect of the first year is to be made whether the life continue or not, its value is $\frac{v}{2}$. Adding,

$$\frac{v}{2}\left(\frac{1}{d+\varpi_x}-1\right)+\frac{v}{2}=\frac{v}{2}\left(\frac{1}{d+\varpi_x}\right) \quad . \quad . \quad . \quad (1)$$

and this is therefore the value of an annuity of $\frac{1}{2}$ payable annually during life and one year longer. Adding to (1) the value of the other annuity of $\frac{1}{2}$, viz., $\frac{1}{2}\left(\frac{1}{d+\varpi_x}-1\right)$ we have

$$\frac{v-\frac{\varpi_x}{2}}{d+\varpi_x} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

for the value of the complete annuity of 1; and this is the same result as that arrived at by Mr. Baden in the letter referred to. The amount for which the assurance is to be effected is $\frac{1-\frac{d}{2}}{d+\varpi_x}$.

The formula (1) may also be deduced as follows. Let 1 be the amount invested by the purchaser. The interest on 1 is $i$; and if $f_x$ denote the annual premium on the assurance, then, after the payment of the first premium, there remains $1-f_x$, and the annuity which this should purchase is $i+f_x$. If a payment of such annuity is to be made in respect of the year in which the life fails, the assurance should be for $1-f_x$. Hence, $f_x=\varpi_x(1-f_x)=\frac{\varpi_x}{1+\varpi_x}$ $\therefore 1-f_x=1-\frac{\varpi_x}{1+\varpi_x}=\frac{1}{1+\varpi_x}$, and $i+f_x=i+\frac{\varpi_x}{1+\varpi_x}=\frac{i+(1+i)\varpi_x}{1+\varpi_x}$. The value of an annuity of $\frac{1}{2}$ is $\frac{1}{2}\cdot\frac{1-f_x}{i+f_x}=\frac{1}{2}\frac{1}{i+(1+i)\varpi_x}=\frac{1}{2}\frac{v}{d+\varpi_x}$, which agrees with (1).

It can be readily seen that, as is pointed out by Mr. Baden, Jones's formula, $\frac{1-\varpi_x}{i+\varpi_x}$, gives a correct result when $\varpi_x=i$. It may be worth while to ascertain the difference between it and the more correct formula $\frac{v-\frac{\varpi_x}{2}}{d+\varpi_x}$.

$$\frac{v-\frac{\varpi_x}{2}}{d+\varpi_x}-\frac{1-\varpi_x}{i+\varpi_x}=\frac{vi+\varpi_x v-\frac{\varpi_x}{2}i-\frac{\varpi_x^2}{2}-d-\varpi_x+d\varpi_x+\varpi_x^2}{(d+\varpi_x)(i+\varpi_x)}$$

$$=\frac{d+\varpi_x v-\frac{\varpi_x}{2}i-\frac{\varpi_x^2}{2}-d-\varpi_x v+\varpi_x^2}{(d+\varpi_x)(i+\varpi_x)}$$

$$=\frac{\frac{\varpi_x}{2}(\varpi_x-i)}{(d+\varpi_x)(i+\varpi_x)}=\left(\frac{\varpi_x-i}{i+\varpi_x}\right)\left(\frac{\frac{\varpi_x}{2}}{d+\varpi_x}\right). \quad . \quad . \quad (3)$$

Hence, according as $\varpi_x$ is $>$ or $<$ $i$, Jones's formula produces too small or too great a value. $\frac{\frac{\varpi_x}{2}}{d+\varpi_x}$ represents the value of an annuity-due of $\frac{\varpi_x}{2}$, and also the increase in the value of an annuity of 1 on account of its being complete, since $\frac{v-\frac{\varpi_x}{2}}{d+\varpi_x}=\frac{1}{d+\varpi_x}-1+\frac{\varpi_x}{2}\frac{1}{d+\varpi_x}$.

Therefore the amount of error in computing such additional value by Jones's formula may be found from (3) for given values of $i$ and $\varpi_x$.

Without wishing to devote more time to this subject than its importance warrants, I will consider the following other points connected with Jones's formula $\frac{1-\varpi_x}{i+\varpi_x}$.

1st. What is really the interpretation to be put on it? This may be easily seen on reference to vol. i, Articles 244 and 245, of his work on Annuities, but will perhaps be more obvious if the formula be converted into one more familiar to the reader. This can be done by multiplying its numerator and denominator by $v$. Thus we have

$$\frac{1-\varpi_x}{i+\varpi_x}=\frac{v(1-\varpi_x)}{v(i+\varpi_x)}=\frac{1-d-v\varpi_x}{d+v\varpi_x}=\frac{1}{d+v\varpi_x}-1=\frac{1}{d+\varpi'_x}-1,$$

if $v\varpi_x=\varpi'_x$, or $\varpi'_x$ represent $\varpi_x$ reduced in the ratio $1:1+i$.

The latter formula coincides with the well known one, $\frac{1}{d+\varpi_x}-1$, except that $\varpi'_x$ is substituted for $\varpi_x$; or, in other words, Jones, in designing to alter the formula for a curtate annuity so as to obtain one for a complete annuity, really retained the same formula, only making a modification in one of its terms, viz., $\varpi_x$.

2nd. The increase in the value of the annuity on account of its being complete, if Jones's formula be used, is

$$\frac{1}{d+v\varpi_x}-1-\left(\frac{1}{d+\varpi_x}-1\right)=\frac{d\varpi_x}{(d+\varpi_x)(d+v\varpi_x)}.$$

Differentiating the Napierian logarithm of the latter quantity, viz., $\log_e d+\log_e \varpi_x-\log_e (d+\varpi_x)-\log_e (d+v\varpi_x)$, to $x$ or $\varpi_x$, and equating the result to 0, according to the theory of maxima and minima, we have

$$\frac{1}{\varpi_x}-\frac{1}{d+\varpi_x}-\frac{v}{d+v\varpi_x}=0$$

$$\frac{d}{\varpi_x(d+v\varpi_x)}-\frac{1}{d+\varpi_x}=0$$

$$d^2+d\varpi_x-d\varpi_x-v\varpi_x^2=0$$

$$\varpi_x=\frac{d}{\sqrt{v}}=\sqrt{id}.$$

Differentiating again: $\mathrm{d}$ being the symbol of differentiation,

$$\frac{\mathrm{d}\left(\frac{1}{\varpi_x}-\frac{1}{d+\varpi_x}-\frac{v}{d+v\varpi_x}\right)}{\mathrm{d}x}=-\left\{\frac{1}{\varpi_x^2}-\frac{1}{(d+\varpi_x)^2}-\frac{v^2}{(d+v\varpi_x)^2}\right\}\frac{\mathrm{d}\varpi_x}{\mathrm{d}x}$$

$$=-\left\{\frac{(d+v\varpi_x)^2-v^2\varpi_x^2}{\varpi_x^2(d+v\varpi_x)^2}-\frac{1}{(d+\varpi_x)^2}\right\}\frac{\mathrm{d}\varpi_x}{\mathrm{d}x}$$

$\left(\text{or since from above } \frac{d}{\varpi_x(d+v\varpi_x)} = \frac{1}{d+\varpi_x}\right)$

$$= -\left\{\frac{(d+v\varpi_x)^2 - v^2\varpi_x^2}{\varpi_x^2(d+v\varpi_x)^2} - \frac{d^2}{\varpi_x^2(d+v\varpi_x)^2}\right\}\frac{d\varpi_x}{dx}$$

$$= -\frac{2dv\varpi_x}{\varpi_x^2(d+v\varpi_x)^2}\,\frac{d\varpi_x}{dx}.$$

The last result is negative. Hence $\log_e \frac{d\varpi_x}{(d+\varpi_x)(d+v\varpi_x)}$ is a maximum, and therefore also $\frac{d\varpi_x}{(d+\varpi_x)(d+v\varpi_x)}$, when $\varpi_x = \sqrt{id}$; that is, the difference between the values of a complete and curtate annuity, if Jones's formula be used for the former, attains a maximum value at the age at which the annual premium $= \sqrt{id}$, and thence decreases as the premium increases; whereas it will be seen that the difference, viz., $\frac{1}{2}\frac{\varpi_x}{d+\varpi_x}$, or $\frac{1}{2}\left(1-\frac{d}{d+\varpi_x}\right)$, between the values of the two annuities, when the formula (2), $\frac{v-\frac{\varpi_x}{2}}{d+\varpi_x}$, is used for the value of the complete annuity, increases as $\varpi_x$ increases.

Since the value of a complete annuity as above deduced is equal to that of a curtate annuity increased by $\frac{1}{2}\frac{\varpi_x}{d+\varpi_x}$, it may appear at first sight that there is no difference between the two annuities, or between the methods of carrying out the transactions in respect of them, except in such additional payment of $\frac{1}{2}\frac{\varpi_x}{d+\varpi_x}$ being made for the complete annuity. In reference to this point, the following details as to the values, &c., of the two annuities are furnished.

| | Value. | Total original cost of Annuity. | Amount of Assurance. | Premium. | Interest. | Premium + Interest. |
|---|---|---|---|---|---|---|
| Complete annuity | $\frac{v-\frac{\varpi_x}{2}}{d+\varpi_x}$ | $\frac{v\left(1+\frac{\varpi_x}{2}\right)}{d+\varpi_x}$ | $\frac{1-\frac{d}{2}}{d+\varpi_x}$ | $\varpi_x\left(\frac{1-\frac{d}{2}}{d+\varpi_x}\right)$ | $d\left(\frac{1+\frac{\varpi_x}{2}}{d+\varpi_x}\right)$ | 1 |
| Curtate annuity | $\frac{1}{d+\varpi_x}-1$ | $\frac{v}{d+\varpi_x}$ | $\frac{1}{d+\varpi_x}$ | $\frac{\varpi_x}{d+\varpi_x}$ | $\frac{d}{d+\varpi_x}$ | 1 |

It will thus be seen that not only the values and costs of the two annuities differ, but also the sums assured, premiums, and interest.

In the case of a complete annuity of 1, $\frac{1-\frac{d}{2}}{d+\varpi_x}$, representing the sum to be assured and $\frac{1}{2}$ the proportion of the annuity in respect of the year of death, the purchaser will receive at that time

$\dfrac{1-\frac{d}{2}}{d+\varpi_x}+\dfrac{1}{2}=\dfrac{1+\frac{\varpi_x}{2}}{d+\varpi_x}=\left\{\dfrac{v\left(1+\frac{\varpi_x}{2}\right)}{d+\varpi_x}\right\}(1+i)$. But $\dfrac{v\left(1+\frac{\varpi_x}{2}\right)}{d+\varpi_x}$ is the original cost of the annuity: therefore he will be repaid the latter with a year's interest on it, as should be the case. Supposing, however, the assurance to be effected for $\dfrac{1}{d+\varpi_x}$, as in the case of a curtate annuity, since $\frac{1}{2}$ will be received as before for the proportion of the annuity in respect of the year of death, the purchaser will receive altogether $\dfrac{1}{d+\varpi_x}+\dfrac{1}{2}$, or $\dfrac{1+\frac{\varpi_x}{2}+\frac{d}{2}}{d+\varpi_x}$, that is, a larger sum than in the preceding case. The assurance for $\dfrac{1}{d+\varpi_x}$ is therefore greater than is necessary.

I am, Sir,
Your obedient servant,
THOMAS CARR.

*7 Royal Exchange,*
4 *March* 1874.

JOURNAL

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AND

ASSURANCE MAGAZINE.

## *A comparison of Reserves brought out by the Use of different Data in the Valuation of the Liabilities of a Life Office. By* JAMES VALENTINE, *Assistant Actuary of the Northern Assurance Company.*

[Read before the Institute, 27 April 1874.]

IT is not intended in this paper to do more than go briefly again over part of the ground covered by Mr. Manly, in his admirable essay, printed in the 14th volume of the *Journal* of this Institute, on the Values of Policies as found by means of various Tables and Methods of Valuation.

Assuming an office to have been started with sufficient rates of premium, it will be universally admitted that no part of the duties afterwards devolving upon the actuary is more important than the determination of the data upon which the Reserves are to be calculated at the periodical investigations into the state of its affairs. The importance of this subject has led to its consideration by the leading men in our profession, many of whose contributions towards its elucidation are to be found in the pages of the *Journal*, and I will not presume to enter into it here further than by instituting a new comparison between the results arrived at by the use of different data.

A similar task was executed by Mr. Manly, in the essay to which I have referred, in the most complete manner, as far as the materials at his command at the time would permit, but the new Experience Tables of Mortality since published by the Institute present so valuable a field for the prosecution of fresh inquiries in the same direction, that it is worth while, I think, to undertake the work again.

Those who are familiar with Mr. Manly's essay will remember that he constructed a table, which may be said to represent a model office, showing assumed amounts of policies taken out at various ages and remaining in force at the end of stated periods, and that, having done so, he formed a second table giving the reserves, in the case of such model office, at different stages in its career, according to various data. In continuation of the second table I have calculated the corresponding reserves required by the Institute Experience Table $H^{M}$, reckoning interest at 3 per cent., $3\frac{1}{2}$ per cent., and 4 per cent. respectively; and also those by the $H^{M(5)}$ Table, with $H^{M}$ pure premiums, at the same rates of interest. Mr. Manly's second table likewise contains a comparison of the several reserves worked out by him with the Carlisle 3 per cent. as a standard. To make a new comparison with a truer standard, formed from the more modern and better data which we now have at hand, is my present intention.

What, then, should be our standard of comparison?

Firstly, as regards mortality. Surely, if we can find a table of the mortality actually experienced among a body of assured lives, numerous enough and drawn from the records of a sufficient number of offices, to give them a representative character—and still more, if it appear that such table is confirmed by another and independent table answering the same conditions—it will need no argument to show that we have there a true exponent for this part of our standard. Other tables, formed from statistics of the whole population, or of particular districts of the country, or even from the experience of individual offices, altho' they may be framed with skill and be very useful in some ways, cannot, it seems to me, have the slightest pretentions, for general life office purposes, to an equal degree of value, to say nothing of superiority. Now, in the new Institute Mortality Experience, I think we find all that is above desired. It is based upon a very large number of facts, contributed by 20 offices, while, as regards confirmation by another table similarly constructed, this is supplied in a striking manner by the

17 Offices' Experience. It is, I believe, well known that a remarkable degree of closeness exists between the latter and the new $H^M$ Table, but it may not be out of place here to illustrate the fact by contrasting a few examples of values of policies brought out by each table.

*Comparison between the Values of Policies for £100 taken out at various ages, and in force for various terms, according to the Seventeen Offices' Experience and the* $H^M$ *Tables of Mortality respectively, reckoning Interest at 4 per cent.*

| Age when Assured. | Duration of Policy. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 10 Years. | | 20 Years. | | 30 Years. | | 40 Years. | |
| | Values by 17 Offices' Experience Table. | Values by Institute $H^M$ Table. | Values by 17 Offices' Experience Table. | Values by Institute $H^M$ Table. | Values by 17 Offices' Experience Table. | Values by Institute $H^M$ Table. | Values by 17 Offices' Experience Table. | Values by Institute $H^M$ Table. |
| 30 | 10·79 | 11·01 | 25·33 | 25·34 | 42·27 | 42·31 | 59·44 | 59·78 |
| 40 | 16·30 | 16·11 | 35·28 | 35·18 | 54·53 | 54·80 | 71·04 | 71·46 |
| 50 | 22·68 | 22·73 | 45·68 | 46·12 | 65·40 | 65·99 | | |
| 60 | 29·75 | 30·27 | 55·25 | 55·98 | | | | |

A glance at the table will show how close the values are to one another, the differences in most cases being very slight, and nowhere reaching 2 per cent. of the $H^M$ values, as I have found by actual trial. As between the new Experience and the old, it will probably be agreed by all that the preference must be given to the former; seeing that it is based on a greater number of facts—at the higher ages especially, it possesses this important advantage—and that it is founded entirely on the observation of lives, whereas the other was in a great measure based on the returns of policies. With respect, therefore, to the element of mortality, I conclude that the Institute Experience ought to form our standard. For the valuation of the sums assured and future premiums, I assume the $H^{M(5)}$ Table (which excludes the experience of the first 5 years). This table seems the best fitted, at the date of the investigation, to measure the mortality to be henceforth expected, the more so the older the office becomes, as it is likely that at each succeeding investigation the proportion of new lives to the whole number of lives on the books will be less. For the calculation of the pure premiums, however, the $H^{M(5)}$ Table is not appropriate, as, thro' the exclusion of part of the experience, it would not bring out premiums representing the original prime cost of the assurances. For this purpose, recourse must be had, I think, to the $H^M$ Table, which includes the *entire* experience of the mortality of assured lives.

Secondly, as to the rate of interest. I think that we must assume a rate which, while offering a good prospect of being maintained, at least for a reasonable time, approaches as near as possible to that realized by offices on their investments. Using a table of mortality and a rate of interest as nearly true as possible, we ascertain the actual present surplus (whether the whole should be divided it is not necessary to discuss here), and we also arrive, in the value of the loading reserved, at the best estimate obtainable of the capacity of the office to make future profits or meet unforeseen contingencies. Some offices, as we all know, adopt a lower rate of interest; and speaking from the single point of view of safety, there can be, of course, no objection to their doing so, as long as none of the other data assumed are inconsistent with the experience which may be looked for, that is to say, provided a table of mortality is taken which brings out sufficiently high policy values, and that the portion of the loading reserved for future expenses alone (apart from future profits) is not less than the actual working charges. Should, however, an office assume a lower rate, with the knowledge that an insufficient table of mortality has been taken, or that the portion of the loading reserved for expenses does not cover the actual out-go under this head, leaving the errors to balance one another, as is, I fear, often done, the position then, I think, becomes untenable, for the office at once introduces a great degree of uncertainty into its calculations, and is manifestly not justified in saying that it bases its valuations on the professed rate of interest. On the whole, I think that 4 per cent. may be fairly taken as the rate.

Thus, for our complete standard, I arrive at the Table combining $H^{M(5)}$, with $H^M$ pure premiums, and 4 per cent. interest. Accordingly I have made a comparison between the reserve required by the use of these data and each of the reserves brought out by the various other data employed. In addition, altho' I think the above is the standard for our purpose, as there are perhaps some actuaries who might consider 4 per cent. too high a rate of interest, I have made another comparison, in which the test value has been taken as $H^{M(5)}$, $H^M$ pure premiums, and interest at 3½ per cent. The following table shows the results at which I have arrived:—

| Data. | Age of Office. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 5 Years. | | | 10 Years. | | | 15 Years. | | |
| | Reserve. | Percentage greater (+) or less (−) than Reserve by HM(5), Interest 3½ per Cent., HM pure Premiums. | Percentage greater (+) or less (−) than Reserve by HM(5), Interest 4 per Cent., HM pure Premiums. | Reserve. | Percentage greater (+) or less (−) than Reserve by HM(5), Interest 3½ per Cent., HM pure Premiums. | Percentage greater (+) or less (−) than Reserve by HM(5), Interest 4 per Cent., HM pure Premiums. | Reserve. | Percentage greater (+) or less (−) than Reserve by HM(5), Interest 3½ per Cent., HM pure Premiums. | Percentage greater (+) or less (−) than Reserve by HM(5), Interest 4 per Cent., HM pure Premiums. |
| Institute of Actuaries' Experience $H^{M(5)}$ | | | | | | | | | |
| $H^M$ pure premiums 3 per cent. | 90,461 | + 5·837 | +11·933 | 231,875 | + 5·453 | +11·139 | 391,181 | + 5·097 | +10·400 |
| Do. do. 3½ „ | 85,472 | ... | + 5·760 | 219,884 | ... | + 5·392 | 372,208 | ... | + 5·045 |
| Do. do. 4 „ | 80,817 | − 5·446 | ... | 208,635 | − 5·116 | ... | 354,332 | − 4·803 | |
| Institute of Actuaries' Experience $H^M$ 3 „ | 81,126 | − 5·085 | + ·382 | 216,505 | − 1·537 | + 3·772 | 372,182 | − ·007 | + 5·038 |
| Do. do. 3½ „ | 76,421 | −10·589 | − 5·439 | 204,922 | − 6·805 | − 1·780 | 353,664 | − 4·982 | − ·188 |
| Do. do. 4 „ | 72,041 | −15·714 | −10·859 | 194,080 | −11·735 | − 6·976 | 336,242 | − 9·663 | − 5·105 |
| Carlisle ... ... ... ... 3 „ | 74,493 | −12·845 | − 7·825 | 200,261 | − 8·924 | − 4·014 | 346,380 | − 6·939 | − 2·244 |
| Do. ... ... ... ... 3½ „ | 69,930 | −18·184 | −13·471 | 188,948 | −14·069 | − 9·436 | 328,177 | −11·830 | − 7·381 |
| Do. ... ... ... ... 4 „ | 65,695 | −23·139 | −18·712 | 178,387 | −18·872 | −14·498 | 311,105 | −16·416 | −12·200 |
| Seventeen Offices' Experience ... 3 „ | 81,262 | − 4·926 | + ·551 | 216,660 | − 1·466 | + 3·846 | 372,303 | + ·026 | + 5·072 |
| Do do. ... 3½ „ | 76,370 | −10·649 | − 5·503 | 204,932 | − 6·800 | − 1·775 | 353,678 | − 4·978 | − ·184 |
| Do. do. ... 4 „ | 72,007 | −15·754 | −10·901 | 194,140 | −11·708 | − 6·948 | 336,347 | − 9·635 | − 5·076 |
| English Life Table, (No. 3) ... 3 „ | 78,518 | − 8·136 | − 2·845 | 209,640 | − 4·659 | + ·481 | 360,584 | − 3·123 | + 1·765 |
| Do. do. ... 3½ „ | 74,029 | −13·388 | − 8·399 | 198,562 | − 9·697 | − 4·828 | 342,824 | − 7·895 | − 3·248 |
| Do. do. ... 4 „ | 69,854 | −18·273 | −13·565 | 188,200 | −14·409 | − 9·795 | 326,133 | −12·379 | − 7·958 |
| English Life Table, (No. 2) ... 3 „ | 78,425 | − 8·245 | − 2·960 | 209,811 | − 4·581 | + ·564 | 361,296 | − 2·932 | + 1·965 |
| Do. do. ... 4 „ | 69,667 | −18·491 | −13·797 | 188,135 | −14·439 | − 9·826 | 326,472 | −12·288 | − 7·863 |
| Davies' Equitable Table ... ... 3 „ | 71,741 | −16·065 | −11·230 | 191,222 | −13·035 | − 8·346 | 328,999 | −11·609 | − 7·149 |
| Do. do. ... ... 3½ „ | 67,383 | −21·164 | −16·623 | 180,375 | −17·968 | −13·545 | 311,460 | −16·321 | −12·100 |
| Do. do. ... ... 4 „ | 63,351 | −25·881 | −21·612 | 170,286 | −22·556 | −18·381 | 295,068 | −20·725 | −16·726 |
| Edmonds' Mean Mortality... ... 3 „ | 78,261 | − 8·437 | − 3·163 | 209,747 | − 4·610 | + ·533 | 361,736 | − 2·814 | + 2·090 |
| Government Males (1829) ... ... 4 „ | 68,661 | −19·668 | −15·041 | 186,290 | −15·278 | −10·710 | 323,873 | −12·986 | − 8·596 |
| Hypothetical Tables. 1. Northampton ... ... 3 „ | 68,108 | −20·315 | −15·726 | 181,980 | −17·238 | −12·776 | 314,008 | −15·636 | −11·381 |
| 2. Do. ... ... 4 „ | 60,637 | −29·056 | −24·970 | 163,228 | −25·766 | −21·764 | 283,466 | −23·842 | −19·999 |
| 3. Mr. Manly's Table No. 19* ... ... | 70,474 | −17·547 | −12·798 | 190,115 | −13·538 | − 8·877 | 329,725 | −11·414 | − 6·945 |
| 4. Do. No. 20† ... ... | 78,565 | − 8·081 | − 2·787 | 210,479 | − 4·277 | + ·884 | 362,961 | − 2·484 | + 2·435 |
| 5. Do. No. 21‡ ... ... | 81,879 | − 4·204 | + 1·314 | 218,747 | − ·517 | + 4·847 | 376,315 | + 1·103 | + 6·204 |

* Table formed from Carlisle with constant addition of ·0037 to the premium of £1.
† Do. do. with loading of 13 per cent.
‡ Do. do. do. 25 per cent.

TABLE *showing a comparison of the Reserves brought out by Institute Experience* $H^{M(5)}$, *&c.*—(continued.)

| Data. | | Age of Office. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 20 Years. | | | 25 Years. | | | 30 Years. | | |
| | | Reserve. | Percentage greater (+) or less (−) than Reserve by $HM^{(5)}$, Interest 3½ per Cent., HM pure Premiums. | Percentage greater (+) or less (−) than Reserve by $HM^{(5)}$, Interest 4 per Cent., HM pure Premiums. | Reserve. | Percentage greater (+) or less (−) than Reserve by $HM^{(5)}$, Interest 3½ per Cent., HM pure Premiums. | Percentage greater (+) or less (−) than Reserve by $HM^{(5)}$, Interest 4 per Cent., HM pure Premiums. | Reserve. | Percentage greater (+) or less (−) than Reserve by $HM^{(5)}$, Interest 3½ per Cent., HM pure Premiums. | Percentage greater (+) or less (−) than Reserve by $HM^{(5)}$, Interest 4 per Cent., HM pure Premiums. |
| Institute of Actuaries' Experience $H^{M(5)}$ $H^M$ pure premiums | 3 per cent. | 551,950 | + 4·770 | + 9·721 | 702,498 | + 4·483 | + 9·127 | 814,588 | + 4·273 | + 8·692 |
| Do. do. | 3½ „ | 526,821 | ... | + 4·725 | 672,355 | ... | + 4·445 | 781,206 | ... | + 4·238 |
| Do. do. | 4 „ | 503,050 | − 4·512 | ... | 643,743 | − 4·256 | ... | 749,446 | − 4·066 | |
| Institute of Actuaries' Experience $H^M$ | 3 „ | 530,818 | + ·759 | + 5·520 | 680,152 | + 1·160 | + 5·656 | 791,662 | + 1·338 | + 5·633 |
| Do. do. | 3½ „ | 506,152 | − 3·923 | + ·617 | 650,467 | − 3·255 | + 1·045 | 758,731 | − 2·877 | + 1·239 |
| Do. do. | 4 „ | 482,847 | − 8·347 | − 4·016 | 622,318 | − 7·442 | − 3·328 | 727,429 | − 6·884 | − 2·938 |
| Carlisle ... ... ... ... | 3 „ | 496,512 | − 5·753 | − 1·300 | 638,482 | − 5·038 | − ·817 | 744,746 | − 4·667 | − ·627 |
| Do. ... ... ... ... | 3½ „ | 472,144 | −10·379 | − 6·144 | 609,028 | − 9·419 | − 5·392 | 711,964 | − 8·864 | − 5·001 |
| Do. ... ... ... ... | 4 „ | 449,199 | −14·734 | −10·705 | 581,196 | −13·558 | − 9·716 | 680,912 | −12·838 | − 9·145 |
| Seventeen Offices' Experience ... | 3 „ | 530,653 | + ·727 | + 5·487 | 679,552 | + 1·070 | + 5·563 | 790,624 | + 1·206 | + 5·494 |
| Do. do. ... | 3½ „ | 505,909 | − 3·970 | + ·568 | 649,807 | − 3·354 | + ·942 | 757,635 | − 3·017 | + 1·093 |
| Do. do. ... | 4 „ | 482,732 | − 8·369 | − 4·039 | 621,813 | − 7·517 | − 3·406 | 726,499 | − 7·003 | − 3·062 |
| English Life Table, (No. 3) ... | 3 „ | 514,619 | − 2·316 | + 2·300 | 659,861 | − 1·858 | + 2·504 | 768,430 | − 1·635 | + 2·533 |
| Do. do. ... | 3½ „ | 490,901 | − 6·818 | − 2·415 | 631,249 | − 6·114 | − 1·941 | 736,632 | − 5·706 | − 1·710 |
| Do. do. ... | 4 „ | 468,521 | −11·066 | − 6,864 | 604,155 | −10·143 | − 6·149 | 706,450 | − 9·569 | − 5·736 |
| English Life Table, (No. 2) ... | 3 „ | 515,961 | − 2·061 | + 2·566 | 661,763 | − 1·575 | + 2·800 | 770,607 | − 1·357 | + 2·823 |
| Do. do. ... | 4 „ | 469,386 | −10·902 | − 6·691 | 605,512 | − 9·942 | − 5·939 | 708,037 | − 9·366 | − 5·525 |
| Davies' Equitable Table ... ... | 3 „ | 470,165 | −10·754 | − 6·537 | 604,101 | −10·151 | − 6·158 | 705,426 | − 9·700 | − 5·874 |
| Do. do. ... ... | 3½ „ | 446,534 | −15·240 | −11·235 | 575,364 | −14·426 | −10·622 | 673,289 | −13·814 | −10·162 |
| Do. do. ... ... | 4 „ | 424,357 | −19·449 | −15·643 | 548,285 | −18·453 | −14·829 | 642,930 | −17·700 | −14·213 |
| Edmonds' Mean Mortality ... ... | 3 „ | 517,280 | − 1·811 | + 2·829 | 664,131 | − 1·223 | + 3·167 | 773,803 | − ·948 | + 3·250 |
| Government Males (1829) ... ... | 4 „ | 465,435 | −11·652 | − 7·477 | 599,875 | −10·780 | − 6·814 | 700,858 | −10·285 | − 6·483 |
| Hypothetical Tables. 1. Northampton ... ... | 3 „ | 450,449 | −14·497 | −10·456 | 580,854 | −13·609 | − 9·769 | 680,181 | −12·932 | − 9·242 |
| 2. Do. ... ... ... | 4 „ | 409,049 | −22·355 | −18·686 | 530,241 | −21·137 | −17·632 | 623,412 | −20·199 | −16·817 |
| 3. Mr. Manly's Table No. 19 ... | ... | 474,000 | −10·026 | − 5·775 | 611,055 | − 9·117 | − 5·078 | 714,077 | − 8·593 | − 4·720 |
| 4. Do. No. 20 ... | ... | 518,854 | − 1·512 | + 3·142 | 666,211 | − ·914 | + 3·491 | 775,605 | − ·717 | + 3·491 |
| 5. Do. No. 21 ... | ... | 536,770 | + 1·889 | + 6·703 | 687,287 | + 2·221 | + 6·764 | 799,122 | + 2·293 | + 6·628 |

TABLE *showing a comparison of the Reserves brought out by Institute Experience* $H^{M(5)}$, *&c.*—(continued).

| Data. | Age of Office. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | 35 Years. | | | 40 Years. | | | 45 Years. | | |
| | Reserve. | Percentage greater (+) or less (−) than Reserve by $H^{M(5)}$, Interest 3½ per Cent., $H^M$ pure Premiums. | Percentage greater (+) or less (−) than Reserve by $H^{M(5)}$, Interest 4 per Cent., $H^M$ pure Premiums. | Reserve. | Percentage greater (+) or less (−) than Reserve by $H^{M(5)}$, Interest 3½ per Cent., $H^M$ pure Premiums. | Percentage greater (+) or less (−) than Reserve by $H^{M(5)}$, Interest 4 per Cent., $H^M$ pure Premiums. | Reserve. | Percentage greater (+) or less (−) than Reserve by $H^{M(5)}$, Interest 3½ per Cent., $H^M$ pure Premiums. | Percentage greater (+) or less (−) than Reserve by $H^{M(5)}$, Interest 4 per Cent., $H^M$ pure Premiums. |
| Institute of Actuaries' Experience $H^{M(5)}$ $H^M$ pure premiums 3 per cent. | 890,380 | + 4·126 | + 8·387 | 937,764 | + 4·025 | + 8·177 | 964,088 | + 3·964 | + 8·051 |
| Do. do. 3½ " | 855,099 | ... | + 4·093 | 901,482 | ... | + 3·992 | 927,328 | ... | + 3·931 |
| Do. do. 4 " | 821,480 | − 3·932 | ... | 866,879 | − 3·838 | ... | 892,252 | − 3·783 | |
| Institute of Actuaries' Experience $H^M$ 3 " | 867,182 | + 1·413 | + 5·563 | 914,448 | + 1·438 | + 5·487 | 940,719 | + 1·444 | + 5·432 |
| Do. do. 3½ " | 832,347 | − 2·661 | + 1·323 | 878,609 | − 2,537 | + 1·353 | 904,402 | − 2·472 | + 1·362 |
| Do. do. 4 " | 799,181 | − 6·539 | − 2·714 | 844,456 | − 6·326 | − 2·587 | 869,774 | − 6·206 | − 2·520 |
| Carlisle ... ... ... ... 3 " | 816,895 | − 4·468 | − ·558 | 862,122 | − 4·366 | − ·549 | 887,279 | − 4·319 | − ·557 |
| Do. ... ... ... ... 3½ " | 782,135 | − 8·533 | − 4·789 | 826,303 | − 8·340 | − 4·681 | 850,944 | − 8·237 | − 4·630 |
| Do. ... ... ... ... 4 " | 749,156 | −12·390 | − 8·804 | 792,284 | −12·113 | − 8·605 | 816,416 | −11·960 | − 8·499 |
| Seventeen Offices' Experience ... 3 " | 865,810 | + 1·253 | + 5·397 | 912,902 | + 1·267 | + 5·309 | 939,119 | + 1·272 | + 5·253 |
| Do. do. ... 3½ " | 830,911 | − 2·829 | + 1·148 | 876,995 | − 2·716 | + 1·167 | 902,731 | − 2·653 | + 1·174 |
| Do. do. ... 4 " | 797,912 | − 6·688 | − 2·869 | 843,004 | − 6·487 | − 2·754 | 868,264 | − 6·369 | − 2·688 |
| English Life Table, (No. 3) ... 3 " | 842,006 | − 1·531 | + 2·499 | 888,048 | − 1·490 | + 2·442 | 913,632 | − 1·477 | + 2·396 |
| Do. do. ... 3½ " | 808,322 | − 5·470 | − 1·602 | 853,358 | − 5·338 | − 1·560 | 878,455 | − 5·270 | − 1·546 |
| Do. do. ... 4 " | 776,299 | − 9·215 | − 5·500 | 820,347 | − 9·000 | − 5·368 | 844,963 | − 8·882 | − 5·300 |
| English Life Table, (No. 2) ... 3 " | 844,289 | − 1·264 | + 2·777 | 890,376 | − 1·232 | + 2·710 | 915,988 | − 1·223 | + 2·660 |
| Do. do. ... 4 " | 777,967 | − 9·020 | − 5·297 | 822,046 | − 8·812 | − 5·172 | 846,684 | − 8·696 | − 5·107 |
| Davies' Equitable Table ... ... 3 " | 775,038 | − 9·363 | − 5·653 | 819,382 | − 9·107 | − 5·479 | 844,412 | − 8·941 | − 5·362 |
| Do. do. ... ... 3½ " | 740,858 | −13·360 | − 9·814 | 784,106 | −13·020 | − 9·548 | 808,608 | −12·802 | − 9·375 |
| Do. do. ... ... 4 " | 708,514 | −17·142 | −13·752 | 750,690 | −16·727 | −13·403 | 774,673 | −16·462 | −13·178 |
| Edmonds' Mean Mortality... ... 3 " | 848,035 | − ·826 | + 3·233 | 894,462 | − ·779 | + 3·182 | 920,262 | − ·762 | + 3·139 |
| Government Males (1829) ... ... 4 " | 769,591 | −10·000 | − 6·317 | 812,948 | − 9·821 | − 6·221 | 837,214 | − 9·718 | − 6·169 |
| Hypothetical Tables. 1. Northampton ... ... 3 " | 748,767 | −12·435 | − 8·851 | 792,601 | −12·078 | − 8·568 | 817,378 | −11·857 | − 8·392 |
| 2. Do. ... ... ... 4 " | 688,286 | −19·508 | −16·214 | 730,117 | −19·009 | −15·777 | 753,925 | −18·699 | −15·503 |
| 3. Mr. Manly's Table No. 19 ... ... | 784,289 | − 8·281 | − 4·527 | 828,471 | − 8·099 | − 4·431 | 853,118 | − 8·003 | − 4·386 |
| 4. Do. No. 20 ... ... | 849,609 | − ·642 | + 3·424 | 895,830 | − ·627 | + 3·340 | 921,468 | − ·632 | + 3·274 |
| 5. Do. No. 21 ... ... | 874,564 | + 2·276 | + 6·462 | 921,549 | + 2·226 | + 6·307 | 947,555 | + 2·181 | + 6·198 |

I have extended the table to show the results for a Society of the standing of 50, 55, and 60 years respectively, but it is hardly worth while to give the figures. It will be sufficient to say that the comparative results differ but little from those of an office 45 years old.

A few words now upon the comparison of other results with the standard proposed. The figures speak for themselves; much comment is, therefore, unnecessary.

The reserves at 4 per cent. interest by *all* other tables of mortality are below the standard.

The reserves at 3½ per cent. interest by the Carlisle, Davies' Equitable, and English No 3 Tables, are below the standard throughout; by the $H^M$ and the 17 Offices' Tables below it up to and including 15 years, but above it for a Society more than 15 years old.

The reserves at 3 per cent. interest by the Carlisle, Davies' Equitable, Northampton, and Hypothetical Table No. 3, are also below the standard throughout; but by the English Tables, by Edmonds' Mean Mortality, and the Hypothetical Table No. 4, the reserves at this rate of interest are above the standard, except for the first period of 5 years, and by the $H^M$, 17 Offices', and Hypothetical Table No. 5, above the standard throughout.

From the Returns made by Offices to the Board of Trade, it appears that the Carlisle and Davies' Equitable Tables are frequently employed for valuation purposes. It may not be unimportant, therefore, to ask special attention to the reserves brought out by these tables. Tested by our standard, it will be seen that these are invariably too low. Even when the calculations are made at 3 per cent. interest, this is found to be the case. If our standard, then, be near the truth, these tables are not appropriate for valuations. And yet, when they are adopted along with interest at 3 per cent. or 3½ per cent., we find it stated in valuation reports that not only is the table of mortality employed a suitable one, but that the office has an unvalued source of profit in the difference between the rate of interest assumed, and that which may be fairly expected to be realized. Such statements can only mislead as to the real position of an office, and it is to be hoped that they will soon cease to be made thro' the recognition and adoption of the new Institute Experience as a true measure of mortality. Altho' the frequency with which the above tables are used has caused me to specially mention them, my remarks, of course, apply equally well to all tables which bring out reserves under what I have set up as our standard.

Speaking of valuation reports calls to my mind what no doubt everybody has noticed, that they sometimes contain a statement to the effect that the number of deaths which has actually happened has approached very closely to the number expected according to the table by which the liabilities have been valued, and there seems to be a tendency to jump to the conclusion that the table therefore is an appropriate one. It is possible, however, as is well known, that this conclusion may be very untrue. Take, for instance, the case of an office with a good many old lives on its books, which uses the Carlisle Table. It is not unlikely that the total number of deaths will be much the same as that expected by the table. If, however, a division be made between the younger and the older lives, drawing the line say at age 60, the agreement will cease. The actual deaths among the younger lives will be less than the number expected, while, among the older lives, the reverse will hold good, and the Carlisle Table, although showing a total mortality approaching the truth, will be seen to be inapplicable for the valuation of policies, because it is not correct at particular periods of life.

I have only further to say, that it is necessary to bear in mind that no account of reversionary bonuses has been taken in the comparisons which have been made. The position of an office reckoning a large amount of such bonuses among its liabilities would be materially affected according as the table employed in its calculations brought out values for reversionary payments greater or less than the standard. To take one example. The Northampton Table brings out too small policy values, but, on the other hand, too high values for reversionary payments, and, therefore, if an office using that table had a considerable proportion of its liabilities in the form of reversionary bonuses, the result would be that the deficiency on the one part of its reserves and the excess on the other would so far balance one another. But this introduction of a fresh element of uncertainty into the calculations is only another argument in favour of the employment of true data, by which alone the real position of an office can be satisfactorily ascertained.

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The following summary of the discussion which followed the reading of the paper is abridged from the *Insurance Record.*

The CHAIRMAN said that the subject of the paper was perhaps the most important that could engage the attention of actuaries, and as the matter must have been thought over by many gentlemen in the room, he would be glad to hear the result of those thoughts from any of the members present.

Mr. H. A. Smith remarked that there are many instances of the older established companies being obliged in the last few years to retrace their steps in the rate of bonus-giving payments; and it would have been well for them if in their early days they had had such data before them as are now available. If they had taken pencil in hand and computed what would have been the premiums necessary, according to the bonuses they were declaring, it might have kept them in the right way. It would be a useful exercise to some of the younger members to work out such a problem as the following:—Suppose an office to be established with a large enough number of lives on its books to form an average, then suppose the books to be closed, not allowing for any new lives. It would be found—supposing the premiums required were in excess of the demands upon them—that the surplus in the first periodical term of investigation would yield a very large bonus; at each periodical investigation afterwards it would become "smaller by degrees and beautifully less." This has only been kept out of the sight of many of our contemporaries by the large influx of new lives still coming in. He hoped there would be no more extravagant bonuses, and no more retrograding from the large amounts that have been so frequently declared.

Mr. Day stated that several offices are beginning to think about a change of basis in their valuations. In some of the large Scotch offices there is a practice of having what is called "a check valuation", and the Carlisle 3 per-cent table has ceased to be that infallible guide that it was formerly considered. He thought that the standard of valuation proposed by Mr. Valentine, was of somewhat too complicated a character for general adoption—*i.e.*, taking the $H^{M(5)}$, combining it with the $H^{M}$, and then assuming the 4 per-cent rate of interest. A simpler method, and one more intelligible to the public, would be the $H^{M}$ $3\frac{1}{2}$ per-cent. With regard to the basis adopted—that of the imaginary office in Mr. Manly's paper—the probability is, that the lives enter at younger ages than Mr. Manly assumed. It would be very interesting if Mr. Valentine or Mr. Manly would take up the subject further, and work it out with the greater information furnished by the returns made under the 5th and 6th Schedules of the Life Assurance Companies' Act. There were certain other matters arising out of Mr. Valentine's paper which are very interesting; and one of those is the confirmation obtained by the comparison—bringing out results of an almost identical character—of the two experience tables—*i.e.*, the Institute Experience table and the Seventeen Offices Experience table. One conclusion to be derived from the paper is, that there will be in the future few actuaries who will be content to recommend the adoption of the hitherto generally accepted valuation by the Carlisle 3 per-cent, or Davies's Equitable.

Mr. Macfadyen said that if by calling the combined $H^{M}$ and $H^{M(5)}$ 4 per-cent table the standard, Mr. Valentine means that it is the best to adopt in all cases, he must demur to that conclusion. He was not prepared to admit that 4 per-cent interest should be used universally by companies realizing that rate. In a large number of offices there are strong reasons for thinking 3 per-cent preferable.

For instance, if bonuses are declared in reversion, they are necessarily cumulative, and may be, through office system, tontine in addition. Now, the adoption of a 3 instead of a 4 per-cent rate in valuation, will allow the difference retained to fructify in the society's coffers at the compound interest rates realized, and thus form a provision for the increasing weight of the bonus. No doubt the same result might be arrived at by reserving something for this and other burdens out of the profits on a 4 per-cent valuation; but when a surplus assumes a tangible shape, it may become a rather difficult thing to prevent policy-holders and shareholders clutching it. With reference to the use of $H^{M(5)}$ for present ages, some offices have doubled their business in five or six years, and the use of $H^{M(5)}$ in such cases would cut off profit they were fairly entitled to divide.

Mr. MANLY could not but feel gratified that Mr. Valentine had taken his paper as the basis of his calculations. At the time he wrote that essay he was very young in experience, and wrote as conclusive doctrine many things which he had since come to look upon as only partially true. He had no basis on which to form his model office. The only figures he could get at were those of a recently published table of the Equitable Office, which had been going down for many years, and some figures published in a report of the old Consolidated Office, which had not been going sufficiently long to afford a very good standard for a model office. Since that time we have had the combined experience of the twenty offices and three returns to the Board of Trade, either of which would have afforded much better data for the construction of a model office. He was not altogether satisfied that what Mr. Valentine had taken as the true basis is true. If we follow out the actual mortality of those who enter at different ages, we shall find that the premium that ought to be charged to those entering at these ages is not represented by the $H^M$ premiums, but, on the other hand, the annuity after 5 years is very close indeed to the $H^{M(5)}$ at all ages; so that, although it would be quite correct to use the $H^{M(5}$ after the policy has been in force five years, it would not be correct to use the $H^M$ pure premiums. If we take all those entering at one age, say 20, or one group of ages, say 20 to 25 years of age, and trace the rate of mortality from year to year, we shall find that it increases very rapidly as compared with the collective experience. Now the number entering at these young ages is small compared with those entering at 30 and 35, so that when the light mortality of the large number of new entrants at 30 and 35 is combined with the heavier mortality of the very small number of those left after five or ten years out of the entrants at 20 or 25, the heavy mortality is entirely lost in the greater amount of light mortality, so that the general experience produces a premium too small for the young ages; and as the same causes work in the opposite direction at the older ages, the greater amount of heavy mortality then swamping the new and light mortality, the premium deduced from the whole experience is too heavy at those ages. He was therefore not fully prepared to endorse Mr. Valentine's opinion, that the $H^M$ pure premiums and the $H^{M(}$ annuities are the proper basis to work upon.

Mr. MACFADYEN wished to point out one obvious reason why the $H^M$ table may not represent an office's true premium. Even if the lives selected by an office were identical in value with those of their ages in the new $H^M$ table, unless their entrances and exits corresponded in time and numbers with that table, the $H^M$ premium would not *theoretically* be the rate they should pay. To take an illustration: Let us compare the mortality table to a tank into which boiling water flows constantly from feed pipes, and as constantly part is drawn off by waste pipes. The water uniformly enters at 212 degrees of temperature—that is, the lives are newly selected—and if not constantly fed, the contents of the tank would, say in five minutes, cool to $H^{M(5)}$. Now, if we consider the premium as the temperature of the water in the tank, it will be seen that any individual office will not have the temperature of its tank identical with that of the $H^M$ tank, unless its feeders and waste pipes inject and eject the water, in corresponding volumes and at corresponding times as the $H^M$ tank—and that, too, notwithstanding the fact that the lives assured entered in each tank at precisely the same thermal value—namely, boiling hot.

Mr. BADEN said that he understood Mr. Valentine's paper to be purely on the subject of the proper method of estimating reserves when the premium has been fixed. Now, whether the premium is fixed at $H^M$, or at $H^{M(5)}$, or any other standard, the fact as to whether the estimate is correct or otherwise makes no difference to Mr. Valentine's conclusions, which are based upon the experience of the Institute, —that policies after a certain number of years are of a certain value; and what they had to discuss was, whether that standard of reserve which he had put before them is approximately correct or not, and not whether the premiums upon which the insurance has been in the first place taken are correct or not.

Mr. P. GRAY observed that both Professor De Morgan and the late President, Mr. Jellicoe, long ago contended for the propriety of making valuations upon true data. Ascertain data as near the truth as you can, use your best judgment, and form your valuation accordingly, and you are then in the best position to decide what is the proper course to be adopted. Using data known to be erroneous, in the expectation that errors in one direction will be compensated by other errors in the opposite direction, is surely a clumsy way of seeking to arrive at the truth. He further mentioned that Mr. Hardy has afforded the means in his Valuation Tables of applying Mr. Valentine's method with great facility. The premiums are determined by the $H^M$ table, which is to represent the whole body of assurers, and the valuation is made then, after a certain lapse of time, by a table which leaves out of account certain extra influences which operate during the earlier years of the insurance.

Mr. WALFORD doubted whether the Experience and the Institute mortality tables represent correctly the great bulk of insured life. The offices which contributed to them were offices in every way of a highly respectable class, but they did not represent altogether the progressive class. He should be glad to see an attempt made some day or other to get at the result of the experience of twenty progressive offices, and see how far they would harmonize with those already published.

Mr. BADEN protested against the young offices generally being called "progressive."

The CHAIRMAN thought "pushing" would be a better word.

Mr. Curtis said that he represented a progressive office. Its mortality had never yet been published, but he was now working at it, and he found it on the whole far more favourable than that of the Seventeen Offices; although certainly in the far East of Europe the mortality is not very favourable. He hoped to be able to submit to the Institute next session the result of his investigations. He thought it eminently desirable that all offices should contribute the experience of their mortality at fixed periods.

Mr. Hodge wanted to say a word for the poor old Carlisle table. Everything goes out of fashion. When he first began to notice the insurance world, the Carlisle table was sneered at for its novelty, and considered quite an innovation; and now it is worn out and about to be cast aside; but it was certainly a most valuable contribution to insurance, and a wonderful exhibition of the skill of Dr. Heysham, who got it up. The insuring world was deeply indebted to him for it; and although his services are now superseded by more extensive and more accurately obtained data, we ought to remember with gratitude his exertions in contributing that table for the use of the insurance world.

The Chairman welcomed in Mr. Valentine a new contributor to the *Journal*, and took the opportunity of saying that the Council are always glad to receive papers from members who have not yet contributed. In this way the younger members are enabled to bring themselves before the public to some extent, and to become better known to their contemporaries. If Mr. Valentine had somewhat shaken their old traditions, he did not think that his object had been to depreciate the Carlisle table, which was rather an evidence of the value of Mr. Milne's judgment in substituting that table for the then generally received Northampton table. Nor does it detract in any way from our estimation of Mr. Milne, that we find after the lapse of some sixty years that better data can be substituted. But looking at Mr. Valentine's tables, he thought that those who trust to the Carlisle table as sufficient for the valuation of the liabilities of a life assurance company, are leaning upon a broken reed. Seeing that Mr. Valentine's standard (which he considers a reasonably true one, and in which interest is reckoned at 4 per-cent), gives a reserve of nearly 4 per-cent above the Carlisle 3 per-cent valuation for young ages, it is quite time to reconsider the Carlisle table as a basis for calculations. He was very much disposed to agree with the dictum of Mr. De Morgan, followed by Mr. Jellicoe, that the object in making valuations and estimates of any description whatever, should be to arrive at the truth. The former had said, in his essay on "Probabilities", published in Lardner's *Cyclopædia*,—"It is not safety, but truth, that is our object in valuations." And every actuary, in making an estimate of the liabilities of an insurance company, should endeavor to satisfy his own mind as to what he really believes will happen, according to the best estimate he can form; but whether the whole of the resulting surplus is to be divided, is another question altogether. Therefore he could not help thinking that Mr. Valentine's standard is in a rough way as good as can be suggested. Up to the present time, the element of the effect of selection has been wholly eliminated from valuations. He thought that in future it must be taken into account, but we must be very careful not to run into extremes. He was very much struck in reading in the report, a few days ago, of a company of long standing that because in fifty years it has made $4\frac{1}{2}$ per-cent on its investments, and makes its valuations at 4 per-cent, therefore it will make a division of such a surplus as would result from a $3\frac{1}{2}$ per-cent valuation. This was hardly logical. Extreme nicety in our estimates was out of place. Formerly we were taught that there were planets and fixed stars,—the former moving, the latter stationary. Now, we know that these fixed stars also move; but yet, for many purposes, the old distinction is convenient. So, probably, Mr. Manly would prefer to compute premiums by keeping separate the entrants at each age; but at several ages this would be impracticable from paucity of numbers. Again, Mr. Sprague, with his singular ingenuity, proposed to prove that the effect of selection was discernable for a number of years. It may be so; but as the evidence is clear that it is to a great extent exhausted in five years, the introduction of the $H^{M(5)}$ table is surely a great improvement, although no one contends for its absolute accuracy. Again, many instances of the hypothetical or reassurance method of valuation are to be found in the parliamentary returns, accompanied by a statement that provision is made for expenses and profits by assuming a rate of mortality higher, and a rate of interest lower than is expected to be realized. But at the older ages, where the liability is heaviest, it will often be found that the mortality assumed is not higher than that realized.

Mr. VALENTINE would be glad to see the new tables which Mr. Manly is calculating from the Institute experience. In the meantime, however, we have no better pure premium at hand than that afforded by the $H^M$ table. Mr. Macfadyen had advocated the use of 3 per-cent interest in valuations, instead of 4 per-cent, on the ground that he would always thereby reserve part of the surplus for future investigations. Surely we could value at 4 per-cent, and yet reserve part of the surplus if we were so minded: and by valuing at 4 per-cent and reserving a part, we should have the advantage of knowing at each investigation how we stand, which we do not know when we value at 3 per-cent. In the case of an office with a very large proportion of new business, the $H^{M(5)}$ table would probably bring out too high reserves; but it is impossible to fix on any table which will give in all cases exactly accurate results. The utmost we can do is to arrive at approximations which shall be as near the truth as possible.

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## *On the Computation of Annuities on Mr. Makeham's Hypothesis. By* EMORY MCCLINTOCK, *Actuary of the Northwestern Mutual Life Insurance Company, of Milwaukee, U.S.*

IT is now some seven years since Mr. Makeham pointed out that, if our mortality tables were graduated by his formula, $l_x = d.g^{q^x}.s^x$, "all summations connected with questions of population, the expectation of life, as well as the values of annuities, would be reduced" to the form $\int_v^\infty \epsilon^{-v}.v^{n-1}.dv$. Since that time Mr. Makeham's formula, though not representing the law of mortality so closely as some others which have been or can be presented, has nevertheless, on account of its comparative simplicity and its great practical advantages, made manifest progress in the favor of the profession. Messrs. Woolhouse and Brown, among others, have constructed life tables in accordance with it, and it is understood that many actuaries favor its general adoption.

Mr. Makeham has recently published a table by the aid of which an annuity can be calculated on any number of lives, by any given constants of mortality, at any given rate of interest (*J.I.A.*, xvii, 312). I give below a method by which the same results can be obtained, without any other table than that of the well-known gamma-function given on the next page.

As regards convenience in practice, my impression is that most computers will prefer the method now proposed to the use of Mr. Makeham's table. Be this as it may, the present method will certainly be found useful in the following cases:

1. It may be used in connection with Mr. Makeham's table, the one result verifying the other.

2. It covers all the cases to which Mr. Makeham's table does not extend, and to which it cannot be made to extend in the manner in which he at first supposed it could, as, for example, on single lives at 3 per cent., or on four or more lives at 6 per cent.

3. By using the more extended standard tables of the gamma-function, we may obtain annuities correct to 6, 7, or more places. Mr. Makeham's table extends to but 5 places.

| $r$ | $\log \Gamma_{1+r}$ | $\Delta$ | $r$ | $\log \Gamma_{1+r}$ | $\Delta$ | $r$ | $\log \Gamma_{1+r}$ | $\Delta$ |
|---|---|---|---|---|---|---|---|---|
| ·00 | 00000 | 247 | ·24 | 95833 | 101 | ·48 | 94731 | 10 |
| ·01 | 99753 | 240 | ·25 | 95732 | 96 | ·49 | 94741 | 13 |
| ·02 | 99513 | 233 | ·26 | 95636 | 91 | ·50 | 94754 | 18 |
| ·03 | 99280 | 227 | ·27 | 95545 | 86 | ·51 | 94772 | 22 |
| ·04 | 99053 | 219 | ·28 | 95459 | 81 | ·52 | 94794 | 26 |
| ·05 | 98834 | 213 | ·29 | 95378 | 76 | ·53 | 94820 | 30 |
| ·06 | 98621 | 206 | ·30 | 95302 | 71 | ·54 | 94850 | 34 |
| ·07 | 98415 | 200 | ·31 | 95231 | 66 | ·55 | 94884 | 37 |
| ·08 | 98215 | 194 | ·32 | 95165 | 61 | ·56 | 94921 | 42 |
| ·09 | 98021 | 187 | ·33 | 95104 | 57 | ·57 | 94963 | 45 |
| ·10 | 97834 | 181 | ·34 | 95047 | 52 | ·58 | 95008 | 49 |
| ·11 | 97653 | 175 | ·35 | 94995 | 47 | ·59 | 95057 | 53 |
| ·12 | 97478 | 168 | ·36 | 94948 | 43 | ·60 | 95110 | 57 |
| ·13 | 97310 | 163 | ·37 | 94905 | 37 | ·61 | 95167 | 60 |
| ·14 | 97147 | 157 | ·38 | 94868 | 34 | ·62 | 95227 | 64 |
| ·15 | 96990 | 151 | ·39 | 94834 | 29 | ·63 | 95291 | 68 |
| ·16 | 96839 | 145 | ·40 | 94805 | 24 | ·64 | 95359 | 71 |
| ·17 | 96694 | 140 | ·41 | 94781 | 20 | ·65 | 95430 | 75 |
| ·18 | 96554 | 133 | ·42 | 94761 | 16 | ·66 | 95505 | 78 |
| ·19 | 96421 | 129 | ·43 | 94745 | 11 | ·67 | 95583 | 82 |
| ·20 | 96292 | 123 | ·44 | 94734 | 7 | ·68 | 95665 | 85 |
| ·21 | 96169 | 117 | ·45 | 94727 | −3 | ·69 | 95750 | 89 |
| ·22 | 96052 | 112 | ·46 | 94724 | +1 | ·70 | 95839 | 92 |
| ·23 | 95940 | 107 | ·47 | 94725 | 6 | ·71 | 95931 | 96 |

The problem is as follows. Given $\log g$, $\log q$, $a = \log_\epsilon s$, $\delta = \log_\epsilon (1+i)$, it is required to find the annuity on a given combination of lives. I give the solution for one life; in the case of $k$ lives, aged $x$, $x+k_1$, $x+k_2$, &c., the solution is the same, except that for $a$ we must substitute $k.a$, and for $g$, $g' = g^{1+q^{k_1}+q^{k_2}+\ldots\ldots}$.—The characteristic of $\log \Gamma_{1+r}$ is $-1$, or 9.

Let

$$r = 1 - \frac{a+\delta}{\log_\epsilon q} = 1 - \mathrm{M}\cdot\frac{a+\delta}{\log q} = 1 - \frac{\log s + \log(1+i)}{\log q},$$

$$\log \epsilon^v = \mathrm{M}.v = q^x.\log\frac{1}{g},$$

$$t = \epsilon^v.v^{1-r}.\Gamma_{1+r},$$

$$v_2 = v\cdot\frac{v}{1+r},\quad v_3 = v_2\cdot\frac{v}{2+r},\quad v_4 = v_3\cdot\frac{v}{3+r},\ \text{\&c.}$$

Then

$$\bar{a}_x = \frac{r - t + v + v_2 + \ldots\ldots}{r.(a+\delta)}.$$

By way of illustration, I take the two examples given by Mr. Makeham (*J.I.A.*, xvii, 309, 310), taking his values of $n$ and $z$, and deriving from them $r$ and $v$ by the relations,

$$r = 1 - \mathrm{M}.n,$$

$$\log \epsilon^v = 10^z.$$

In these cases the working out of the formula would, using 5-place logarithms, be as follows:

| | *Ex.* 1. | *Ex.* 2. |
|---|---|---|
| $\log \epsilon^v$ . . . . . . . | 0·0065313 | 0·012303 |
| $(1-r).\log v$ . . . . | 8·90400 | 9·10290 |
| $\log \Gamma_{1+r}$ . . . . . . | 9·94809 | 9·94760 |
| $\log t$ . . . . | 8·85862 | 9·06280 |
| $r$. . . . . . . . . | 0·39873 | 0·42039 |
| $-t$ . . . . . . . . | $\bar{1}$·92779 | $\bar{1}$·88444 |
| $v$. . . . . . . . . | 0·015038 | 0·028329 |
| $v_2$ . . . . . . . . | 0·00016 | 0·00057 |
| numerator . . . | 0·34172 | 0·33373 |
| log numerator . . . | 9·53367 | 9·52340 |
| colog $r$ . . . . . . . | 0·39932 | 0·37635 |
| colog $(a+\delta)$ . . . . | 1·25665 | 1·25485 |
| log annuity . . | 1·18964 | 1·15460 |
| Mr. Makeham obtains . | 1·18964 | 1·15462 |

These examples would, as given, be unintelligible except in connection with Mr. Makeham's paper. The basing of the work on the quantities $n$ and $z$ is of course unnecessary, and has only been done on this occasion to show the connection of the two methods. The following is a new and distinct example of the present method, beginning at the very beginning.

In the $\mathrm{H}^{\mathrm{MF}}$ tables published by Mr. Brown (*J.I.A.*, xvi, 428), the constants are, $\log g = -{\cdot}00041$, $\log q = {\cdot}04$, $\log s = {\cdot}00286$; required the 3 per cent. continuous annuity at age 40. Here $\delta = {\cdot}029559$, $\log_\epsilon s = {\cdot}006585 = a$, and $a + \delta = {\cdot}036144$.

| | |
|---|---|
| $40 \log q$ . . | 1·6 |
| $\log^2 \frac{1}{g}$ . . . | 6·61278 |
| $\log^2 \epsilon^v$ . . . | 8·21278 |

whence $\log \epsilon^v = {\cdot}016322$,
and $\log_\epsilon \epsilon^v = v = {\cdot}037583$

| | |
|---|---|
| $\log v^{1-r}$ . . | 9·44080 |
| $\log \epsilon^v$ . . . | 0·01632 |
| $\log \Gamma_{1+r}$ . . | 9·95153 |
| $\log t$ . . | 9·40865 |

| | |
|---|---|
| log numerator | 9·59084 |
| colog $r$ . . . | 0·21640 |
| colog $(a+\delta)$ . | 1·44196 |
| $\log \bar{a}_{40}$ . | 1·24920 |

whence,

$$\bar{a}_{40} = 17{\cdot}750$$

| | |
|---|---|
| log Modulus . | 9·63778 |
| $\log (a+\delta)$ . . | 8·55804 |
| colog $\log q$ . . | 1·39794 |
| $\log (1-r)$ . | 9·59376 |
| $\log^2 v$ . $(-)$ | 0·15381 |
| $\log^2 v^{1-r} (-)$ | 9·74757 |

| | |
|---|---|
| $r$ . . . . | ·60757 |
| $-t$ . . . . | $\bar{1}$·74376 |
| $v$ . . . . | ·03758 |
| $v_2$ . . . . | ·00088 |
| $v_3$ . . . . | ·00001 |
| numerator | ·38980 |

*Verification by Mr. Brown's table:*

| | |
|---|---|
| $a_{40}$ . . . | 17·254 |
| $\cdot 5 - \frac{\mu+\delta}{12}$ . | ·497 |
| $\bar{a}_{40}$ . . . | 17·751 |

The demonstration of the formula remains to be given. Mr. Makeham has shown (*J.I.A.*, xvii, 308) that

$$\bar{a}_x = \frac{1}{\epsilon^{-v} . v^{r-1} . \log_\epsilon q} \cdot \int_v^\infty \epsilon^{-v} . v^{r-2} . dv ,$$

his symbol $m$ being replaced by its equivalent, $r-1$. The following property of the gamma-function is well known:

$$\int \epsilon^{-v} . v^n . dv = n . \int \epsilon^{-v} . v^{n-1} . dv - \epsilon^{-v} . v^n .$$

By means of this property may easily be derived the modified formula,

$$\bar{a}_x = \frac{r+v}{r.(a+\delta)} - \frac{1}{\epsilon^{-v}.v^{r-1}.r.(a+\delta)} \cdot \int_v^\infty \epsilon^{-v}.v^r.dv\,,$$

it being noticed that $\log_\epsilon q = \frac{a+\delta}{1-r}$. By expansion and integration,

$$\int_v^\infty \epsilon^{-v}.v^r.dv = \mathrm{C} - v^{1+r}.\left(\frac{1}{1+r} - \frac{v}{2+r} + \frac{v^2}{2.(3+r)} - \frac{v^3}{2.3.(4+r)} + \dots\right)$$

Multiplying this series by $\epsilon^{-v}$, and also by the expansion of $\epsilon^v$, its value remains unaltered, and we have

$$\int_v^\infty \epsilon^{-v}.v^r.dv = \mathrm{C} - \epsilon^{-v}.v^{1+r}\left\{\frac{1}{1+r} + \frac{v}{(1+r).(2+r)} + \frac{v^2}{(1+r).(2+r).(3+r)} + \dots\right.$$
$$= \mathrm{C} - \epsilon^{-v}.v^{r-1}.(v_2 + v_3 + v_4 + \dots).$$

If $v=0$, all of the second member vanishes except the constant, whence $\mathrm{C} = \Gamma_{1+r} = \int_0^\infty \epsilon^{-v}.v^r.dv$. Dividing the equation now by $\epsilon^{-v}.v^{r-1}$, we have

$$\frac{1}{\epsilon^{-v}.v^{r-1}} \cdot \int_v^\infty \epsilon^{-v}.v^r.dv = \Gamma_{1+r}.\epsilon^v.v^{1-r} - v_2 - v_3 - \dots$$
$$= t - v_2 - v_3 - \dots$$

Substituting this result in the expression last found for $\bar{a}_x$, we obtain directly the desired formula,

$$\bar{a}_x = \frac{r - t + v + v_2 + \dots}{r.(a+\delta)}.$$

The series for $\int \epsilon^{-v}.v^r.dv$ may also be obtained by integration by parts, or by applying Leibnitz's Theorem for the determination of the $n$th differential coefficient of the product of two functions, making $n = -1$.

If $r<0$, $\Gamma_{1+r}$ must be found by means of the well-known relation, $\Gamma_{1+r} = \frac{\Gamma_{2+r}}{1+r}$. If $r$, whether positive or negative, approaches closely to zero, the nearer it does so, the smaller will be the number of decimal places to which the resulting annuity can be depended upon; and when $r=0$, the formula is reduced to the form $\frac{0}{0}$. In determining $\Gamma$ from the table, interpolation by first differences will usually be sufficiently accurate; but towards the beginning of the table second differences will have to be resorted to.

In place of $v$, $v_2$, $v_3$, &c., may be substituted a corresponding series of powers of $u=\epsilon^v-1$, thus:—

$$u_2=\frac{1-r}{1+r}\cdot\frac{u^2}{2},$$

$$u_3=-\frac{1-r}{2+r}\cdot\frac{u^3}{3},$$

$$u_4=\frac{1-r}{3+r}\cdot\frac{u^4}{4}\text{ (nearly)},$$

and so on. A series of powers of $y=\log_\epsilon(1+v)$ might also be employed, but being less convergent than the others would be practically valueless. These series may be derived from the series of powers of $v$ by expansion and substitution of $v=\log_\epsilon(1+u)$ and $v=\epsilon^y-1$, or they may be obtained in the same manner as the original formula, by transformation of the integral, expansion, integration, and multiplication.

The series of powers of $u$ is less symmetrical and less convenient for calculation than that of $v$, but it is usually more convergent, requiring a smaller number of terms to give a close approximation to accuracy. On the whole, however, I am disposed to prefer the use of the powers of $v$. To illustrate the comparative convergence of the two series, it will suffice to quote the three examples already given.

| | | |
|---|---|---|
| $v$ =·01504 | $v$ =·02833 | $v$ =·03758 |
| $v_2$=·00016 | $v_2$=·00057 | $v_2$=·00088 |
| | | $v_3$=·00001 |
| ·01520 | ·02890 | ·03847 |
| $u$ =·01515 | $u$ =·02873 | $u$ =·03830 |
| $u_2$=·00005 | $u_2$=·00017 | $u_2$=·00018 |
| ·01520 | ·02890 | ·03848 |

## *On the Value of a complete Annuity when payable by* m *equal Instalments in each Year.* *By* THOMAS CARR, F.I.A.

IF the death take place in the $(t+1)$th interval in the year of death, then the present value of the $t$ instalments of $\frac{1}{m}$ each paid in that year previously to the death, is $=\frac{\text{A}_k}{vm}\,(v^{\frac{1}{m}}+v^{\frac{2}{m}}+\,\ldots\,+v^{\frac{t}{m}})$

$=\dfrac{A_k v^{\frac{1}{m}}}{vm}\cdot\dfrac{1-v^{\frac{t}{m}}}{1-v^{\frac{1}{m}}}$. Supposing the deaths equally distributed over the year, multiplying the latter formula by $\frac{1}{m}$ and summing it for all values of $t$ from 1 to $m-1$, both inclusive, we have

$$\frac{A_k v^{\frac{1}{m}}}{m^2 v(1-v^{\frac{1}{m}})}\left(m-1-\frac{v^{\frac{1}{m}}-v}{1-v^{\frac{1}{m}}}\right)=\frac{A_k v^{\frac{1}{m}}}{m^2 v(1-v^{\frac{1}{m}})}\left(m-\frac{vi}{1-v^{\frac{1}{m}}}\right),$$

or putting $\log_e(1+i)=\delta$, whence $1+i=e^\delta$ and $i=e^\delta-1$;

$$=\frac{A_k e^\delta}{m(e^{\frac{\delta}{m}}-1)}\left(1-\frac{e^\delta-1}{me^\delta}\cdot\frac{e^{\frac{\delta}{m}}}{e^{\frac{\delta}{m}}-1}\right)$$

$$=\frac{A_k i}{m(e^{\frac{\delta}{m}}-1)}\left\{\frac{e^\delta}{e^\delta-1}-\frac{e^{\frac{\delta}{m}}}{m(e^{\frac{\delta}{m}}-1)}\right\}$$

$$=\frac{A_k i}{m(e^{\frac{\delta}{m}}-1)}\left(\frac{m-1}{m}+\frac{1}{e^\delta-1}-\frac{\frac{1}{m}}{e^{\frac{\delta}{m}}-1}\right);$$

or using the well known development by the numbers of Bernoulli;

$$=\frac{A_k i}{m(e^{\frac{\delta}{m}}-1)}\left(\frac{m-1}{2m}+\frac{m^2-1}{12m^2}\delta-\frac{m^4-1}{720m^4}\delta^3+\ldots\right)\quad.\ .\ (a)$$

The value of the proportional part of $\frac{1}{m}$, from the last payment to the day of death, is (see *Journal*, vol. xiii, page 363)

$$A_k i\left(\frac{1}{\delta^2}-\frac{1}{e^{\frac{\delta}{m}}-1}\frac{1}{m\delta}\right)=\frac{A_k i}{m(e^{\frac{\delta}{m}}-1)}\left\{\frac{m(e^{\frac{\delta}{m}}-1)}{\delta^2}-\frac{1}{\delta}\right\}$$

$$=\frac{A_k i}{m(e^{\frac{\delta}{m}}-1)}\left(\frac{1}{2m}+\frac{\delta}{6m^2}+\frac{\delta^2}{24m^3}+\frac{\delta^3}{120m^4}+\ldots\right).$$

Adding the latter to $(a)$ we have the value of the total sum paid under the annuity in the year of death

$$=\frac{A_k i}{m(e^{\frac{\delta}{m}}-1)}\left(\frac{1}{2}+\frac{m^2+1}{12m^2}\delta+\frac{\delta^2}{24m^3}-\frac{m^4-7}{720m^4}\delta^3+\ldots\right)\quad.\ .\ (\beta)$$

which is found to be

$$= \mathrm{A}_k\left(\frac{1}{2} + \frac{4m^2-3m+1}{12m^2}i - \frac{m^2+m-1}{24m^2}i^2 + \frac{14m^4+15m^3-10m^2-3}{720m^4}i^3 - \ldots\right).*$$

The formula ($\beta$) may be written thus:

$$\frac{\mathrm{A}_k\delta}{m(e^{\frac{\delta}{m}}-1)}\frac{i}{\delta}\left(\frac{1}{2} + \frac{\delta}{12} - \frac{\delta^3}{720} + \ldots\ldots + \frac{\delta}{12m^2} + \frac{\delta^2}{24m^3} + \frac{7\delta^3}{720m^4} + \ldots\right)$$

which, since $\frac{i}{\delta} = 1 + \frac{\delta}{2} + \frac{\delta^2}{6} + \frac{\delta^3}{24} + \ldots\ldots,$

$$\frac{\mathrm{A}_k\delta}{m(e^{\frac{\delta}{m}}-1)}\left(\frac{1}{2} + \frac{\delta}{3} + \frac{\delta^2}{8} + \frac{\delta^3}{30} + \ldots\ldots + \frac{\delta}{12m^2} + \frac{m+1}{24m^3}\delta^2 + \frac{10m^2+15m+7}{720m^4}\delta^3 + \ldots\right)$$

(or remembering that $\frac{\frac{\delta}{m}}{e^{\frac{\delta}{m}}-1} = 1 - \frac{\delta}{2m} + \frac{\delta^2}{12m^2} - \frac{\delta^4}{720m^4} + \ldots\ldots$)

$$= \mathrm{A}_k\left(1 - \frac{\delta}{2m}\right)\left(\frac{1}{2} + \frac{\delta}{3} + \frac{\delta^2}{8} + \frac{\delta^3}{30} + \ldots\ldots\right)$$

$$+ \mathrm{A}_k\left(\frac{\delta^2}{12m^2} - \ldots\ldots\right)\left(\frac{1}{2} + \frac{\delta}{3} + \frac{\delta^2}{8} + \frac{\delta^3}{30} + \ldots\ldots\right)$$

$$+ \mathrm{A}_k\left(\frac{\delta}{12m^2} + \frac{\delta^2}{24m^2} + \frac{10m^2-3}{720m^4}\delta^3 + \ldots\ldots\right)$$

$$= \mathrm{A}_k\left(1 - \frac{\delta}{2m}\right)\left(\frac{1}{2} + \frac{\delta}{3} + \frac{\delta^2}{8} + \frac{\delta^3}{30} + \ldots\ldots\right)$$

$$+ \mathrm{A}_k\left(\frac{\delta}{12m^2} + \frac{\delta^2}{12m^2} + \frac{10m^2-1}{240m^4}\delta^3 + \ldots\ldots\right)$$

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* If this formula be compared with that given in the paper by Mr. T. B. Sprague, M.A., "On the Value of Apportionable Annuities" &c., in vol. xiii, page 363, of the *Journal*, for the value of the proportionate payment to the day of death, or, as it is there called, the correction, in the case of an annuity payable annually, viz., $\mathrm{A}_k\left(\frac{1}{2} + \frac{i}{6} - \frac{i^2}{24} + \frac{i^3}{45} - \ldots\ldots\right)$ it will be seen that as the correction is the only payment in that case in the year of death, the value of the total payment made in that year remains the same with the exception of small quantities involving $i$, $i^2$, &c., whether the annuity be payable annually or $m$ times a year. Supposing the latter to be the case, and that the death occurs at the expiration of the portion $\frac{t}{m} + \frac{1}{mn}$ of the year of death, then $t$ payments of $\frac{1}{m}$ each have been already made in that year, and the correction is $\frac{1}{mn}$. These are, together, $= \frac{t}{m} + \frac{1}{mn}$. But if the annuity be payable annually, and T denote the correction in that case, then $\mathrm{T} = \frac{t}{m} + \frac{1}{mn}$, since $\frac{t}{m} + \frac{1}{mn}$ is the proportion of the year elapsed from the previous payment to the date of death. The same is true

(or since $A_k=\frac{1-ia_k}{1+i}$, and $\frac{1}{1+i}, \frac{i}{1+i}$, are respectively $=e^{-\delta}$ and $1-e^{-\delta}$,)

$$=A_k\left(1-\frac{\delta}{2m}\right)\left(\frac{1}{2}+\frac{\delta}{3}+\frac{\delta^2}{8}+\frac{\delta^3}{30}+\dots\right)$$

$$+\frac{\delta}{12m^2}-\frac{\delta^3}{240m^4}+\dots-a_k\left(\frac{\delta^2}{12m^2}+\frac{\delta^3}{24m^2}+\dots\right)\quad . \; . \quad (\gamma)$$

and this is the present value of the total sum payable in the year of death. When $m$ is infinite or the annuity is payable momently, this expression is reduced to

$$A_k\left(\frac{1}{2}+\frac{\delta}{3}+\frac{\delta^2}{8}+\frac{\delta^3}{30}+\dots\right).$$

Also since

$$\frac{1}{m}\left\{1+(1+i)^{\frac{1}{m}}+(1+i)^{\frac{2}{m}}+\dots+(1+i)^{\frac{m-1}{m}}\right\}=\frac{i}{m\left\{(1+i)^{\frac{1}{m}}-1\right\}}=\frac{i}{m(e^{\frac{\delta}{m}}-1)},$$

the value of the annuity in respect of the number of completed years it continues, that is, excluding the year of death, is

$$\frac{i}{m(e^{\frac{\delta}{m}}-1)}a_k=\frac{\delta}{m(e^{\frac{\delta}{m}}-1)}\frac{i}{\delta}a_k=\left(1-\frac{\delta}{2m}\right)\frac{i}{\delta}a_k+a_k\left(\frac{\delta^2}{12m^2}+\frac{\delta^3}{24m^2}+\dots\right).\quad . \; . \quad ($$

This becomes $\frac{i}{\delta}a_k$ when $m$ is infinite, since in that case $m(e^{\frac{\delta}{m}}-1)=\delta$.

Adding to $\frac{i}{\delta}a_k$ the expression found above,

$$A_k\left(\frac{1}{2}+\frac{\delta}{3}+\frac{\delta^2}{8}+\frac{\delta^3}{30}+\dots\right),$$

we have

$$\frac{i}{\delta}a_k+A_k\left(\frac{1}{2}+\frac{\delta}{3}+\frac{\delta^2}{8}+\frac{\delta^3}{30}+\dots\right)=\bar{a}_k.$$

Also adding together $(\gamma)$ and $(\zeta)$, and omitting terms involving $\delta^3$

for all possible values of $\frac{t}{m}+\frac{1}{mn}$, or T; and if it be borne in mind that T is payable when the death has occurred, whereas the $t$ payments of $\frac{1}{m}$ each are made previously to the death, an explanation is afforded of the fact that the value of the total payments in the year of death in the case of an annuity payable $m$ times a year differs from the value of the correction in the case of an annuity payable annually by quantities involving $i$, $i^2$, &c.

in which $a_k$ does not enter, as well as those which cancel one another, we have

$$\left(1-\frac{\delta}{2m}\right)\frac{i}{\delta}a_k+\left(1-\frac{\delta}{2m}\right)A_k\left(\frac{1}{2}+\frac{\delta}{3}+\frac{\delta^2}{8}+\frac{\delta^3}{30}+\ldots\right)+\frac{\delta}{12m^2}$$

$$=\left(1-\frac{\delta}{2m}\right)\left\{\frac{i}{\delta}a_k+A_k\left(\frac{1}{2}+\frac{\delta}{3}+\frac{\delta^2}{8}+\frac{\delta^3}{30}+\ldots\right)\right\}+\frac{\delta}{12m^2},$$

whence

$$\mathring{a}_k^{(m)}=\left(1-\frac{\delta}{2m}\right)\bar{a}_k+\frac{\delta}{12m^2}.$$

If now we compare this formula with (23), page 377, vol. xiii of the *Journal,* where $\bar{a}_k=a_k+\frac{1}{2}-\frac{\mu+\delta}{12}$, we see that the formula there given is applicable when $\bar{a}_k$ is deduced on the assumption that the deaths are uniformly distributed over the year.

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## *An Estimate of the Degrees of the* Mortality *of Mankind, drawn from curious* Tables *of the* Births *and* Funerals *at the City of* Breslaw; *with an Attempt to ascertain the Price of* Annuities *upon* Lives. *By Mr.* E. HALLEY, *R.S.S.*

Reprinted from the *Philosophical Transactions,* vol. xvii, page 579, (Numb. 196, for January 1693.)

THE Contemplation of the *Mortality* of *Mankind,* has besides the *Moral,* its *Physical* and *Political* Uses, both which have been some years since most judiciously considered by the curious Sir *William Petty,* in his *Natural* and *Political* Observations on the Bills of *Mortality* of *London,* owned by Captain *John Graunt.* And since in a like Treatise on the Bills of *Mortality* of *Dublin.* But the Deduction from those Bills of *Mortality* seemed even to their Authors to be defective: First, In that the *Number* of the People was wanting. Secondly, That the *Ages* of the People dying was not to be had. And Lastly, That both *London* and *Dublin* by reason of the great and casual Accession of *Strangers* who die therein, (as appeared in both, by the great Excess of the *Funerals* above the *Births*) rendred them incapable of being Standards for this purpose; which requires, if it were possible, that the People we treat of should not at all be changed, but die where they were born, without any Adventitious Increase from Abroad, or Decay by Migration elsewhere.

This *Defect* seems in a great measure to be satisfied by the late curious Tables of the Bills of *Mortality* at the City of *Breslaw,* lately communicated to this Honourable Society by Mr. *Justell,* wherein both the *Ages* and *Sexes* of all that die are monthly delivered, and compared with the number of the *Births,* for Five Years last past, *viz.* 1687, 88, 89, 90, 91, seeming to be done with all the Exactness and Sincerity possible.

This City of *Breslaw* is the Capital City of the Province of *Silesia;* or, as the *Germans* call it, *Schlesia,* and is scituated on the Western Bank of the River *Oder,* anciently called *Viadrus;* near the Confines of *Germany* and *Poland,* and very nigh the Latitude of *London.* It is very far from the Sea, and as much a *Mediterranean* Place as can be desired, whence the Confluence of Strangers is but small, and the Manufacture of Linnen employs chiefly the poor People of the place, as well as of the Country round about; whence comes that sort of Linnen we usually call your *Sclesie Linnen;* which is the chief, if not the only Merchandize of the place. For these Reasons the People of this City seem most proper for a *Standard;* and the rather, for that the *Births* do, a small matter, exceed the *Funerals.* The only thing wanting is the Number of the whole People, which in some measure I have endeavoured to supply by comparison of the *Mortality* of the People of all Ages, which I shall from the said Bills trace out with all the Acuracy possible.

It appears that in the Five Years mentioned, *viz.* from 87 to 91 inclusive, there were *born* 6193 Persons, and *buried* 5869; that is, born *per Annum* 1238, and *buried* 1174; whence an *Encrease* of the People may be argued of .64 *per Annum,* or of about a 20th part, which may perhaps be ballanced by the Levies for the *Emperor's* Service in his Wars. But this being contingent, and the Births certain, I will suppose the People of *Breslaw* to be encreased by 1238 *Births* annually. Of these it appears by the same Tables, that 348 do die *yearly* in the *first Year* of their *Age,* and that but 890 do arrive at a full *Year's Age;* and likewise, that 198 do die in the *Five Years* between 1 and 6 compleat, taken at a *Medium;* so that but 692 of the Persons *born* do survive *Six* whole *Years.* From this *Age* the Infants being arrived at some degree of Firmness, grow less and less *Mortal;* and it appears that of the whole People of *Breslaw* there die *yearly,* as in the following Table, wherein the upper Line shews the *Age*, and the next under it the *Number* of Persons of that Age *dying yearly.*

7 . 8 9 . . 14 . 18 . 21 . 27 . 28 . . 35 . 36 .
11 . 11 . 6 . 5½ . 2 . 3½ 5 6 4½ 6½ 9 . 8 . 7 . 7 . 8 . 9½

42 . 45 49 54 . 55 . 56 . 63 70 71 . 72
8 . 9 . 7 . 7 . 10 11 . 9 . 9 . 10 . 12 9½ 14 9 . 11 9½

77 81 84 . 90 91 . 98 . 99 . 100.
6 . 7 . 3 . 4 . 2 . 1 . 1 . 1 . 0 . ⅕ . ⅗

And where no *Figure* is placed over, it is to be understood of those that die between the Ages of the preceding and consequent *Column.*

From this Table it is evident, that from the Age of 9 to about 25 there does not die above 6 *per Annum* of each *Age*, which is much about one *per Cent.* of those that are of those *Ages:* And whereas in the 14, 15, 16, 17 *Years* there appear to die much fewer, as 2 and 3½, yet that seems rather to be attributed to Chance, as are the other Irregularities in the Series of Ages, which would rectifie themselves, were the number of Years much more considerable, as 20 instead of 5. And by our own Experience in *Christ-Church Hospital,* I am informed there die of the *Young Lads,* much about one *per Cent. per Annum,* they being of the foresaid *Ages.* From 25 to 50 there seem to die from 7 to 8 and 9 *per Annum* of each Age; and after that to 70, they growing more *crasie,* though the number be much diminished, yet the *Mortality encreases,* and there are found to die 10 or 11 of each Age *per Annum:* From thence the number of the *Living* being grown very small, they gradually decline till there be none left to *die:* as may be seen at one View in the Table.

From these Considerations I have formed the *adjoyned Table,* whose Uses are manifold, and give a more just *Idea* of the *State* and *Condition* of *Mankind,* than any thing yet extant that I know of. It exhibits the *Number* of *People* in the City of *Breslaw* of all Ages, from the *Birth* to extream *Old Age,* and thereby shews the Chances of *Mortality* at all *Ages,* and likewise how to make a certain Estimate of the value of *Annuities* for *Lives,* which hitherto has been only done by an imaginary *Valuation:* Also the *Chances* that there are that a *Person* of any *Age* proposed does live to any other *Age* given; with many more, as I shall hereafter shew. This *Table* does shew the *number* of *Persons* that are living in the *Age* current annexed thereto, as follows:

| Age. Curt. | Persons. | Age. Curt. | Persons. | Age. Curt. | Persons. | Age. Curt. | Persons. | Age. Curt. | Persons. | Age. Curt. | Persons. |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1000 | 8 | 680 | 15 | 628 | 22 | 586 | 29 | 539 | 36 | 481 |
| 2 | 855 | 9 | 670 | 16 | 622 | 23 | 579 | 30 | 531 | 37 | 472 |
| 3 | 798 | 10 | 661 | 17 | 616 | 24 | 573 | 31 | 523 | 38 | 463 |
| 4 | 760 | 11 | 653 | 18 | 610 | 25 | 567 | 32 | 515 | 39 | 454 |
| 5 | 732 | 12 | 646 | 19 | 604 | 26 | 560 | 33 | 507 | 40 | 445 |
| 6 | 710 | 13 | 640 | 20 | 598 | 27 | 553 | 34 | 499 | 41 | 436 |
| 7 | 692 | 14 | 634 | 21 | 592 | 28 | 546 | 35 | 490 | 42 | 427 |
| 43 | 417 | 50 | 346 | 57 | 272 | 64 | 202 | 71 | 131 | 78 | 58 |
| 44 | 407 | 51 | 335 | 58 | 262 | 65 | 192 | 72 | 120 | 79 | 49 |
| 45 | 397 | 52 | 324 | 59 | 252 | 66 | 182 | 73 | 109 | 80 | 41 |
| 46 | 387 | 53 | 313 | 60 | 242 | 67 | 172 | 74 | 98 | 81 | 34 |
| 47 | 377 | 54 | 302 | 61 | 232 | 68 | 162 | 75 | 88 | 82 | 28 |
| 48 | 367 | 55 | 292 | 62 | 222 | 69 | 152 | 76 | 78 | 83 | 23 |
| 49 | 357 | 56 | 282 | 63 | 212 | 70 | 142 | 77 | 68 | 84 | 20 |

| Age. | Persons. |
|---|---|
| 7 | 5547 |
| 14 | 4584 |
| 21 | 4270 |
| 28 | 3964 |
| 35 | 3604 |
| 42 | 3178 |
| 49 | 2709 |
| 56 | 2194 |
| 63 | 1694 |
| 70 | 1204 |
| 77 | 692 |
| 84 | 253 |
| 100 | 107 |
| | 34000 |
| Sum Total. | |

Thus it appears, that the whole People of *Breslaw* does consist of 34000 *Souls,* being the Sum *Total* of the Persons of all Ages in the *Table:* The first use hereof is to shew the Proportion of *Men* able to bear *Arms* in any *Multitude,* which are those between 18 and 56, rather than 16 and 60; the one being generally too weak to bear the *Fatigues* of *War* and the Weight of *Arms,* and the other too crasie and infirm from *Age,* notwithstanding particular Instances to the contrary. Under 18 from the *Table,* are found in this City 11997 Persons, and 3950 above 56, which together make 15947. So that the Residue to 34000 being 18053 are Persons between those *Ages.* At least one half thereof are Males, or 9027: So that the whole Force this City can raise of *Fencible Men,* as the *Scotch* call them, is about 9000, or $\frac{9}{34}$, or somewhat more than a quarter of the *Number* of *Souls,* which may perhaps pass for a Rule for all other places.

The *Second Use* of this Table is to show the differing degrees of *Mortality,* or rather *Vitality* in all *Ages;* for if the number of Persons of any *Age* remaining after one year, be divided by the difference between that and the number of the Age proposed, it shews the *odds* that there is, that a Person of that Age does not die in a *Year.* As for Instance, a Person of 25 *Years* of *Age* has the odds of 560 to 7 or 80 to 1, that he does not *die* in a *Year:* Because that of 567, living of 25 years of Age, there do die no more than 7 in a *Year,* leaving 560 of 26 Years old.

So likewise for the *odds,* that any Person does not die before he attain any proposed *Age:* Take the *number* of the remaining

Persons of the Age proposed, and divide it by the difference between it and the number of those of the *Age* of the Party proposed; and that shews the *odds* there is between the Chances of the Party's living or dying. As for Instance; What is the *odds* that a Man of 40 lives 7 Years: Take the number of Persons of 47 years, which in the Table is 377, and subtract it from the number of Persons of 40 years, which is 445, and the *difference* is 68: Which shews that the *Persons dying* in that 7 years are 68, and that it is 377 to 68 or 5½ to 1, that a Man of 40 does live 7 Years. And the like for any other *number* of *Years.*

*Use* III. But if it be enquired at what number of *Years,* it is an even Lay that a Person of any *Age* shall die, this Table readily performs it: For if the *number* of Persons *living* of the *Age* proposed to be *halfed,* it will be found by the *Table* at what Year the said *number* is reduced to half by *Mortality;* and that is the Age, to which it is an even Wager, that a Person of the *Age* proposed shall arrive before he *die.* As for Instance; a Person of 30 Years of *Age* is proposed, the number of that Age is 531, the half thereof is 265, which number I find to be between 57 and 58 Years; so that a Man of 30 may reasonably expect to live between 27 and 28 Years.

*Use* IV. By what has been said, the *Price* of *Insurance* upon *Lives* ought to be regulated, and the difference is discovered between the *price* of ensuring the *Life* of a *Man* of 20 and 50, for Example: it being 100 to 1 that a Man of 20 dies not in a year, and but 38 to 1 for a Man of 50 Years of Age.

*Use* V. On this depends the Valuation of *Annuities* upon *Lives;* for it is plain that the *Purchaser* ought to pay for only such a part of the value of the *Annuity,* as he has Chances that he is living; and this ought to be computed yearly, and the Sum of all those yearly Values being added together, will amount to the value of the *Annuity* for the *Life* of the Person proposed. Now the present value of Money payable after a term of years, at any given rate of Interest, either may be had from Tables already computed; or almost as compendiously, by the Table of Logarithms: For the Arithmetical Complement of the Logarithm of Unity and its yearly Interest (that is, of 1, 06 for Six *per Cent.* being 9, 974694) being multiplied by the number of years proposed, gives the present value of One Pound payable after the end of so many years. Then by the foregoing Proposition, it will be as the number of Persons living after that term of years, to the number dead; so are the Odds that any one Person is Alive or Dead. And by consequence, as the Sum of both or the number of

Persons living of the Age first proposed, to the number remaining after so many years, (both given by the Table) so the present value of the yearly Sum payable after the term proposed, to the Sum which ought to be paid for the Chance the person has to enjoy such an Annuity after so many Years. And this being repeated for every year of the person's Life, the Sum of all the present Values of those Chances is the true Value of the Annuity. This will without doubt appear to be a most laborious Calculation, but it being one of the principal Uses of this Speculation, and having found some *Compendia* for the Work, I took the pains to compute the following Table, being the short Result of a not ordinary number of Arithmetical Operations: It shews the Value of Annuities for every Fifth Year of Age, to the Seventieth, as follows.

| Age. | Years Purchase. | Age. | Years Purchase. | Age. | Years Purchase. |
|---|---|---|---|---|---|
| 1 | 10,28 | 25 | 12,27 | 50 | 9,21 |
| 5 | 13,40 | 30 | 11,72 | 55 | 8,51 |
| 10 | 13,44 | 35 | 11,12 | 60 | 7,60 |
| 15 | 13,33 | 40 | 10,57 | 65 | 6,54 |
| 20 | 12,78 | 45 | 9,91 | 70 | 5,32 |

This shews the great Advantage of putting Money into the present *Fund* lately granted to their Majesties, giving 14 *per Cent. per Annum*, or at the rate of 7 years purchase for a Life; when young Lives, at the usual rate of Interest, are worth above 13 years Purchase. It shows likewise the Advantage of young Lives over those in Years; a Life of Ten Years being almost worth 13½ years purchase, whereas one of 36 is worth but 11.

*Use* V. Two Lives are likewise valuable by the same Rule; for the number of Chances of each single Life, found in the Table, being multiplied together, become the Chances of the Two Lives. And after any certain Term of Years, the Product of the two remaining Sums is the Chances that both the Persons are living. The Product of the two Differences, being the numbers of the Dead of both Ages, are the Chances that both the Persons are dead. And the two Products of the remaining Sums of the one Age multiplied by those dead of the other, shew the Chances that there are that each Party survives the other: Whence is derived the Rule to estimate the value of the Remainder of one Life after another. Now as the Product of the Two Numbers in the Table for the Two Ages proposed, is to the difference between that Product and the Product of the two numbers of Persons deceased in any space of time, so is the value of a Sum of Money to be paid after so much time, to the value thereof under the Contingency of Mortality. And as the aforesaid Product of the two Numbers answering to the Ages proposed, to the Product of the Deceased of one Age multiplied by those remaining alive of the other; So the Value of a Sum of Money to be paid after any time proposed, to the value of the Chances that the one Party has that he survives the other whose number of Deceased you made use of, in the second Term of the proportion. This perhaps may be better understood, by putting $N$ for the number of the younger Age, and $n$ for that of the Elder; $Y$, $y$, the deceased of both Ages respectively, and $R$, $r$, for the Remainders; and $R+Y=N$ and $r+y=n$. Then shall $Nn$ be the whole number of Chances; $Nn-Yy$ be the Chances that one of the two Persons is living, $Yy$ the Chances that they are both dead; $Ry$ the Chances that the elder Person is dead and the younger living; and $rY$ the Chances that the elder is living and the younger dead. Thus two Persons of 18 and 35 are proposed, and after 8 years these Chances are required. The Numbers for 18 and 35 are 610 and 490, and there are 50 of the First Age dead in 8 years, and 73 of the Elder Age. There are in all $610 \times 490$ or 298900 Chances; of these there are $50 \times 73$ or 3650, that they are both dead. And as 298900, to $298900-3650$ or 295250: So is the present value of a Sum of Money to be paid after 8 years, to the present value of a Sum to be paid if either of the two live. And as $560 \times 73$, so are the Chances that the Elder is dead, leaving the Younger; and as $417 \times 50$, so are the Chances that the Younger is dead, leaving the Elder. Wherefore as $610 \times 490$ to $560 \times 73$, so is the present value of a Sum to be paid at eight years end, to the Sum to be paid for the Chance of the Youngers Survivance; and as $610 \times 490$ to $417 \times 50$, so is the same present value to the Sum to be paid for the Chance of the Elders Survivance.

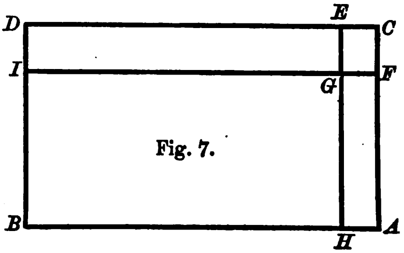

Fig. 7.

This possibly may be yet better explained by expounding these Products by Rectangular Parallelograms, as in *Fig.* 7. wherein *AB* or *CD* represents the number of persons of the younger Age, and *DE*, *BH*, those remaining alive after a certain term of years; whence *CE* will answer the number of those dead in that time: So *AC*, *BD* may represent the number of the Elder Age; *AF*, *BI* the Survivors after the same term; and *CF*, *DI*, those of that Age that are dead at that time. Then shall the whole Parallelogram *ABCD* be *Nn,* or the Product of the two Numbers of persons, representing such a number of Persons of the two Ages given; and by what was said before, after the Term proposed the Rectangle *HD* shall be as the number of Persons of the younger Age that survive, and the Rectangle *AE* as the number of those that die. So likewise the Rectangles *AI, FD* shall be as the Numbers, living and dead, of the other Age. Hence the Rectangle *HI* shall be as an equal number of both Ages surviving. The Rectangle *FE* being the Product of the deceased, or *Yy,* an equal number of both dead. The Rectangle *GD* or *Ry,* a number living of the younger Age, and dead of the Elder: And the Rectangle *AG* or *rY* a number living of the Elder Age, but dead of the younger. This being understood, it is obvious, that as the whole Rectangle *AD* or *Nn* is to the *Gnomon FABDEG* or $Nn-Yy$, so is the whole number of Persons or Chances to the number of Chances that one of the two Persons is living: And as *AD* or *Nn* is to *FE* or *Yy,* so are all the Chances, to the Chances that both are dead; whereby may be computed the value of the Reversion after both Lives. And as *AD* to *GD* or *Ry,* so the whole number of Chances, to the Chances that the younger is living and the other dead; whereby may be cast up what value ought to be paid for the Reversion of one Life after another, as in the case of providing for Clergy-mens Widows and others by such Reversions. And as *AD* to *AG* or *rY,* so are all the Chances, to those that the Elder survives the younger. I have been the more particular, and perhaps tedious, in this matter, because it is the Key to the Case of Three Lives, which of itself would not have been so easie to comprehend.

VII. If Three Lives are proposed, to find the value of an Annuity during the continuance of any of those three Lives. The Rule is, *As the Product of the continual multiplication of the Three Numbers, in the Table, answering to the Ages proposed, is to the difference of that Product and of the Product of the Three Numbers of the deceased of those Ages, in any given term of Years; So is the present value of a Sum of Money to be paid certainly after so many Years, to the present value of the same Sum to be paid, provided one of those three Persons be living at the Expiration of that term.* Which proportion being yearly repeated, the Sum of all those present values will be the value of an Annuity granted for three such Lives. But to explain this, together with all the Cases of Survivance in three Lives: Let *N* be the Number in the Table for the Younger Age, $n$ for the Second, and $\nu$ for the Elder Age; let $Y$ be those dead of the Younger Age in the term proposed, $y$ those dead of the Second Age, and $v$ those of the Elder Age; and let $R$ be the Remainder of the younger Age, $r$ that of the middle Age, and $\rho$ the Remainder of the Elder Age. Then shall $R+Y$ be equal to $N$, $r+y$ to $n$, and $\rho+v$ to $\nu$, and the continual Product of the three Numbers $Nn\nu$ shall be equal to the continual Product of $R+Y\times r+Y\times \rho+v$, which being the whole number of Chances for three Lives is compounded of the eight Products following. (1) $Rr\rho$, which is the number of Chances that all three of the Persons are living. (2) $r\rho\mathrm{Y}$, which is the number of Chances that the two Elder Persons are living, and the younger dead. (3) $R\rho y$ the number of Chances that the middle Age is dead, and the younger and Elder living. (4) $Rrv$ being the Chances that the two younger are living, and the elder dead. (5) $\rho Yy$ the Chances that the two younger are dead, and the elder living. (6) $rYv$ the Chances that the younger and elder are dead, and the middle Age living. (7) $Ryv$, which are the Chances that the younger is living, and the two other dead. And Lastly and Eightly, $Yyv$, which are the Chances that all three are dead. Which latter substracted from the whole number of Chances $Nn\nu$, leaves $Nn\nu-Yyv$ the Sum of all the other Seven Products; in all of which one or more of the three Persons are surviving.

To make this yet more evident, I have added *Fig.* 8. wherein

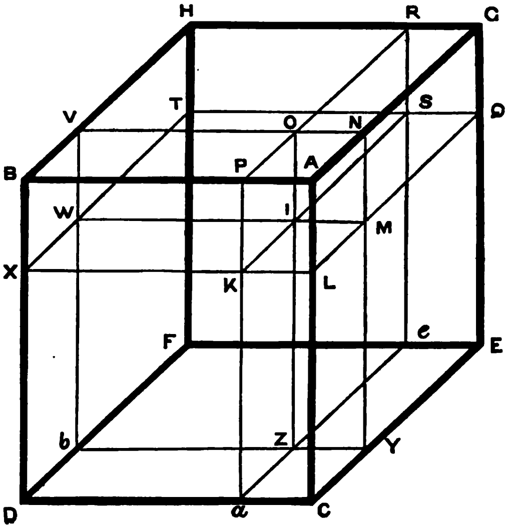

Fig. 8.

these Eight several Products are at one view exhibited. Let the rectangled Parallelepipedon *ABCDEFGH* be constituted of the sides *AB, GH,* &c. proportional to *N* the number of the younger Age; *AC, BD,* &c. proportional to *n;* and *AG, CE,* &c. proportional to the number of the Elder, or *ν*. And the whole Parallelepipedon shall be as the Product *Nnν,* or our whole number of Chances. Let *BP* be as *R,* and *AP* as *Y;* let *CL* be as *r,* and *Ln* as *y;* and *GN* as *ρ,* and *NA* as *v;* and let the Plain *PRea* be made parallel to the plain *ACGE;* the plain *NVbY* parallel to *ABCD;* and the plain *LXTQ* parallel to the plain *ABGH.* And our first Product *Rrρ* shall be as the Solid *STWIFZeb.* The Second, or *rρY* will be as the Solid *EYZeQSMI.* The Third, *Rρy,* as the Solid *RHOVWIST.* And the Fourth, *Rrv,* as the Solid *ZabDWXIK.* Fifthly, *ρYy,* as the Solid *GQRSIMNO.* Sixthly, *rYv,* as *IKLMGYZA.* Seventhly, *Ryv,* as the Solid *IKPOBXVW.* And Lastly, *AIKLMNOP* will be as the Product of the 3 numbers of persons dead, or *Yyv.* I shall not apply this in all the cases thereof for brevity sake; only to shew in one how all the rest may be performed, let it be demanded what is the value of the Reversion of the younger Life after the two elder proposed. The proportion is as the whole number of Chances, or *Nnν* to the Product *Ryv,* so is the certain present value of the Sum payable after any term proposed, to the value due to such Chances as the younger person has to bury both the elder, by the term proposed; which therefore he is to pay for. Here it is to be noted, that the first term of all these Proportions is the same throughout, *viz. Nnν.* The Second changing yearly according to the Decrease of *R, r, ρ,* and Encrease of *Y, y, v.* And the third are successively the present values of Money payable after one, two, three, &c. years, according to the rate of Interest agreed on. These numbers, which are in all cases of Annuities of necessary use, I have put into the following Table, they being the Decimal values of one Pound payable after the number of years in the Margent, at the rate of 6 *per Cent.*

| Years. | Present value of 1*l*. | Years. | Present value of 1*l*. | Years. | Present value of 1*l*. |
|---|---|---|---|---|---|
| 1 | 0,9434 | 19 | 0,3305 | 37 | 0,1158 |
| 2 | 0,8900 | 20 | 0,3118 | 38 | 0,1092 |
| 3 | 0,8396 | 21 | 0,2941 | 39 | 0,1031 |
| 4 | 0,7921 | 22 | 0,2775 | 40 | 0,0972 |
| 5 | 0,7473 | 23 | 0,2618 | 45 | 0,0726 |
| 6 | 0,7050 | 24 | 0,2470 | 50 | 0,0543 |
| 7 | 0,6650 | 25 | 0,2330 | 55 | 0,0406 |
| 8 | 0,6274 | 26 | 0,2198 | 60 | 0,0303 |
| 9 | 0,5919 | 27 | 0,2074 | 65 | 0,0227 |
| 10 | 0,5584 | 28 | 0,1956 | 70 | 0,0169 |
| 11 | 0,5268 | 29 | 0,1845 | 75 | 0,0126 |
| 12 | 0,4970 | 30 | 0,1741 | 80 | 0,0094 |
| 13 | 0,4688 | 31 | 0,1643 | 85 | 0,0071 |
| 14 | 0,4423 | 32 | 0,1550 | 90 | 0,0053 |
| 15 | 0,4173 | 33 | 0,1462 | 95 | 0,0039 |
| 16 | 9,3936 | 34 | 0,1379 | 100 | 0,0029 |
| 17 | 0,3714 | 35 | 0,1301 | | |
| 18 | 0,3503 | 36 | 0,1227 | | |

It were needless to advertise, that the great trouble of working so many Proportions will be very much alleviated by using Logarithms; and that instead of using *Nnv*—*Yyv* for the Second Term of the Proportion in finding the value of Three Lives, it may suffice to use only *Yyv*, and then deducting the Fourth Term so found out of the Third, the Remainder shall be the present value sought; or all these Fourth Terms being added together, and deducted out of the value of the certain Annuity for so many Years, will leave the value of the contingent Annuity upon the Chance of Mortality of all those three Lives. For Example; Let there be Three Lives of 10, 30, and 40 years of Age proposed, and the Proportions will be thus:

As 661 in 531 in 445 or 156190995, or *Nnv*

to 8 in 8 in 9, or 576, or *Yyv* for the first year, so 0,9434. to 0,00000348
to 15 in 16 in 18, or 4320, for the second year, so 0,8900. to 0,00002462
to 21 in 24 in 28, or 14112, for the third year, so 0,8396. to 0,00008128
to 27 in 32 in 38, for the fourth year, so 0,7921. to 0,00016650
to 33 in 41 in 48, for the fifth year, so 0,7473. to 0,00031071
to 39 in 50 in 58, for the sixth year, so 0,7050. to 0,00051051

And so forth to the 60th year, when we suppose the elder Life of Forty certainly to be expired; from whence till Seventy we must compute for the First and Second only, and from thence to Ninety for the single youngest Life. Then the Sum Total of all these Fourth Proportionals being taken out of the value of a certain Annuity for 90 Years, being 16, 58 years Purchase, shall leave the just value to be paid for an Annuity during the whole term of the Lives of three Persons of the Ages proposed. And note, that it will not be necessary to compute for every year singly, but that in most cases every 4th or 5th year may suffice, interpoling for the intermediate years *secundum artem.*

It may be objected, that the different *Salubrity* of places does hinder this Proposal from being *universal;* nor can it be denied. But by the number that die, being 1174. *per Annum* in 34000, it does appear that about a 30th part die yearly, as Sir *William Petty* has computed for *London;* and the number that die in Infancy, is a good Argument that the Air is but indifferently salubrious. So that by what I can learn, there cannot perhaps be one better place proposed for a Standard. At least 'tis desired that in imitation hereof the Curious in other Cities would attempt something of the same nature, than which nothing perhaps can be more useful.

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## *Some further Considerations on the* Breslaw *Bills of Mortality. By the same Hand with the former.*

*Phil. Trans.* vol. xvii. p. 653 (No. 198, for March 1693).

*SIR,*—What I gave you in my former Discourse on these Bills, was chiefly designed for the Computation of the Values of Annuities on Lives, wherein I believe I have performed what the short Period of my Observations would permit, in relation to exactness, but at the same time do earnestly desire, that their Learned Author Dr. *Newman* of *Breslaw* would please to continue them after the same manner for yet some years further, that so the casual Irregularities and apparent Discordance in the Table *p.* 599. may by a certain number of Chances be rectified and ascertain'd.

Were this *Calculus* founded on the Experience of a very great number of Years, it would be very well worth the while to think of Methods for facilitating the Computation of the Value of two, three, or more Lives; which as proposed in my former, seems (as I am inform'd) a Work of too much Difficulty for the ordinary Arithmetician to undertake. I have sought, if it were possible, to find a Theorem that might be more concise than the Rules there laid down, but in vain; for all that can be done to expedite it, is by Tables of Logarithms ready computed, to exhibit the *Rationes* of $N$ to $Y$ in each single Life, for every third, fourth or fifth Year of Age, as occasion shall require; and these Logarithms being added to the Logarithms of the present Value of Money payable after so many Years, will give a Series of Numbers, the Sum of which will shew the Value of the Annuity sought. However for each Number of this Series two Logarithms for a single Life, three for two Lives, and four for three Lives, must necessarily be added together. If you think the matter, under the uncertainties I have mentioned, to deserve it, I shall shortly give you such a Table of Logarithms as I speak of, and an Example or two of the use thereof: but by Vulgar Arithmetick the labour of these Numbers were immense; and nothing will more recommend the useful Invention of Logarithms to all Lovers of Numbers, than the advantage of Dispatch in this and such like Computations.

Besides the uses mentioned in my former, it may perhaps not be an unacceptable thing to infer from the same Tables, how unjustly we repine at the shortness of our Lives, and think ourselves wronged if we attain not Old Age; whereas it appears hereby, that the one half of those that are born are dead in Seventeen years time, 1238 being in that time reduced to 616. So that instead of murmuring at what we call an untimely Death, we ought with Patience and unconcern to submit to that Dissolution which is the necessary Condition of our perishable Materials, and of our nice and frail Structure and Composition: And to account it as a Blessing that we have survived, perhaps by many Years, that Period of Life, whereat the one half of the whole Race of Mankind does not arrive.

A second Observation I make on the said Table, is that the Growth and Encrease of Mankind is not so much stinted by anything in the Nature of the *Species*, as it is from the cautious difficulty most People make to adventure on the state of *Marriage*, from the prospect of the Trouble and Charge of providing for a Family. Nor are the poorer sort of People herein to be blamed, since their difficulty of subsisting is occasion'd by the unequal Distribution of Possessions, all being necessarily fed from the Earth, of which yet so few are Masters. So that besides themselves and Families, they are yet to work for those who own the Ground that feeds them: And of such does by very much the greater part of Mankind consist; otherwise it is plain, that there might well be four times as many Births as we now find. For by computation from the Table, I find that there are nearly 15000

Persons above 16 and under 45, of which at least 7000 are Women capable to bear Children. Of these notwithstanding there are but 1238 born yearly, which is but little more than a sixth part: So that about one in six of these Women do breed yearly; whereas were they all married, it would not appear strange or unlikely, that four of six should bring a Child every year. The Political Consequences hereof I shall not insist on, only the Strength and Glory of a King being in the multitude of his Subjects, I shall only hint, that above all things, Celibacy ought to be discouraged, as, by extraordinary Taxing and Military Service: And those who have numerous Families of Children to be countenanced and encouraged by such Laws as the *Jus trium Liberorum* among the *Romans*. But especially, by an effectual Care to provide for the Subsistence of the Poor, by finding them Employments, whereby they may earn their Bread, without being chargeable to the Publick.

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*The following remarks by Dr. Farr on Halley's table, are extracted from the Appendix to the Registrar-General's Sixth Report, p.* 558.

Halley's Life Table consists of two columns, headed "Age Current" and "Persons". The latter is the column P of the new table, and not, as I at first supposed, the column D. The $P_0$ is however placed against the "Age Current" 1, the $P_1$ against "Age Current" 2, &c.; he thus contrived to make one column do duty as the columns D and P of modern tables: 1000 *living in the first year* of life (1—0) constitute the basis of the construction, which is a population table, and at the same time serves to find the values of annuities on lives at the "current ages" 1, 2, 3, 4, which correspond with the "mean ages" ½, 1½, 2½, 3½, &c.

A table founded on the Breslau observations, having 1000 children born for its basis, would stand thus (*vide* second table): for

| HALLEY'S TABLE. | | Age according to the present Notation. |
|---|---|---|
| Age Curt. | Persons. | |
| 1 | 1000 | ½ |
| 2 | 855 | 1½ |
| 3 | 798 | 2½ |
| 4 | 760 | 3½ |
| 5 | 732 | 4½ |
| 6 | 710 | 5½ |
| 7 | 692 | 6½ |
| * | * | * |
| 84 | 20 | 84½ |

*Constructed from Halley's facts.*

| Age. | D. |
|---|---|
| 0 | 1000 |
| 1 | 719 |
| * | * |
| 6 | 559 |

**Halley stated in his paper, that in the five years 6193 persons were born, and 5869 buried, in Breslau; giving an annual average of 1238·6 births and 1173·8 deaths, or 64·8 fewer deaths than births. In constructing his table he assumed the annual births to be 1238; and as 1238 were living at the beginning, and 890 at the end of the year, took 1000 for the number living through the year (1 — 0), which is 64 less than 1064 = ½(1238 + 890).**

# HOME AND FOREIGN INTELLIGENCE.

## NORWICH UNION LIFE INSURANCE SOCIETY.

*Established* 1808.

### QUINQUENNIAL VALUATION AND DECLARATION OF BONUS, 30 JUNE 1871.

### *Report of the Actuary.*

"The valuation has been made upon the same principles as were adopted at the last three divisions of profit—namely, by the Equitable Experience Table, and at 3½ per-cent interest. As these elements of calculation have been so long in use, I have not felt justified in departing from them on the present occasion. I might have preferred (if commencing *de novo*) to use a rate of mortality based upon the experience of the Society itself, or if the tables now in preparation by the Institute of Actuaries had been completed, upon the mortality there ascertained; but such a course would have greatly delayed the announcement of the bonus, and I have not thought it desirable at the present time to make any alteration in the previous practice. Thus valued, the funds of the Norwich Union Society, exclusive of the Amicable Fund, present a surplus of £142,078. 15*s*. 9*d*., of which, after setting aside £214. 17*s*. 7*d*., the amount of the excess of premium received in the New Series since 1869, which belongs alone to the policies issued therein, four-fifths, or £113,491. 2*s*. 7*d*., is applicable for division amongst the policyholders, and £28,372. 15*s*. 7*d*. remains, according to the provisions of the Deed of Settlement, to accumulate until the next valuation in 1876. This surplus thus ascertained will give reversionary bonuses to the holders of existing policies in the Old Series, of 7½ per-cent upon all the premiums paid since the commencement of the policies, amounting to £2,333,240. 17*s*. 7*d*., and when added to the excess of premium reserved, as already explained, cash bonuses of 21·87, or nearly £22 per-cent upon the premiums paid in the New Series, which will be convertible into much larger reversionary additions or equivalent benefits.

"In this fund it is satisfactory to find the abatement which was anticipated in the excessive mortality which occurred in the five years ending June 1866. In that period the claims paid, including bonuses, amounted to no less than £1,135,487. 3*s*. 7*d*., while during the last five years these payments have fallen to the sum of £962,565. 11*s*. 5*d*. During the latter period I find that new policies have been issued insuring £1,328,800. 6*s.* 6*d.*, at an annual premium of £39,084. 7*s.* 1*d.*, in addition to £6,491. 12*s.* 8*d.* received as gross or single premiums.

"By the Norwich Union and Amicable Societies' Act, 1866, an annual valuation is required to be made of the liabilities and assets of the Amicable Fund in a prescribed form, the rate of mortality used being that of the Amicable experience to 1861, and the rate of interest 3½ per-cent, and the value of the Amicable share in respect of non-assentient insurers, now representing a very small number, is fixed by this calculation. The effect of the Act is to make the Norwich Union Society purchasers of the assenting policies at a fixed rate of £227 per share, or assurance of £200. Upon the accounts of this year the value has risen to £230. 4*s.*, and the difference between this value and the fixed rate of £227 upon the assentient assurances, together with the saving of expenses, now exceeding £2,000 per annum, represents the profit arising at the present time upon the transfer. The terms of the special Act prevent any immediate dealing with this fund, which therefore in no way affects the present bonus; but the position of the fund will be best understood by observing that the balance in its favour has risen from £67,185 in 1867, to £74,203 in 1871, notwithstanding that the amount insured upon the policies, exclusive of additions, has decreased during that period from £1,055,828 to £826,935. This favourable aspect of affairs arises from the low rate of mortality which occurs in a very select class of insurers, and from the careful investment of the assets, by virtue of which the surplus interest upon a fund amounting to nearly half a million, beyond the 3½ per-cent assumed in the valuation, goes far from year to year to provide the addition of £27 upon the £200 share.

"The rate of interest received upon the entire Amicable assets during the last year was £4. 12*s.* 7*d.* per-cent. I am of opinion that this fund places the Society in a much stronger position as regards the valuation than it has occupied at any previous time during the last twenty-five years, and creates an additional reserve equivalent to the assumption of a less favourable rate of mortality and interest in the valuation of the Norwich Union policies.

"The annuities have been valued by the experience of the Government male and female annuitants, and at 4 per-cent interest, rates which may safely be accepted as anticipating the most probable results of the future.

"The valuation has been attended with more than ordinary labour and difficulty; for not only, as at former divisions, have the liabilities of the Society been calculated, but it has been necessary to re-cast the accounts into the form required by the Life Assurance Companies' Act, 1870. I have endeavoured in so doing to comply with the letter and spirit of that enactment; but to accomplish this was no easy task, since the forms given in the schedule to the Act are not those contemplated by the practice of the office, and the funds with which we have to deal comprise two separate accounts calculated upon different principles, and balanced up to different periods of the year. If successfully achieved, the work will be a precedent for future years, and it now presents at the earliest possible opportunity a full and complete explanation of the affairs and position of the office in accordance with the provision of the Act."

* * * * *

The following additional particulars are taken from the report of a verbal explanation given by Mr. Bunyon at the Meeting.

With regard to the bonus, the amount of surplus which we have ascertained upon the gross valuation of accounts, is £142,000. This arises upon the Norwich Union accounts alone; and it will be seen that this is very nearly one year's premium out of the five years' premiums. This is a very important fact; because if you take offices whose scale of premiums is at all similar to ours, one out of five will be found to be a very fair and considerable surplus to make upon any quinquennium. In the apportionment of that surplus, we have had to bring into effect the particular principles upon which this office has been founded; and in the Old Series undoubtedly the old insurers obtain the largest proportion of that surplus, since here the addition is made, in every case, in proportion to all the premiums paid from the commencement of the policy. But when we turn to the New Series, we find that the insurers there receive very large additions—I think as large additions as any person can obtain from any other company. Not that the insurers in the new series get any greater benefit than the insurers in the old series; on the contrary, the new series to a certain extent relinquishes part of its profits in favour of the old, since on the present occasion we do not give a bonus equal to 10 per-cent upon all the premiums paid. The division of profits which falls to the new series is made in such a manner that those persons who have been insured in it obtain additions to their policies in proportion to the premiums they have paid during the five years which precede the declaration of a bonus. Of course in this new series, as it has only been running three years, no person can possibly have had a policy which could have had any claim in respect of any premiums previously paid, because there was no policy existing before the five years; but upon every succeeding declaration of bonus, the same rule will apply, and the same measure of bonus may be expected upon every insurance. Bonuses average from 1½ to 2 per-cent per annum on the sum assured; and if the advantage given to the old series could be divided in the same way, there is no doubt that the profit which is applied in mass to the old series would have given equal advantages to them all. In consequence, however, of the tontine principle of division in the old series, the new insurer there gets a smaller sum than the old insurer. The old insurers will receive a larger sum at the present time; but as they fall off, the new insurers will take their places. The result is that the advantages are equally and justly given, and the reason why the advantage in the new series appears larger than in the old, arises not from the new series being in any way favoured or getting any preference over the old, but entirely in the mode in which it is allotted. In one respect, indeed, the new series is entitled to more than the old, because they pay higher premiums. Their premiums are about 8 per-cent higher than the premiums paid by the old series; and that 8 per-cent is reserved entirely for their benefit. But with that exception, the advantage upon the division is certainly given as equally as possible to both series.

* * * * *

We make our valuation upon what is called a gross premium valuation; but on the other hand, we make our reserve by retaining such a proportion of the premiums as would be an equivalent to our making the reserve by a net premium valuation. When the net premium valuation is made, a deduction is made from the premiums, in the first instance, of an amount which may have been assumed to have been added to the premiums in order to provide for the expenses, fluctuations, and profits. When a gross valuation is made, the whole of the premiums are valued; but in an office like this, a deduction is then made, which is equivalent to the fund which it is necessary to reserve for those expenses, profits, and fluctuations. In the Norwich Union Office our tables have not been uniform during our existence. We began with exceedingly low rates of premium, and they have been adjusted from time to time, so that there is not one scale in existence over the whole. That would be a reason against using what is called the net valuation, because if you have two policies in the same position, there is no reason why you should not make the same reserve upon both. The consequence has been that it has been thought proper, during the last twenty-five years, always to make a reserve by the percentage premiums upon the gross amount of those premiums, taking such a proportion as is most nearly equivalent to the reserve which ought to be retained by a net premium valuation upon the average. That reserve is 18 per-cent of the premiums, and when to that reserve —I speak especially of profit policies—we add the one-fifth resulting surplus, which in the present case we are obliged to add by the Deed of Settlement, we then get a reserve of nearly 20 per-cent on the premiums on the old series; 14 per-cent upon non-profit premiums; and 28 per-cent upon premiums in the new series. A further reserve is found in the interest realized beyond 3½ per-cent on the insurance, and 4 per-cent on the annuity funds. If we take 20 per-cent upon the gross amount of premiums we receive, that is £30,000 a year. If, again, we ascertain what interest and dividends we are receiving over and above the hypothetical 3½ per-cent which is used in the valuation, we have, I find, on turning to the figures of last year, a surplus in that year of £21,303. The result is that the reserves of this office, quite apart from any question of mortality, and quite apart from any question of profit which may arise to us from other causes out of the Amicable Fund, will now go on for the future as something over £50,000 a year to meet expenses, profits, and fluctuations.

*The following are taken from the Returns under the Fifth and Sixth Schedules of the Life Assurance Companies Act,* 1870.

The valuation is made up to the 30th of June 1871, for the Norwich Union Fund, and to the 5th of April 1871, for the Amicable Fund; in accordance with the Deed of Settlement, Royal Charters, and Acts of Parliament respectively affecting the same.

The distribution of profits is determined by the Deed of Settlement of the Norwich Union Life Insurance Society, and by the Society's Act, 1868, and by the Amicable and Norwich Union Societies' Act, 1866.

In the Norwich Union all profit policies, except those effected in the current year, are entitled to participate. In the Old Series the policies participate by receiving a reversionary bonus at each time of division, proportioned in amount to the premiums paid from the date of the policy. In the New Series the surplus is divided in proportion to the premiums paid since the last preceding bonus, and then converted in each case into a reversionary addition. For this latter series the bonus may be applied to relieve the policy from future payments of premium.

The tables used in the valuation for assurances are the Equitable Experience Tables (Davies), 3½ per-cent, for assurances contingent upon a single life, and the Carlisle Tables, 3½ per-cent, for assurances involving more than one life. For annuities the Government Male and Female Annuity Tables, 1860, with 4 per-cent interest. The Amicable risks are valued by the Amicable Experience Table, 1861, interest 3½ per-cent.

On the Norwich Union Account the reserve is, in the first instance, 18 per-cent of the present value of the future premiums on profit policies, and 12 per-cent upon non-profit policies, and one-fifth of the resulting surplus at each division; and the excess of the rates in the new above that of the old series, which is not valued. The resulting reserve will then be nearly 20 per-cent of the premiums of the old series, 14 per-cent upon non-profit premiums, and 28 per-cent upon the premiums in the new series. A further reserve is found in the interest realized beyond 3½ per-cent on the Assurance, and 4 per-cent on the Annuity Funds; also in an anticipated surplus upon the Amicable policies, now, with few exceptions, non-participating policies under the Act of 1866. In the Amicable Fund, the reserve is found in the rates of mortality and interest, and in the mode of calculating the Charter share prescribed by the Amicable Charters and the special Acts affecting the same.

The total profit made by the Society upon the Norwich Union Account in 5 years ending the 30th June 1871, was £142,078. 15*s.* 9*d.* Of this sum, after deducting £214. 17*s.* 7*d.* excess of premiums paid in the new series since the 1st of January 1869, £113,491. 2*s.* 7*d.* is distributed in manner aforesaid, and £28,372. 15*s.* 7*d.* is allowed to fall back into the coffers of the office to fructify until the next division. 9,088 policies, assuring £4,800,530. 6*s.* 4*d.*, participated.

### SPECIMENS OF BONUS. (OLD SERIES.)

*Exclusive of previous additions.*

| Age. | Annual Premium. | 5 Years. | 10 Years. | 15 Years. | 20 Years. | 25 Years. | 30 Years. | 40 Years. | 50 Years. | 60 Years. |
|---|---|---|---|---|---|---|---|---|---|---|
| 20 | 1·975 | ·7406 | 1·4812 | 2·2218 | 2·9624 | 3·7030 | 4·4436 | 5·9248 | 7·406 | 8·8872 |
| 30 | 2·440 | ·915 | 1·83 | 2·745 | 3·660 | 4·575 | 5·49 | 7·3200 | 9·15 | 10·98 |
| 40 | 3·100 | 1·1625 | 2·325 | 3·4875 | 4·650 | 5·8125 | 6·975 | 9·3000 | 11·625 | |
| 50 | 4·300 | 1·6125 | 3·225 | 4·8375 | 6·4500 | 8·0625 | 9·675 | 12·900 | | |

### SPECIMENS OF BONUS. (NEW SERIES.)

| Age. | Annual Premium. | 3 Years. |
|---|---|---|
| 20 | 2·1238 | 4·439 |
| 30 | 2·5875 | 4·485 |
| 40 | 3·325 | 4·816 |
| 50 | 4·62 | 5·557 |

### *Consolidated Revenue Account of the Norwich Union Life Insurance Society.*

| | £ | s. | d. |
|---|---|---|---|
| Amount of Funds on 1st July 1866 | 1,727,413 | 7 | 5 |
| Premiums after deducting reassurance premiums | 768,365 | 8 | 0 |
| Consideration for annuities granted | 52,809 | 5 | 11 |
| Interest and dividends | 393,503 | 8 | 6 |
| Rents | 8,212 | 2 | 10 |
| Fees for registration of assignments | 210 | 4 | 7 |
| Profits on securities realized | 36,843 | 14 | 7 |
| | £2,987,357 | 11 | 10 |

| | £ | s. | d. |
|---|---|---|---|
| Claims under policies after deducting sums reassured | 807,279 | 0 | 9 |
| Bonuses thereon | 155,286 | 10 | 8 |
| Surrenders | 25,234 | 5 | 1 |
| Annuities | 156,248 | 14 | 11 |
| Commission | 36,889 | 11 | 5 |
| Medical fees | 1,981 | 19 | 0 |
| Expenses of management | 53,851 | 7 | 1 |
| Income tax | 3,574 | 6 | 9 |
| *Amount of Funds at 30th June 1871 | 1,747,011 | 16 | 2 |
| | £2,987,357 | 11 | 10 |

* NOTE.—In addition to this item, there was a sum of £14,258 0*s.* 2*d.* for current interest due, increasing the Total Funds to £1,761,269 16*s.* 4*d.*

### *Summary and Valuation of the Policies* (see p. 271).

### *Valuation Balance Sheet of the Norwich Union Fund of the Norwich Union Life Insurance Society, as at the* 30*th June* 1871.

| DR. | £ |
|---|---|
| To net liability under assurance and annuity transactions, as per Summary and Valuation | 1,619,191 |
| To surplus | 142,079 |
| | £1,761,270 |

| CR. | £ |
|---|---|
| By total funds, as per Balance Sheet, to 1st January 1871 | 1,737,175 |
| By additions thereto, after deducting claims unpaid, and annuities due to this date | 24,095 |
| | £1,761,270 |

### *Summary and Valuation of the Amicable Policies* (see p. 272).

### *Valuation Balance Sheet of the Amicable Fund of the Norwich Union Life Insurance Society as at the 5th April* 1871.

| DR. | £ |
|---|---|
| To *net Liability* under Assurance transactions | 438,476 |
| To Surplus under the Society's Act, 1866, not divisible | 22,561 |
| | 461,037 |

| CR. | £ |
|---|---|
| By *Life Assurance Fund* as per Balance Sheet | 461,037 |
| | 461,037 |

| | Number of Policies. | Sums Assured and Bonuses. | Office Yearly Premiums. | Value by the Equitable Experience Table. Interest 3½ per-cent. | | |
|---|---|---|---|---|---|---|
| | | | | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Liability. |
| ASSURANCES. | | | | | | |
| *With Participation in Profits.* | | £ | £ | £ | £ | £ |
| For whole term of life | 8,853 | 4,678,031 | 129,843 | 2,652,739 | 1,568,329 | 1,084,410 |
| Joint life | 96 | 37,560 | 1,487 | 22,195 | 17,496 | 4,699 |
| Longest life | 74 | 54,988 | 783 | 23,073 | 12,921 | 10,152 |
| With premiums payable for limited terms | 44 | 19,029 | 591 | 7,814 | 5,686 | 2,127 |
| Payable at a given age | 13 | 4,881 | 202 | 3,795 | 1,098 | 2,697 |
| Survivorship assurances | 8 | 6,041 | 57 | 1,148 | 432 | 716 |
| Extra premiums payable | .. | .. | 2,470 | .. | | |
| Total assurances *with profits* | 9,088 | 4,800,530 | 135,433 | 2,710.764 | 1,605,962 | 1,104,801 |
| *Without Participation in Profits.* | | | | | | |
| For whole term of life | 113 | 194,271 | 5,994 | 94,994 | 90,658 | 4,336 |
| Joint life | 2 | 5,500 | 245 | 3,178 | 3,124 | 54 |
| Longest life | 4 | 7,800 | 156 | 2,997 | 2,747 | 250 |
| With premiums payable for limited terms | 13 | 6,750 | 369 | 2,446 | 832 | 1,613 |
| Payable at a given age | 160 | 37,900 | 1,538 | 22,536 | 15,939 | 6,597 |
| Survivorships | 46 | 56,196 | 636 | 5,052 | 3,980 | 1,072 |
| Issue risks | 10 | 42,382 | .. | 1,522 | .. | 1,522 |
| Short periods | 74 | 42,077 | 776 | 3,116 | 3,100 | 17 |
| Extra premiums payable | .. | .. | 155 | | | |
| Total assurances *Without Profits* | 422 | 392,876 | 9,869 | 135,841 | 120,380 | 15,461 |
| TOTAL ASSURANCES *With and Without Profits* | 9,510 | 5,193,406 | 145,302 | 2,846,605 | 1,726,342 | 1,120,262 |
| *Deduct* reassurances | .. | 46,819 | 1,646 | 22,040 | 19,518 | 2,521 |
| Extra premium ditto | .. | .. | 15 | | | |
| *Net amount* of assurances | 9,510 | 5,146,586 | 143,640 | 2,824,565 | 1,706,824 | 1,117,741 |
| ANNUITIES. | | | | | | |
| Immediate Single lives—Males | 258 | 12,034 | .. | 73,707 | | |
| Females | 519 | 16,314 | .. | 117,944 | | |
| Joint Lives | 2 | 390 | .. | 3,030 | | |
| Longest life | 21 | 821 | .. | 8,169 | | |
| Total of immediate annuities | 800 | 29,559 | .. | 202,850 | | |
| Of which are *reassured* | .. | 1,048 | .. | 5,161 | | |
| | | 28,511 | .. | 197,689 | .. | 197,689 |
| Survivorship annuities | 18 | 1,643 | 393 | 5,749 | 2,399 | 3,350 |
| Total resulting liability | | | | | £ | 1,318,780 |
| Reserve on the above | | | | | | 300,410 |
| NET LIABILITY | | | | | £ | 1,619,191 |

NOTE.—The Net Yearly Premiums are not ascertained.

*Summary and Valuation of the Amicable Policies of the Norwich Union Life Insurance Society, as at the 5th April* 1871.

| | Number of Policies. | Sums Assured and Bonuses. | Office Yearly Premiums. | Valuations by the Amicable Mortality Tables. Interest 3½ per-cent. | | |
|---|---|---|---|---|---|---|
| | | | | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Liability. |
| ASSURANCES. | | | | | | |
| WHOLE TERM OF LIFE. | | £ | £ | £ | £ | £ |
| I. Charter and bonus policies . . . . . . . | 820 | 845,293 | 21,622 | 594,423 | 172,793 | 421,630 |
| Joint lives, payable on decease of survivor of 2 lives | 2 | 2,043 | 21 | 1,398 | 144 | 1,253 |
| Survivorship . . . . . . . . . | 1 | 1,135 | 20 | 406 | 86 | 321 |
| Extra premiums . . . . . . . . . | ... | ... | 142 | | | |
| Total assurances (issued) *with* profits . . . | 823 | 848,471 | 21,805 | 596,227 | 173,023 | 423,204 |
| II. (issued) WITHOUT PARTICIPATION, whole term of life | 78 | 68,535 | 2,414 | 39,900 | 24,711 | 15,189 |
| Joint lives, payable on death of survivor of 2 lives . | 1 | 300 | 4 | 151 | 68 | 83 |
| Extra premiums . . . . . . . . . | ... | ... | 56 | | | |
| | 79 | 68,835 | 2,474 | 40,051 | 24,779 | 15,272 |
| TOTAL ASSURANCES . . . . . . | 902 | 917,306 | 24,279 | 636,278 | 197,802 | 438,476 |

Reassurances, none. Annuities, none.—The net yearly premiums are not ascertained.

This calculation assumes the value of the Charter Share of £200 at £227, in accordance with the Amicable and Norwich Union Societies Act, 1866.

The average rate of interest at which the Life Assurance Fund of the Company was invested at the close of each year, during the period since the last investigation, is shown in the following table:—

AVERAGE RATE OF INTEREST ON INVESTMENTS.

| Year. | Norwich Union Fund. | Amicable Fund. |
|---|---|---|
| 1867 | 4·316 per-cent | 4·148 per-cent |
| 1868 | 4·437 „ | 4·394 „ |
| 1869 | 4·345 „ | 4·06 „ |
| 1870 | 4·345 „ | 4·44 „ |
| 1871 | 4·483 „ | 4·631 „ |

*Policies for surrender are valued by a Table constructed for the Society, the minimum surrender values being* 25 *per-cent of the premiums paid in the Old, and* 33 *per-cent of the premiums paid in the New Series.*

EXAMPLES OF VALUES. (OLD SERIES.)

| Age. | 10 Years. | 20 Years. | 30 Years. | 40 Years. |
|---|---|---|---|---|
| 20 | 6·67 | 13·92 | 22·52 | 33·6 |
| 30 | 8·44 | 17·94 | 30·14 | 41·26 |
| 40 | 11·45 | 25·20 | 37·88 | 47·63 |
| 50 | 17·61 | 32·54 | 44·08 | 51·48 |

*In addition to which,* 75 *per-cent is allowed of the Office Reserve for Bonuses.*

*The extra premiums comprises the excess of premiums charged upon the New Series Policies above those charged in the Old Series, an addition of* 10 *per centum on the premiums for insurances on single lives for* £51,157. 15*s.* 7*d. effected at Oporto, Climate Risks, and additions made in cases where the life insured has been considered under the average. These extra premiums, as also any sums charged for Voyage or Climate Risks, are not valued, but are considered to form a fund applicable for the purposes for which they were designed, as they arise. As a rule, unhealthy lives are not insured in this Office. In valuing the Amicable Policies, where any extra premium has been charged, the life has been treated as existing at an advanced age.*

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## CLERICAL, MEDICAL, AND GENERAL LIFE ASSURANCE SOCIETY.

*Established* 1824.

### NINTH DIVISION OF PROFITS.

### BONUS REPORT, 1872.

In conformity with the provisions of the Society's Deed of Constitution and Special Act of Parliament, the Directors have called the present Meeting of Proprietors and Assured, in order to report the transactions of the quinquennial period which terminated on the 30th June 1871, and the financial condition of the Society at that date.

Concurrently with this report, the returns to the Board of Trade required by the "Life Assurance Companies Act, 1870", will be issued to the members of the Society. These returns, though valuable for presenting to critical observation the whole system and condition of the Society, are, however, too voluminous and too technical in many of their details to be readily understood by those not conversant with the matters of which they treat. They therefore neither relieve the Directors of the duty, nor deprive them of the pleasure, of explaining the principles and illustrating the results, both immediate and prospective, of the investigation to which they chiefly relate.

Taking first a retrospective survey of the 5 years, the following facts present themselves:

**1° AS TO INCOME:**

The NEW ASSURANCES were **2,150** in number, for an aggregate sum of **£1,356,303**, at premiums amounting to **£44,664** per annum.

The YEARLY REVENUE was increased by over **£21,000** per annum, and reached **£236,563** on the 30th June 1871.

The INTEREST yielded by the whole of the Funds, whether invested or uninvested, was **£4 : 5 : 0** per-cent on the average of the entire period, being fully 3*s*. per-cent more than that realized in the previous period. This increase was obtained not only without loss, but without the smallest impairment of security.

**2° AS TO OUTGOINGS:**

The CLAIMS which accrued by the death of 795 persons, assured by 977 Policies, amounted to **£646,481**. The corresponding payments in the previous 5 years were £643,885; and in the 5 years, 1856–61, £464,280. The mortality in the quinquennium just expired was very favourable to the Society, the payments having been below those estimated by fully £85,000, and the deaths which occasioned them fewer by 92 than the number expected.

The EXPENSES incurred in conducting the business were fractionally less than in the previous period, and fell below 7½ per-cent on the revenue.

The subsisting assurances on the 30th June were **8,679** in number, assuring, with their bonus additions, the sum of **£5,445,028**. The Assurance Fund, accumulated to meet these liabilities, was at the same date **£1,826,459**; being larger than in 1866 by **£264,700**, notwithstanding the payment in 1867 of £86,868 on account of bonus.

The assurances above enumerated, and annuities for £1,059 per annum, constituting together the whole engagements of the Society, were the subject of the present investigation.

In their valuation, the Carlisle rate of mortality, with 3 per-cent interest, was employed throughout, and net premiums only having been taken into account, the whole of what is technically called "loading", exceeding £30,000 per annum, remains as a provision for future expenses, contingencies, and profit.

According to the custom of the office, each policy, as well as any bonus attached to it, was valued separately. On this occasion, however, advantage was taken of a classification of the policies according to the ages of the lives assured as they stood in the office books on the 30th June, made to meet the requirements of the Life Assurance Companies Act, to value the liabilities in the mass, and so to test the results derived from the ordinary method above described. It is satisfactory to know that the two valuations produced results so nearly identical as to establish beyond all doubt the accuracy of both.

The Liabilities thus deduced, and also the Assets of the Society, are set out in detail in the annexed statements. From these it will be seen that, after providing for all known claims and expenses,

| | | | |
|---|---|---|---|
| The Assurance Fund at the date of Valuation was . . . . | £1,826,458 | 10 | 9 |
| And the total Liability calculated on the above basis . . . . | 1,477,179 | 17 | 3 |
| Leaving a Surplus of . . . . | **£349,278** | **13** | **6** |

Deducting therefrom the permanent Reserve Fund of £50,000, pursuant to sec. 32 of the Society's Special Act of Parliament, there remains to represent the profit of the 5 years the sum of **£299,278 : 13 : 6**, an amount equal to **26** per-cent of the total revenue from all sources during the 5 years, and exceeding by **£59,831 : 6 : 5** the surplus of any previous quinquennium.

This surplus is matter for hearty and unmixed congratulation, and justifies the preference shown by the Board for a well-selected business tending to profit. It must, however, be remembered that although owing in the main to ordinary recurring causes, and to sources of profit having every prospect of permanence, it is, nevertheless, certain that its unprecedented enlargement is due to a condition of mortality favourable beyond previous experience, to be probably compensated under the law of averages by an increase of deaths hereafter beyond those allowed for in the calculations.

Deeming it prudent to provide for such a contingency, the Directors have, under the advice of their actuary, set aside the sum of £25,000 for this purpose. Of the remaining £274,278 : 13 : 6, they now recommend the division of **£270,000**, a sum greater by £33,000 than any previously divided, and sufficient to give to the shareholders £9 a share, and to the assured the largest bonus ever allotted to them. After this distribution, the sum in reserve will equal 28·42 per-cent of the gross amount at risk, against 26·20 per-cent in 1866, and 25·36 per-cent in 1861.

The Directors feel that in thus following in that course of caution and prudence which has made the office what it is, they will have the full concurrence of all the members of the Society. For the continued use, indeed, of those rigorous principles of valuation which have so long formed the foundation of the Society's prosperity,—for keeping intact the loading provided by the assured for expenses and contingencies which lie wholly in the future, and for adhering in the calculations to the low rate of 3 per-cent interest on which the assured have been content that their premiums should be based,—the Directors claim no higher merit than that of keeping faith with the policyholders by giving them the security for which they have paid. But in respect of the special reserves now made, they are, perhaps, entitled to indulge in a feeling of gratification that whilst, as they hope, satisfying every reasonable expectation of present bonus, they are doing all in their power to maintain unimpaired the profits of the future, to the benefit alike of present and future assured.

Of the sum now to be divided, five-sixths, or **£225,000**, will fall to the assured, and will produce a reversionary addition to the policies of **£323,871**. In this sum all with-profit policies in force on the 30th June last will share in proportion to their respective contributions to the funds of the Society since the last division.

This reversionary bonus will average **49** per-cent, or vary according to age from **34** to **89** per-cent on the premiums received in the quinquennium on all the policies amongst which it will be distributed. The average on the last occasion was 45 per-cent, and the range from 32 to 85 per-cent.

The cash bonus, which is the present value of the reversionary bonus and therefore the true measure of the allotment, will average **29** per-cent on the like payments, as against 26 per-cent at the last division, and 28 per-cent in 1862, which was the highest previous percentage.

Options for reducing the premiums temporarily or permanently will also be given as before. In reference to the latter, it may be useful to continue a comparison heretofore made as to the number of cases in which the premiums may be wholly extinguished by total or partial application of bonus, the policies retaining, nevertheless, all their present right of sharing in future profits. The number on this occasion is **538**: in 1867 it was 350, and in 1862, 176.

With this illustration, the statement of the Directors is complete. In conclusion, they venture to express the hope that, in thus again exhibiting the abundant resources and the undiminished fruitfulness of the Society, they will have done something to re-establish amongst the public at large the belief, once rightly held, though of late somewhat discredited, that of all monetary institutions none are more secure or more uniformly profitable than Assurance Societies conducted with integrity and prudence.

*Summary and Valuation of the Policies* (see p. 278).

### *BALANCE SHEET on the* 30*th June* 1871.

| LIABILITIES. | | | | £ | s. | d. |
|---|---|---|---|---|---|---|
| Shareholders' Capital paid up ... | £50,000 | 0 | 0 | | | |
| Assurance Fund... ... ... | 1,826,458 | 10 | 9 | | | |
| Total Funds ... ... ... ... ... ... | | | | 1,876,458 | 10 | 9 |
| Claims admitted or announced, but not paid... ... | | | | 30,745 | 18 | 0 |
| Dividends to Shareholders due ... ... ... | | | | 2,515 | 0 | 0 |
| Due for Rates, Income Tax, Commission and Sundry Expenses ... ... ... ... ... ... | | | | 3,198 | 9 | 2 |
| | | | | **£1,912,917** | **17** | **11** |

| ASSETS. | | | | £ | s. | d. |
|---|---|---|---|---|---|---|
| Mortgages on Freehold Landed Property within the United Kingdom | | | | 486,829 | 10 | 1 |
| Loans on the Society's Policies (within their surrender value) | | | | 68,847 | 0 | 0 |
| Loans of Half-Premiums on Credit Policies | | | | 22,269 | 0 | 3 |
| Investments— | | | | | | |
| In British Government Securities | | | | 150,021 | 12 | 6 |
| „ Colonial Government Securities | | | | 73,148 | 16 | 11 |
| „ Foreign Government Securities | | | | 32,286 | 14 | 10 |
| „ English Railway Debentures and Debenture Stocks | | | | 640,000 | 0 | 0 |
| „ Indian Government Guaranteed Railway Stocks | | | | 90,871 | 11 | 1 |
| „ House Property (the Society's Freehold Office) | | | | 12,750 | 0 | 0 |
| Other Investments— | | | | | | |
| Loans on Life Interests in Land and in Government Securities | | | | 146,500 | 0 | 0 |
| Loans on Security of County, District, and Borough Rates | | | | 86,880 | 10 | 8 |
| Value of Bonuses on Policies belonging to the Society at other Offices | | | | 14,387 | 14 | 2 |
| Agents' Quarterly Balances | | | | 17,968 | 17 | 1 |
| Outstanding Premiums | | | | 7,636 | 19 | 7 |
| Outstanding Interest due and payable | £1,868 | 8 | 10 | | | |
| Do. accrued, but not yet payable | 26,045 | 3 | 8 | | | |
| | | | | 27,913 | 12 | 6 |
| Cash— | | | | | | |
| On Deposit | £25,000 | 0 | 0 | | | |
| In hand and on Current Account | 9,605 | 18 | 3 | | | |
| | | | | 34,605 | 18 | 3 |
| | | | | £1,912,917 | 17 | 11 |

*The following further particulars are extracted from the returns under the Fifth and Sixth Schedules of the Life Assurance Companies Act:—*

The principles upon which the Valuation was made were determined by resolution of the Directors under authority of Clause 90 of the Society's Deed of Settlement, which directs "that each account and calculation to be respectively taken and made . . . . for ascertaining the amount of 'the Consolidated Fund', and the sum which may with safety be taken out of such fund, shall be taken and made in such manner as the Board of Directors shall think fit."

The principles upon which the distribution of profits among the policyholders was made, are determined by clause 88 of the Deed of Settlement, and Section XXXI of the "Clerical, Medical, and General Life Assurance Company's Act, 1850", which together direct that five equal sixth parts of the sum which at the close of each period of five years may with safety be taken out of the Consolidated Fund, shall be declared as a bonus on all participating policies for the whole period of life which shall be then subsisting, rateably and in proportion to the sums to which the premiums paid in respect of such life policies within such period, and the accumulations of interest thereon at the rate of 3 per-cent per annum, shall severally amount.

The whole of the "loading" on the net (Carlisle) premiums assumed in the valuation (averaging 24·94 per-cent for with-profit, and 14·43 per-cent for without-profit policies) has been reserved as a provision for future expenses and profits.

## *SUMMARY and VALUATION of the Policies as at 30th June* 1871.

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | | VALUATION. | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | VALUE BY CARLISLE TABLE—INTEREST 3 PER-CENT. | | | |
| | Number of Policies. | Sums Assured and Bonuses. | Office Yearly Premiums.* | Net Yearly Premiums. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Yearly Premiums. | Net Liability. |
| **ASSURANCES.** | | | | | | | | |
| I.—***With Participation in Profits.*** | | £ | £ | £ | | | | £ |
| For Whole Term of Life ... ... ... | 7930 | 4,876,598 | 139,117 | 111,375 | The assurances were all valued singly: the whole-term policies by a table of policy values based on net premiums, and the remainder by methods which, in like manner, gave the net or resulting liability without exhibiting the values of the sums assured and the premiums separately. These values, though computed as a test, and found to be almost identical in result with those above referred to, were consequently not made the measure of the liability, and are therefore not here given. | | | 1,372,005 |
| Last Survivor ... ... ... ... ... | 62 | 62,390 | 981 | 766 | | | | 11,695 |
| Joint Lives ... ... ... ... ... | 38 | 21,519 | 913 | 719 | | | | 7,991 |
| Limited Premiums ... ... ... ... | 7 | 12,715 | 348 | 278 | | | | 4,820 |
| Ascending Scale ... ... ... ... | 1 | 5,000 | 119 | 95 | | | | 103 |
| Extra Premiums payable† ... ... ... | ... | ... | 708 | ... | | | | ... |
| Total Assurances with Profits ... ... | 8038 | 4,978,222 | 142,186 | 113,233 | | | | 1,396,614 |
| II.—***Without Participation in Profits.*** | | | | | | | | |
| For Whole Term of Life ... ... ... | 442 | 320,218 | 12,099 | 10,681 | | | | 64,546 |
| Last Survivor ... ... ... ... ... | 7 | 15,200 | 439 | 375 | | | | 2,340 |
| Joint Lives ... ... ... ... ... | 5 | 9,275 | 521 | 441 | | | | 572 |
| Survivorship ... ... ... ... ... | 36 | 29,288 | 484 | 381 | | | | 1,088 |
| Ascending Scale ... ... ... ... | 4 | 11,000 | 329 | 292 | | | | 306 |
| Endowment Assurances ... ... ... | 74 | 20,100 | 839 | 760 | | | | 2,281 |
| Short Term ... ... ... ... ... | 71 | 52,725 | 883 | 697 | | | | 441 |
| Issue ... ... ... ... ... ... | 2 | 9,000 | ... | ... | | | | 90 |
| Extra Premiums payable† ... ... ... | ... | ... | 217 | ... | | | | ... |
| Total Assurances Without Profits ... | 641 | 466,806 | 15,811 | 13,627 | | | | 71,664 |
| Total Assurances... ... ... ... | 8679 | 5,445,028 | 157,997 | 126,860 | | | | 1,468,278 |
| **ANNUITIES.** | | Per Annum | | | | | | |
| Immediate ... ... ... ... ... | 2 | 1,059 | ... | ... | | | | 8,902 |
| **Total of the results** ... ... ... ... ... ... | | | **157,997** | **126,860** | | | | **1,477,180** |

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* From these premiums are excluded the permanent abatements made by application of bonuses (£2,564 : 15 : 6), and the premiums commuted (£1,387 : 0 : 11), together £3,951 : 16 : 5 per annum.

### *Consolidated Revenue Account since the last valuation.*

| | £ | s. | d. |
|---|---|---|---|
| Amount of Funds on the 30th June 1866 ... ... | 1,611,758 | 10 | 10* |
| Premiums (no reassurances) ... ... ... ... | 772,454 | 11 | 0 |
| Interest and dividends ... ... ... ... ... | 370,407 | 17 | 2 |
| Fines on renewal of lapsed policies ... ... ... | 213 | 6 | 0 |
| Profit on security realized ... ... ... ... | 257 | 2 | 10 |
| | £2,755,091 | 7 | 10 |

| | £ | s. | d. |
|---|---|---|---|
| Claims under policies (no reassurances) ... ... | 646,481 | 6 | 0 |
| Surrenders ... ... ... ... ... ... ... | 34,398 | 3 | 3 |
| Annuities ... ... ... ... ... ... ... | 5,483 | 19 | 2 |
| Commission ... ... ... ... ... ... ... | 29,754 | 3 | 2 |
| Expenses of management ... ... ... ... | 55,460 | 8 | 5 |
| Dividends to shareholders ... ... ... ... | 12,500 | 0 | 0 |
| Bonus to shareholders at quinquennial division in 1867 | 37,500 | 0 | 0 |
| Ditto to assured in commutation of reversionary additions ... ... ... ... ... ... | 49,367 | 19 | 0 |
| Income tax ... ... ... ... ... ... | 7,686 | 18 | 1 |
| Amount of Funds on the 30th June 1871 ... ... | 1,876,458 | 10 | 9 |
| | £2,755,091 | 7 | 10 |

* This amount is shown to agree with the statement of "Assets on 30th June 1866", given in the Bonus Report of 1867, as follows:

| | | | | £ | s. | d. |
|---|---|---|---|---|---|---|
| The "Total Assets" were there shown to be ... ... | | | | 1,669,539 | 14 | 8 |
| From which must be deducted the following sums there stated to be owing by the Society on the 30th June 1866, viz.: | | | | | | |
| Claims admitted or announced, but not paid ... ... ... ... | £53,199 | 18 | 0 | | | |
| Dividends to shareholders due ... | 2,562 | 10 | 0 | | | |
| Due for rates, income tax, commission, and sundry expenses ... | 2,018 | 15 | 10 | 57,781 | 3 | 10 |
| Leaving as the "Amount of Funds on the 30th June 1866" | | | | £1,611,758 | 10 | 10 |

### *Valuation Balance Sheet as at 30th June 1871.*

| Dr. | £ | s. | d. |
|---|---|---|---|
| To Net Liability under Assurance and Annuity Transactions (as per Summary Statement above) ... | 1,477,179 | 17 | 3 |
| To Surplus* ... ... ... ... ... ... ... | 349,278 | 13 | 6 |
| | £1,826,458 | 10 | 9 |

| Cr. | £ | s. | d. |
|---|---|---|---|
| By Life Assurance Fund (as per Balance Sheet) ... | 1,826,458 | 10 | 9 |
| | £1,826,458 | 10 | 9 |

| | |
|---|---|
| * Surplus brought down ... ... ... ... | £349,278 : 13 : 6 |
| Deduct Reserve Fund, pursuant to Sec. 32 of the Society's Special Act of Parliament ... ... | 50,000 : 0 : 0 |
| Leaving as the Profit of the Five Years ... | £299,278 : 13 : 6 |

All with-profit policies, however recently effected, existing at the date up to which the valuation was made, were entitled to share in the profits.

The total amount of profit made by the Society in the five years ending 30th June 1871 (including £830:15:6 unappropriated at the last division), was £299,278:13:6, of which £270,000 was divided.

The amount of profit divided among the policyholders was £225,000. The number of policies which participated was 8038, assuring, exclusive of previous bonus additions, the sum of **£4,617,448:18:0.**

Specimens of bonuses allotted to policies for £100, effected at the under-mentioned ages, according to the various modes in which the bonuses might be received.

| Office Age at Entry. | POLICY IN FORCE. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 5 YEARS. | | | | 10 YEARS. | | | |
| | Reversion. | Cash. | Life Reduction. | 5 Year Reduction. | Reversion. | Cash. | Life Reduction. | 5 Year Reduction. |
| | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. |
| 20 | 7 2 0 | 2 12 0 | 0 2 6 | 0 11 8 | 6 11 0 | 2 13 0 | 0 2 8 | 0 11 9 |
| 30 | 7 18 0 | 3 8 0 | 0 3 9 | 0 15 5 | 7 6 0 | 3 9 0 | 0 4 0 | 0 15 8 |
| 40 | 8 19 0 | 4 11 0 | 0 5 9 | 1 0 9 | 8 3 0 | 4 10 0 | 0 6 4 | 1 0 9 |
| 50 | 10 8 0 | 6 7 0 | 0 10 3 | 1 9 5 | 9 15 0 | 6 9 0 | 0 12 4 | 1 11 6 |

| Office Age at Entry. | POLICY IN FORCE. | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | 15 YEARS. | | | | 20 YEARS. | | | |
| | Reversion. | Cash. | Life Reduction. | 5 Year Reduction. | Reversion. | Cash. | Life Reduction. | 5 Year Reduction. |
| | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. |
| 20 | 6 1 0 | 2 12 0 | 0 2 10 | 0 11 9 | 5 12 0 | 2 13 0 | 0 3 1 | 0 12 0 |
| 30 | 6 14 0 | 3 8 0 | 0 4 4 | 0 15 8 | 6 2 0 | 3 8 0 | 0 4 9 | 0 15 8 |
| 40 | 7 9 0 | 4 11 0 | 0 7 4 | 1 1 1 | 6 19 0 | 4 13 0 | 0 8 10 | 1 2 6 |
| 50 | 9 1 0 | 6 9 0 | 0 14 5 | 1 12 0 | 8 11 0 | 6 10 0 | 0 18 4 | 1 14 3 |

N.B.—As the above options may be exercised any time within six months from the date of the declaration of the Bonus (4th January 1872), the amounts apportioned under the various modes are at present undetermined.

*The Society has no Reassurances.*

On the 30th June 1867, the Investments yielded £4 8 0 per cent.
„ „ 1868, „ „ £4 7 5 „
„ „ 1869, „ „ £4 8 4 „
„ „ 1870, „ „ £4 8 2 „
„ „ 1871, „ „ £4 7 9 „

The average rate of interest yielded during the five years by the whole of the Funds, whether invested or uninvested, was £4:5:1 per-cent per annum.

For whole-term policies, whether with or without participation in profits, granted at ordinary rates, on which three full years' premiums have been paid, and Endowment Assurances granted at ordinary rates, on which two full years' premiums have been paid, the minimum surrender value is 33 per-cent of the total premiums received. It being, however, the practice of the Society to allow on surrender of such Policies not less than two-thirds of the office reserve, if without profits, and three-fourths of such reserve, if with profits (in addition, in the latter case, to the full reserve in respect of any existing bonus), this percentage increases with the duration of the assurances.

For purposes of reserve and distribution of bonus, policies granted on unhealthy lives are treated as though the assumed ages on which the increased premiums are based were the real ages; but for purposes of surrender they are treated as though granted at ordinary rates.

The Society does not transact business other than at European rates.

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## LONDON AND LANCASHIRE LIFE ASSURANCE COMPANY.

*Established* 1862.

### EXTRACT FROM REPORT OF THE DIRECTORS.

The New Assurances for the last three years have been:—

| | Sums Assured. | Annual Premiums. |
|---|---|---|
| 1870 | £144,414 | £5,047 |
| 1871 | 205,582 | 6,208 |
| 1872 | 231,000 | 8,530 |

* * * * *

In accordance with the Deed of Settlement, a valuation of the Company's policies has been made, the result of which shows a surplus of over £11,000.

Of the one-fifth of the surplus shown by the Actuary's Report, which by the provisions of the Deed of Settlement belongs to the Proprietors' Fund, the sum of £2,000, or 4*s*. per share, will be placed to the credit of each share as additional capital, making £1. 4*s*. per share paid up. Upon this increased amount interest will be paid at the rate of £5 per-cent per annum from the 31st December last, being equivalent to £6 per-cent upon the original amount paid up.

### REPORT OF MR. SAMUEL BROWN.

I have carefully examined the Valuation which has been made of the Liabilities and Assets of the London and Lancashire Life Assurance Company, on the 31st December 1872. I have tested the calculations in each branch of the business, and find that they have been carefully computed and subjected to proper checks to ensure accuracy.

The well known Carlisle table, on which the official premiums have been calculated, is taken as the basis of the valuation, and the low rate of interest of 3 per-cent only assumed, so that the utmost confidence may be felt that the future interests of the policyholders have been duly protected.

The value of the whole of the loading on future premiums for profits and expenses has been deducted from the value of the full premiums, that there may be no anticipation of future profits on the present division.

The policies for the whole duration of life, whether participating or non-participating, existing at the same office age, have been brought together and valued at each age. In all other classes, the policies, 415 in number, have been valued separately, and in cases where extra premiums have been charged for peculiar risk of climate, occupation, etc., the liabilities have been increased according to the proportionate part of the premium for the unexpired risk.

With this explanation, the following Balance Sheet shows the position of the company on 31st December 1872:—

### *BALANCE SHEET*, 31*st December* 1872.

| **LIABILITIES.** | £ | *s.* | *d.* |
|---|---|---|---|
| Value of £1,236,798, assured by 2,980 Policies | 579,399 | 8 | 8 |
| Surplus | 11,379 | 1 | 11 |
| | £590,778 | 10 | 7 |

| **ASSETS.** | £ | *s.* | *d.* |
|---|---|---|---|
| Value of £39,183 16*s.* 6*d.*, premiums | 583,118 | 15 | 2 |
| *Less*, Value of Loading for future Profits and Expenses | 86,416 | 0 | 4 |
| | 496,702 | 14 | 10 |
| Value of Reassurances | 11,372 | 18 | 9 |
| Life Assurance Fund, 31st December 1872 | 82,702 | 17 | 0 |
| | £590,778 | 10 | 7 |

Allowing that the securities in which the surplus is invested will bear interest at a much higher rate than that which has been assumed in the valuation, I think we shall be justified in concluding that it will suffice for a Reversionary Bonus of one per-cent per annum on the sums assured by the Whole Life Participating Policies in force.

*The following additional particulars are extracted from the Returns under the Fifth and Sixth Schedules of the Life Assurance Companies Act*, 1870.

The date up to which the valuation has been made is the 31st December 1872.

The valuation and the division of profits have been made on the following principles:—

(*a.*) The ordinary whole life policies *with* and *without* participation have been valued in groups at a common age. The policies for all other kinds of assurances have been valued singly (this company does not grant annuities). The net or pure premiums only have been used in the valuation. Every premium was assumed to be due six months after the date of valuation. In those cases where an extra premium had been charged for foreign residence, voyaging, or occupation, an estimated proportion was set aside to meet the unexpired risk, and in cases of permanent extras the increased age corresponding to the premium actually charged has been taken as the valuation age.

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF POLICIES FOR VALUATION. | | | | VALUATION. | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | VALUE BY CARLISLE TABLE, INTEREST 3 PER-CENT. | | | |
| | Number of Policies. | Sums Assured. | Office Yearly Premiums. | Net Yearly Premiums. | Sums Assured. | Office Yearly Premiums. | Net Yearly Premiums. | Net Liability. |
| ASSURANCES. | | | | | | | | |
| I.—WITH PARTICIPATION IN PROFITS. | | £ | £ | £ | £ | £ | £ | £ |
| For Whole Term of Life . . . . . | 2,181 | 823,394 | 23,907 | 19,933 | 386,472 | 403,363 | 336,136 | 50,336 |
| By Limited Payments . . . . . | 79 | 32,250 | 1,694 | 1,390 | 14,391 | 11,224 | 9,858 | 4,533 |
| Endowment Assurances . . . . . | 139 | 37,200 | 1,909 | 1,590 | 22,245 | 23,871 | 20,127 | 2,117 |
| Joint Lives . . . . . . . | 62 | 16,750 | 685 | 548 | 9,513 | 9,650 | 7,720 | 1,793 |
| Extra Premiums payable . . . . . | ... | ... | 258 | ... | 136 | ... | ... | 136 |
| Total Assurances *With Profits* . . | 2,461 | 909,594 | 28,453 | 23,461 | 432,757 | 448,108 | 373,841 | 58,915 |
| II.—WITHOUT PARTICIPATION IN PROFITS. | | | | | | | | |
| For Whole Term of Life . . . . . | 384 | 244,499 | 7,893 | 7,172 | 125,669 | 118,649 | 107,863 | 17,806 |
| By Limited Payments . . . . . | 22 | 13,215 | 790 | 719 | 7,091 | 5,447 | 5,059 | 2,032 |
| Endowment Assurances . . . . . | 29 | 10,780 | 615 | 559 | 7,618 | 5,674 | 5,281 | 2,337 |
| Joint Lives . . . . . . . | 12 | 3,100 | 151 | 134 | 1,910 | 1,877 | 1,668 | 242 |
| Last Survivor . . . . . . . | 4 | 2,700 | 81 | 72 | 1,267 | 1,359 | 1,208 | 59 |
| Contingent Assurances . . . . . | 11 | 9,200 | 87 | 77 | 2,380 | 2,004 | 1,782 | 598 |
| Term Assurances . . . . . . | 54 | 41,410 | 578 | 462 | 437 | ... | ... | 437 |
| Assurances against Issue . . . . . | 3 | 3,300 | (S.P.)230 | ... | 115 | ... | ... | 115 |
| Extra Premiums payable . . . . | ... | ... | 306 | ... | 155 | ... | ... | 155 |
| Total Assurances *Without Profits* . | 519 | 328,204 | 10,731 | 9,195 | 146,642 | 135,010 | 122,861 | 23,781 |
| Total Assurances . . . . . | 2,980 | 1,237,798 | 39,184 | 32,656 | 579,399 | 583,118 | 496,702 | 82,696 |
| Deduct Reassurances . . . . . | 108 | 115,207 | 3,924 | 3,460 | 53,523 | 46,488 | 42,150 | 11,373 |
| Net Amount of Assurances and Liability | 2,872 | 1,122,591 | 35,260 | 29,196 | 525,876 | 536,630 | 454,552 | 71,323 |

(*b.*) The profits are distributed to the participating policyholders according to the number of years each policy has been in force prior to the date of valuation.

The Table of Mortality used was the "Carlisle" Table.

The rate of interest assumed throughout the whole of the calculations was three per-cent.

The whole of the "loading," *i.e.*, the difference between the premiums payable and the net premiums, has been reserved as a provision for future profits and expenses.

All participating policies in force at the date of the declaration and opened prior to the date of valuation are entitled to share in the profits.

The results of the valuation show that—

The total amount of profit made by the company during the ten years of its existence is £11,372 18*s.* 9*d.*

The amount of profit divided amongst the policyholders is £9,098. 7*s.* 0*d.*, and the number of policies which have participated 2,461, the amount assured under these being £909,594.

The following table exhibits specimens of bonus allotted to policies for £100 effected at different ages and in force for specified periods with the amounts apportioned under the various modes in which the same might be received.

| Age at Entry. | NUMBER OF YEARS IN FORCE. | | | | | |
|---|---|---|---|---|---|---|
| | FIVE YEARS. | | | TEN YEARS. | | |
| | Reversionary Additions. | Permanent Reduction of Premiums. | Cash. | Reversionary Additions. | Permanent Reduction of Premiums. | Cash. |
| | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* |
| 20 | 5 0 0 | 0 1 4 | 1 1 10 | 10 0 0 | 0 3 1 | 2 9 3 |
| 30 | 5 0 0 | 0 1 10 | 1 7 4 | 10 0 0 | 0 4 3 | 3 1 6 |
| 40 | 5 0 0 | 0 2 6 | 1 14 3 | 10 0 0 | 0 6 0 | 3 17 2 |
| 50 | 5 0 0 | 0 3 10 | 2 4 8 | 10 0 0 | 0 10 1 | 5 2 10 |

## *Consolidated Revenue Account for Ten Years, ending* 31*st December* 1872.

| | | £ *s.* *d.* |
|---|---|---|
| Amount of Funds, consisting of Capital paid up, representing a subscribed Capital of £54,730 on 1st January 1863, the beginning of the period . | | 5,473 0 0 |
| Premiums . . . . . . . | 269,208 7 9 | |
| Less Reassurance premiums . . | 46,362 3 10 | |
| | | 222,846 3 11 |
| Interest and Dividends . . . . | | 14,566 12 7 |
| Other Receipts— | | |
| Capital . . . . . . . | | 4,527 0 0 |
| Registration of Assignments and other Fees . . . . . | | 636 6 1 |
| | | £248,049 2 7 |

| | £ | s. | d. |
|---|---|---|---|
| Claims under policies (after deduction of sums reassured) | 60,759 | 19 | 0 |
| Surrenders . . . . . . . . . . . . | 5,573 | 8 | 7 |
| Commission . . . . . . . . . . . . | 17,798 | 13 | 4 |
| Expenses of Management, including Preliminary Expenses . . . . . . . . . . . | 66,596 | 19 | 11 |
| Dividends to Shareholders (being 5 per-cent interest) . | 4,617 | 4 | 9 |
| Amount of Funds on 31st December 1872, the end of the period . . . . . . . . . . | 92,702 | 17 | 0 |
| | £248,049 | 2 | 7 |

*Summary and Valuation of the Policies* (see p. 283).

*Valuation Balance Sheet as at* 31*st December* 1872.

| Dr. | £ | s. | d. |
|---|---|---|---|
| To net liability under Assurance transactions, as per Summary Statement . . . . . . . . | 71,323 | 15 | 1 |
| To Surplus . . . . . . . . . . . | 11,379 | 1 | 11 |
| | £82,702 | 17 | 0 |

| Cr. | £ | s. | d. |
|---|---|---|---|
| By Life Assurance Funds, as per Balance Sheet . . | 82,702 | 17 | 0 |
| | £82,702 | 17 | 0 |

The average rate of interest at which the Life Assurance Fund has been invested each year since the commencement of the company, was £4·088 per-cent.

The Company has no table of Minimum Surrender Values, but a surrender value is allowed on policies effected for the whole term of life, after they have been three years in force, the amount being 30 per-cent and upwards of the premiums paid, according to the circumstances in each case. Endowment Assurances are dealt with in a similar manner. The company has not issued *Endowment Policies*. Policies on unhealthy lives are dealt with as at the advanced ages corresponding to the premium paid.

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## EAGLE INSURANCE COMPANY.

*Established* 1807.

### Report of the Directors

### *For the Year and Quinquennium ending* 30*th June* 1872.

The Report which the Directors have to make to the Proprietors embraces not only the events of the year just expired, such as are laid before Annual General Meetings, but also the position of the Company at the close of its 13th Quinquennium—being the sixty-fifth completed year of its existence.

* * * * *

In regard to the Quinquennial Period, the Directors now beg leave to draw the attention of the Proprietors to the financial position of the Company.

The Life Assurance Companies Act requires several full statements to be sent in to the Board of Trade, and it allows nine months after the closing of the Quinquennium for this purpose. The whole of those statements, which before long will be made public, necessarily require more time than has elapsed since the close of our Quinquennial Period, on the 30th June last; but two have already been prepared, and are annexed to this Report, and as these are the most important for the present information of the Proprietors, the Directors invite attention to them as affording the means for arriving at an accurate conclusion as to the condition of the company's affairs.

The first statement exhibits in full detail the whole number and amount of the company's assurance and annuity risks, and the liabilities under them. The calculations required in compiling it have been made on the basis of the Carlisle table of mortality, and interest at the rate of 4 per-cent; and the reserve for future expenses, for divisions of surplus, fluctuations in the rate of interest, and other contingencies, is obtained, after providing for some guaranteed bonuses on one class of assurances, by a reduction of 24 per-cent from all future premiums on participating, and of 12 per-cent from those on non-participating policies. The statement also shows the amount of reassurances effected with other companies, and the premiums payable on account of them, with the value of such amounts and premiums respectively. The final result shown by the Table is a net liability, after making the provisions mentioned, of £2,749,223.

The second statement appended to this Report is the Balance-Sheet of the company, in the form required by the Act of Parliament, and it will be seen from it that, after providing for the Proprietors' capital, and for the liabilities which had accrued and which were about to mature, this document exhibits a balance in favour of the company of £2,937,599.

In the form then of the valuation Balance-Sheet prescribed by the Act, we have to the credit of the company—

| | |
|---|---|
| The Life Assurance and Annuity Funds, as per Balance-Sheet . | £2,937,599 |
| Less the net liability under Assurance and Annuity transactions | 2,749,223 |
| The Surplus being . . . . . . . . | £188,376 |

Out of this surplus, the Directors propose to divide on the present occasion the sum of £184,654, leaving £3,722 to be carried to the next account.

The time has not sufficed to furnish a tabular statement of additions in particular cases, but it has been approximately ascertained that the additions to the sums assured under participating Policies of twenty years' standing will probably be on the average at a rate of £45 to each £1000 assured, or about 30 per-cent of the Premiums paid during the last five years.

The remaining returns required by the recent Act of Parliament to be furnished to the Board of Trade are now in course of preparation, and will contain all the additional information which can be desired in relation to the company.

It appears from the Balance Sheet that the Shareholders' capital paid up is £167,867. 10*s*.

*The following additional particulars are ex under the Fifth and Sixth Schedules Companies Act* 1870:—

The principles upon which the valuation distributed are determined by the Directors. office premiums, less the deductions mentio from the value of the sums assured, the dif Company's liability. The proportion of profi pating policyholders is eighty per-cent, and ratio indicated by the amount of the "loadin holder since the foregoing division.

The provision for future expenses and profit of twenty-four per-cent from the future annual of twelve per-cent from the future annual non- and of fifteen per-cent from the future annu respect of the special class of assurances with

CONSOLIDATED REVENUE ACCOUNT *for Five June* 1867, *and ending* 30*th J*

| | £ |
|---|---|
| Amount of Funds, 30th June 1867 | 2,897,861 1 |
| Add — Outstanding Premiums, 30th June 1867 . . | 11,874 1 |
| Profit on Sale of Property in Charlotte Row, City . | 32,529 1 |
| Deduct—Value of Reassurances (included in the ab | |
| Premiums . . . . . . | 1,821,419 1 |
| Less—Reassurances . . . | 275,922 1 |
| Interest and Dividends . . . . . . | |
| Balance of Profit on Reversions fallen in—viz., £66,470. 15*s*. 7*d*., less £45,285. 0*s*. 10*d*. writte | |
| Balance of sundry Profits . . . . . . | |
| Claims under Policies after deduction of sums Reassu | |
| Surrenders . . . . . . . . . . | |
| Annuities . . . . . . . . . . | |
| Commission . . . . . . . . . | |
| Expenses of Management . . . . . . | |
| Dividends . . . . . . . . . . | |
| Medical Fees . . . . . . . . . | |
| Income Tax . . . . . . . . . | |
| Shares purchased . . . . . . . | |
| Cash Bonus (1867) . . . . . . . | |
| Proprietors' Bonus (1867) . . . . . . | |
| Balance of Funds 30th June 1872 . . . | |

### *SUMMARY and VALUATION of the POLICIES, as at the 30th June* 1872.

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF POLICIES FOR VALUATION. | | | | VALUATION. Value by the Carlisle Table, Interest 4 per-cent. | | | |
|---|---|---|---|---|---|---|---|---|
| | No. of Policies. | Sums Assured and Bonuses. | Office Yearly Premiums, less Reductions by Bonus. | Estimated Net Yearly Premiums. | Sums Assured and Bonuses. | Office Yearly Premiums. | Estimated Net Yearly Premiums. | Net Liability. |
| | | £ | £ | £ | £ | £ | £ | £ |
| ASSURANCES.<br>I.—WITH PARTICIPATION IN PROFITS.<br>1. *All future Bonus Reductions Guaranteed.* | (a) [illegible] | [illegible] | [illegible] | (a) [illegible] | 777,874 | 391,048 | (a) 333,151 | 444,723 |

| | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| For Whole Term of Life | 2,997 | 1,541,329 | 32,533 | [illegible] | [illegible] | [illegible] | [illegible] | [illegible] |
| Endowment Assurances | 261 | 77,335 | 2,215 | 1,883 | 55,570 | 14,801 | 12,581 | 42,989 |
| Joint Lives | 40 | 13,850 | 392 | 333 | 8,099 | 4,127 | 3,508 | 4,591 |
| Last Survivor | 2 | 2,000 | 66 | 56 | 1,370 | 491 | 417 | 953 |
| Limited Payments | 4 | 4,715 | 72 | 61 | 1,790 | 289 | 246 | 1,544 |
| Ascending Premiums | 2 | 636 | 13 | 11 | 371 | 130 | 111 | 260 |
| Endowments—Premiums Returnable | 1 | 300 | 5 | 4 | 261 | 12 | 10 | 251 |
| Whole Term of Life—Premiums Returnable | 1 | 100 | 1 | 1 | 260 | 14 | 12 | 248 |
| | 3,308 | 1,646,265 | 35,297 | 30,002 | 845,595 | 411,807 | 350,036 | 495,559 |
| 2. *Ordinary Participating.* | | | | | | | | |
| For Whole Term of Life | 10,665 | 6,438 957 | 192,814 | (b) 146,539 | 3,355,976 | 2,202,888 | (b) 1,674,195 | 1,681,781 |
| Endowment Assurances | 376 | 129,405 | 5,944 | 4,517 | 82,758 | 48,016 | 36,492 | 46,265 |
| Joint Lives | 169 | 44,859 | 1,822 | 1,385 | 26,185 | 17,932 | 13,628 | 12,557 |
| Last Survivor | 57 | 50,112 | 691 | 525 | 20,505 | 9,590 | 7,289 | 13,216 |
| Limited Payments | 94 | 69,536 | 240 | 182 | 41,332 | 1,688 | 1,283 | 40,049 |
| Ascending Premiums | 43 | 17,262 | 531 | 404 | 9,444 | 6,465 | 4,914 | 4,531 |
| Descending Premiums | 1 | 1,331 | 11 | 8 | 903 | 88 | 67 | 836 |
| Contingent Assurances | 3 | 2,125 | 70 | 53 | 898 | 448 | 340 | 558 |
| Extra Premiums payable | .. | .. | 2,078 | .. | 1,039 | .. | .. | 1,039 |
| | 11,408 | 6,753,587 | 204,201 | 153,613 | 3,539,040 | 2,287,115 | 1,738,208 | 1,800,832 |
| Total Assurances With Profits | **14,716** | **8,399,852** | **239,498** | **183,615** | **4,384,635** | **2,698,922** | **2,088,244** | **2,296,391** |
| II.—WITHOUT PARTICIPATION IN PROFITS. | | | | | | | | |
| For Whole Term of Life | 2,236 | 2,759,421 | 85,213 | (c) 74,988 | 1,364,546 | 1,012,827 | (c) 891,288 | 473,258 |
| Endowment Assurances | 164 | 33,837 | 1,232 | 1,084 | 25,031 | 7,329 | 6,450 | 18,581 |
| Joint Lives | 153 | 58,731 | 2,735 | 2,407 | 33,345 | 28,320 | 24,922 | 8,423 |
| Last Survivor | 80 | 75,706 | 1,492 | 1,313 | 29,869 | 21,289 | 18,734 | 11,134 |
| Limited Payments | 28 | 29,520 | 94 | 83 | 12,361 | 548 | 482 | 11,879 |
| Ascending Premiums | 141 | 105,072 | 3,466 | 3,050 | 56,707 | 41,264 | 36,312 | 20,395 |
| Descending Premiums | 1 | 400 | 4 | 4 | 252 | 38 | 34 | 219 |
| Contingent Assurances | 90 | 131,358 | 1,862 | 1,639 | 16,358 | 13,557 | 11,930 | 4,428 |
| Endowments—Premiums Returnable | 45 | 13,740 | 214 | 188 | 12,498 | 630 | 554 | 11,944 |
| ,, Premiums not Returnable | 7 | 1,599 | 27 | 23 | 1,009 | 142 | 124 | 884 |
| Endowment Assurances, with Deferred Annuities | 7 | 450 | 37 | 33 | 349 | 143 | 126 | 223 |
| Term Assurances | 57 | 57,982 | 1,011 | 889 | 1,069 | .. | .. | 1,069 |
| Assurances against Issue | 23 | 191,675 | 581 | 511 | 6,697 | .. | .. | 6,697 |
| Sundries | 2 | 10,200 | 9 | 8 | 3,551 | *2,776 | 2,443 | 1,109 |
| Extra Premiums payable | .. | .. | 557 | .. | 278 | .. | .. | 278 |
| Total Assurances Without Profits | **3,034** | **3,469,691** | **98,534** | **86,230** | **1,563,920** | **1,128,863** | **993,399** | **570,521** |
| Total Assurances | 17,750 | 11,869,543 | 338,032 | 269,835 | 5,948,555 | 3,827,785 | 3,081,643 | 2,866,912 |
| Deduct Reassurances | .. | 1,897,901 | 54,989 | 48,390 | 840,188 | 727,712 | 640,386 | 199,802 |
| Net Amount of Assurances | 17,750 | 9,971,642 | 283,043 | 221,445 | 5,108,367 | 3,100,073 | 2,441,257 | 2,667,110 |
| ANNUITIES. | | | | | | | | |
| Immediate | 122 | 9,635 | .. | .. | 76,091 | .. | .. | 76,091 |
| Deferred | 21 | 940 | 181 | 160 | 6,766 | 871 | 766 | 6,000 |
| Contingent | 15 | 1,885 | 271 | 239 | 2,022 | 2,273 | 2,000 | 22 |
| Total of the Results | .. | .. | **283,495** | **221,844** | **5,193,246** | **3,103,217** | **2,444,023** | **2,749,223** |

(a) Office Yearly Premiums, less 15 per-cent. No new business of this description transacted by the Company. (b) Office Yearly Premiums, less 24 per-cent.

(c) Office Yearly Premiums, less 12 per-cent. * This item includes the value of £239. 3s. per annum Deferred Premiums in addition to the Office Yearly Premiums at present payable.

### VALUATION BALANCE SHEET *as at* 30*th June* 1872.

| DR. | £ | s. | d. |
|---|---|---|---|
| To net liability under Assurance and Annuity transactions, as per Summary Statement . . . | 2,749,223 | 0 | 0 |
| To surplus . . . . . . . . . . | 188,376 | 0 | 0 |
| | £2,937,599 | 0 | 0 |

| CR. | £ | s. | d. |
|---|---|---|---|
| By Life Assurance and Annuity Funds, as per Balance Sheet . . . . . . . . . . | 2,937,599 | 0 | 0 |
| | £2,937,599 | 0 | 0 |

Participating policies commence to participate immediately they are effected; but the additions made do not vest till the policies have been five years in force.

The results of this valuation are as follows:—

1. The total amount of profit made by the company is £188,376.
2. The amount of profit divided among the ordinary participating policyholders is £147,723, the policies participating being 11,408 in number, and £6,151,314 in amount. The other class of participating policies, numbering 3,308, and insuring £1,593,936. 15*s*. 8*d*., being entitled to future reduction of premium only, for which a special reserve is made as stated in the summary.
3. The following table exhibits specimens of bonuses allotted to policies for £100, effected at the respective ages of 20, 30, 40, and 50, and having been respectively in force for 5 years, 10 years, and upwards, at intervals of 5 years respectively, together with the amounts apportioned under the various modes in which the bonus may be received.

| No. of Years in force. | AGE AT ENTRY, 20. | | | | | | | | | | | | AGE AT ENTRY, 30. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Cash Bonus | | | Reversionary Bonus. | | | Reduction of Annual Premium for Five Years. | | | Reduction of Annual Premium for Whole Term. | | | Cash Bonus. | | | Reversionary Bonus. | | | Reduction of Annual Premium for Five Years. | | | Reduction of Annual Premium for Whole Term. | | |
| | £ | s. | d. | £ | s. | d. | £ | s | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. |
| 5 | – | 4 | 0 | 8 | 1 | 0 | 0 | 9 | 10 | 0 | 2 | 5 | 2 | 7 | 0 | 7 | 0 | 0 | 0 | 10 | 6 | 0 | 2 | 10 |
| 10 | 2 | 4 | 0 | 7 | 5 | 0 | 0 | 9 | 10 | 0 | 2 | 6 | 2 | 7 | 0 | 6 | 5 | 0 | 0 | 10 | 6 | 0 | 3 | 0 |
| 15 | 2 | 4 | 0 | 6 | 11 | 0 | 0 | 9 | 10 | 0 | 2 | 8 | 2 | 7 | 0 | 5 | 11 | 0 | 0 | 10 | 6 | 0 | 3 | 4 |
| 20 | 2 | 4 | 0 | 5 | 16 | 0 | 0 | 9 | 10 | 0 | 2 | 10 | 2 | 7 | 0 | 4 | 19 | 0 | 0 | 11 | 0 | 0 | 3 | 8 |
| 25 | 2 | 4 | 0 | 5 | 3 | 0 | 0 | 9 | 10 | 0 | 3 | 1 | 2 | 7 | 0 | 4 | 8 | 0 | 0 | 11 | 0 | 0 | 4 | 2 |
| 30 | 2 | 4 | 0 | 4 | 12 | 0 | 0 | 10 | 3 | 0 | 3 | 5 | 2 | 7 | 0 | 3 | 19 | 0 | 0 | 11 | 0 | 0 | 4 | 10 |

| No. of Year. in force. | AGE AT ENTRY, 40. | | | | | | | | | | | | AGE AT ENTRY. 50. | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Cash Bonus. | | | Reversionary Bonus. | | | Reduction of Annual Premium for Five Years. | | | Reduction of Annual Premium for Whole Term. | | | Cash Bonus. | | | Reversionary Bonus. | | | Reduction of Annual Premium for Five Years. | | | Reduction of Annual Premium for Whole Term. | | |
| | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. |
| 5 | 2 | 11 | 0 | 5 | 19 | 0 | 0 | 11 | 3 | 0 | 3 | 7 | 2 | 16 | 0 | 5 | 4 | 0 | 0 | 12 | 11 | 0 | 4 | 11 |
| 10 | 2 | 11 | 0 | 5 | 6 | 0 | 0 | 11 | 10 | 0 | 3 | 11 | 2 | 16 | 0 | 4 | 14 | 0 | 0 | 12 | 11 | 0 | 5 | 8 |
| 15 | 2 | 11 | 0 | 4 | 15 | 0 | 0 | 11 | 10 | 0 | 4 | 5 | 2 | 16 | 0 | 4 | 5 | 0 | 0 | 13 | 6 | 0 | 6 | 9 |
| 20 | 2 | 11 | 0 | 4 | 5 | 0 | 0 | 11 | 10 | 0 | 5 | 2 | 2 | 16 | 0 | 3 | 18 | 0 | 0 | 14 | 1 | 0 | 8 | 3 |
| 25 | 2 | 11 | 0 | 3 | 17 | 0 | 0 | 12 | 4 | 0 | 6 | 2 | 2 | 16 | 0 | 3 | 12 | 0 | 0 | 14 | 8 | 0 | 10 | 5 |
| 30 | 2 | 11 | 0 | 3 | 11 | 0 | 0 | 12 | 10 | 0 | 7 | 7 | 2 | 16 | 0 | 3 | 8 | 0 | 0 | 14 | 8 | 0 | 13 | 6 |

The average rate of interest at which the Life Assurance Fund of the company was invested at the close of each year since the last investigation is—

| | £ | s. | d. | |
|---|---|---|---|---|
| In 1868 . . . . . . | 4 | 4 | 0 | per-cent. |
| „ 1869 . . . . . . | 4 | 4 | 6 | „ |
| „ 1870 . . . . . . | 4 | 5 | 4 | „ |
| „ 1871 . . . . . . | 4 | 8 | 1 | „ |
| „ 1872 . . . . . . | 4 | 8 | 9 | „ |

The following table exhibits the minimum values allowed for the surrender of Policies for £100 effected at various ages, for the whole term of life, and at the expiration of 5 years, 10 years, 15 years, &c.

| Age at Entry. | YEARS IN FORCE. | | | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 Years. | | | 10 Years. | | | 15 Years. | | | 20 Years. | | | 25 Years. | | | 30 Years. | | | 35 Years. | | | 40 Years. | | | 45 Years. | | |
| | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. |
| **20** | 2 | 10 | 0 | 5 | 8 | 0 | 8 | 18 | 0 | 12 | 19 | 0 | 17 | 14 | 0 | 23 | 1 | 0 | 28 | 16 | 0 | 34 | 16 | 0 | 40 | 18 | 0 |
| **25** | 3 | 0 | 0 | 6 | 12 | 0 | 10 | 16 | 0 | 15 | 14 | 0 | 21 | 5 | 0 | 27 | 4 | 0 | 33 | 9 | 0 | 39 | 15 | 0 | 45 | 16 | 0 |
| **30** | 3 | 14 | 0 | 8 | 2 | 0 | 13 | 4 | 0 | 19 | 0 | 0 | 25 | 4 | 0 | 31 | 14 | 0 | 38 | 5 | 0 | 44 | 11 | 0 | 50 | 8 | 0 |
| **35** | 4 | 12 | 0 | 10 | 0 | 0 | 16 | 1 | 0 | 22 | 12 | 0 | 29 | 8 | 0 | 36 | 7 | 0 | 43 | 0 | 0 | 49 | 2 | 0 | 54 | 12 | 0 |
| **40** | 5 | 15 | 0 | 12 | 4 | 0 | 19 | 3 | 0 | 26 | 9 | 0 | 33 | 16 | 0 | 40 | 18 | 0 | 47 | 8 | 0 | 53 | 5 | 0 | 58 | 9 | 0 |
| **45** | 7 | 0 | 0 | 14 | 10 | 0 | 22 | 8 | 0 | 30 | 8 | 0 | 38 | 1 | 0 | 45 | 2 | 0 | 51 | 9 | 0 | 57 | 2 | 0 | 62 | 9 | 0 |
| **50** | 8 | 6 | 0 | 17 | 0 | 0 | 25 | 16 | 0 | 34 | 5 | 0 | 42 | 1 | 0 | 49 | 1 | 0 | 55 | 5 | 0 | 61 | 3 | 0 | | | |

These values are 75 per-cent of the result obtained by a net premium valuation at 4 per-cent Experience mortality, and the surrender value of endowment assurances is determined in like manner.

Endowment Policies are not now issued by this company.

The company transacts no business at other than European rates, such extra premium as may be charged for residence in other climates being discontinued on the return of the assured within the ordinary limits.

Policies on Unhealthy Lives are dealt with for all purposes as if the advanced ages at which the lives have been accepted were the true ages.

# CORRESPONDENCE.

## ON THE FORMULAS FOR THE APPROXIMATE DETERMINATION OF THE RATE OF INTEREST OF AN ANNUITY.

*To the Editor of the Journal of the Institute of Actuaries.*

Sir,—Having recently had occasion to examine the ordinary formulas for the determination, with the help of tables, of the rate of interest involved in the value of a term annuity, I venture to send you an account of certain modifications of them which appear to be desirable, in the hope that it may not be without interest to readers of the *Journal.*

The problem to be considered is as follows. Let $a$ be the present value at an unknown rate of interest, $x$, of an annuity of 1 for $n$ years. Let $a'$ be the value of a similar annuity at a rate of interest, $i$, being the tabulated value which approaches most nearly to $a$. Let $\rho=x-i$. Required, an expression for the value of $\rho$ in terms of the above known quantities, or others depending upon them.

The best known formulas for this purpose are, that to be found at page 42 of Mr. Jones's work on annuities, previously given by Baily, and the more exact one given by Professor De Morgan at page 67 of the eighth volume of the *Journal.* The well known expression for the value of an annuity for $n$ years, at a rate of interest of $i+\rho$, is $\frac{1-(1+i+\rho)^{-n}}{i+\rho}$. Equating this to $a$, and multiplying both sides of the equation by $i+\rho$, we obtain

$$(i+\rho)a=1-(1+i+\rho)^{-n};$$

and from this form of the equation, Mr. Jones, and apparently Professor De Morgan, have derived the formulas given by them. Professor De Morgan's formula may be thus obtained. Expanding the right hand side of the equation by the binomial theorem, and rejecting terms involving powers of $\rho$ above the second, we have

$$(i+\rho)a=1-(1+i)^{-n}+n(1+i)^{-(n+1)}\rho-\frac{n(n+1)}{2}(1+i)^{-(n+2)}\rho^2.$$

Transposing, and writing $v$ for $(1+i)^{-1}$, we have

$$ai=1-v^n-(a-nv^{n+1})\rho-\frac{n(n+1)}{2}v^{n+2}\rho^2,$$

that is,

$$\frac{n(n+1)}{2}v^{n+2}\rho^2+(a-nv^{n+1})\rho-(a'-a)i=0.$$

One of the roots of this equation is

$$\frac{-(a-nv^{n+1})+\sqrt{(a-nv^{n+1})^2+4i(a'-a)\frac{n(n+1)}{2}v^{n+2}}}{2\left\{\frac{n(n+1)}{2}v^{n+2}\right\}}.$$

Multiplying numerator and denominator of this fraction by

$$(a-nv^{n+1})+\sqrt{(a-nv^{n+1})^2+4i(a'-a)\frac{n(n+1)}{2}v^{n+2}},$$

it is reduced to

$$\frac{2i(a'-a)}{(a-nv^{n+1})+\sqrt{(a-nv^{n+1})^2+4i(a'-a)\frac{n(n+1)}{2}v^{n+2}}}.$$

Putting the second term of the denominator into the form

$$\sqrt{(a-nv^{n+1})^2+4(a-nv^{n+1})\frac{n(n+1)}{2}\cdot\frac{i(a'-a)v^{n+2}}{a-nv^{n+1}}},$$

we see that an approximate value is

$$(a-nv^{n+1})+2\frac{n(n+1)}{2}\cdot\frac{i(a'-a)v^{n+2}}{a-nv^{n+1}}.$$

Substituting this value, the root becomes

$$\frac{i(a'-a)}{(a-nv^{n+1})+\dfrac{n(n+1)}{2}\cdot\dfrac{i(a'-a)v^{n+2}}{a-nv^{n+1}}}\quad\ldots\quad(1)$$

This is the formula given by Professor De Morgan, while, as pointed out by him, if the second term of the denominator be neglected, we obtain a formula which by a simple transformation may be shown to be equivalent to that given by Mr. Jones.

It is to be observed, however, that the multiplication of both sides of the fundamental equation, $a=\dfrac{1-(1+i+\rho)^{-n}}{i+\rho}$ by $i+\rho$, which forms the first step in the above process, is objectionable, as involving a considerable sacrifice of accuracy. Its effect is to increase the coefficient of each power of $\rho$ by $\dfrac{1}{i}$th part of the coefficient of the next lower power, a portion of the coefficient of $\rho^2$ thus becoming involved in that of $\rho^3$, and left out of account in obtaining the solution of the equation. A more correct result will be obtained by expanding the right hand side of the equation in ascending powers of $\rho$. This may be done by the binomial theorem, thus,

$$\frac{1-(1+i+p)^{-n}}{i+\rho}=\{1-(1+i+\rho)^{-n}\}(i+\rho)^{-1}$$

$$=\left\{1-v^n+nv^{n+1}\rho-\frac{n(n+1)}{2}v^{n+2}\rho^2\right\}\left\{\frac{1}{i}-\frac{\rho}{i^2}+\frac{\rho^2}{i^3}\right\}$$

$$=\frac{1-v^n}{i}+\frac{nv^{n+1}}{i}\rho-\frac{1-v^n}{i^2}\rho-\frac{n(n+1)v^{n+2}}{2i}\rho^2-\frac{nv^{n+1}}{i^2}\rho^2+\frac{1-v^n}{i^3}\rho^3$$

$$=a'-\frac{1}{i}(a'-nv^{n+1})\rho-\frac{1}{i^2}\left\{\frac{n(n+1)iv^{n+2}}{2}-(a'-nv^{n+1})\right\}\rho^2$$

This gives for the complete equation,

$$\frac{1}{i^2}\left\{\frac{n(n+1)iv^{n+2}}{2}-(a'-nv^{n+1})\right\}\rho^2+\frac{1}{i}(a'-nv^{n+1})\rho-(a'-a)=0.$$

It will be satisfactory to show how this result may be obtained by a different method.

Expanding by the binomial theorem, the common expression for the value of an annuity for $n$ years at a rate of interest, $i$, becomes

$$\frac{1}{i}-\frac{1}{i}\left\{1-ni+\frac{n(n+1)}{2}i^2-\frac{n(n+1)(n+2)}{2.3}i^3+\ldots.\right\}$$

$$=n-\frac{n(n+1)}{2}i+\frac{n(n+1)(n+2)}{2.3}i^2-\ldots.$$

and the difference between such value and the value of a similar annuity at a rate of interest, $i+\rho$, is seen to be

$$-\frac{n(n+1)}{2}\{i-(i+\rho)\}+\frac{n(n+1)(n+2)}{2.3}\{i^2-(i+\rho)^2\}-\ldots.$$

$$=\frac{n(n+1)}{2}\{(i+\rho)-i\}-\frac{n(n+1)(n+2)}{2.3}\{(i+\rho)^2-i^2\}+\ldots.$$

$\rho$ being a small quantity, its powers of a higher order than the second may be neglected, and the expression last obtained then becomes

$$\frac{n(n+1)}{2}\rho-\frac{n(n+1)(n+2)}{2.3}(2i\rho+\rho^2)+\frac{n(n+1)(n+2)(n+3)}{2.3.4}(3i^2\rho+3i\rho^2)$$
$$-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}(4i^3\rho+6i^2\rho^2)+\ldots.$$

The coefficient of $\rho$ in this series is

$$\frac{n(n+1)}{2}-\frac{n(n+1)(n+2)}{2.3}2i+\frac{n(n+1)(n+2)(n+3)}{2.3.4}3i^2$$
$$-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}4i^3+\ldots.$$

which may be written

$$\frac{n(n+1)}{2}2-\frac{n(n+1)(n+2)}{2.3}3i+\frac{n(n+1)(n+2)(n+3)}{2.3.4}4i^2$$
$$-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}5i^3+\ldots.$$

$$-\frac{n(n+1)}{2}+\frac{n(n+1)(n+2)}{2.3}i-\frac{n(n+1)(n+2)(n+3)}{2.3.4}i^2$$
$$+\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}i^3-\ldots.$$

Adding to this the neutral quantity

$$\frac{1}{i^2}-\frac{n}{i}-\frac{1}{i^2}+\frac{n}{i},$$

we obtain

$$\frac{1}{i^2}-\frac{n}{i}+n(n+1)-\frac{n(n+1)(n+2)}{2}i+\frac{n(n+1)(n+2)(n+3)}{2.3}i^2$$
$$-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4}i^3+\ldots.$$

$$-\frac{1}{i^2}+\frac{n}{i}-\frac{n(n+1)}{2}+\frac{n(n+1)(n+2)}{2.3}i-\frac{n(n+1)(n+2)(n+3)}{2.3.4}i^2$$
$$+\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}i^3-\ldots.$$

$$=\frac{1}{i^2}-\frac{n}{i}\left\{1-(n+1)i+\frac{(n+1)(n+2)}{2}i^2-\frac{(n+1)(n+2)(n+3)}{2.3}i^3+\dots\right\}$$

$$-\frac{1}{i^2}\left\{1-ni+\frac{n(n+1)}{2}i^2-\frac{n(n+1)(n+2)}{2.3}i^3+\dots\right\}$$

$$=\frac{1}{i^2}-\frac{nv^{n+1}}{i}-\frac{v^n}{i^2}=\frac{1}{i}\left\{\frac{1-v^n}{i}-nv^{n+1}\right\}$$

$$=\frac{1}{i}(a'-nv^{n+1}).$$

The coefficient of $\rho^3$ in the original equation is

$$-\frac{n(n+1)(n+2)}{2.3}+\frac{n(n+1)(n+2)(n+3)}{2.3.4}3i-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}6i^2+\dots$$

Bearing in mind that the first order of differences of a series of figurate numbers is the series of the next lower order, we may write this expression in the form

$$-\frac{n(n+1)(n+2)}{2.3}3+\frac{n(n+1)(n+2)(n+3)}{2.3.4}6i-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}10i^2+\dots$$

$$+\frac{n(n+1)(n+2)}{2.3}2-\frac{n(n+1)(n+2)(n+3)}{2.3.4}3i+\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}4i^2-\dots$$

$$=-\frac{n(n+1)(n+2)}{2.2.3}6+\frac{n(n+1)(n+2)(n+3)}{2.2.3.4}12i-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.2.3.4.5}20i^2+\dots$$

$$+\frac{n(n+1)(n+2)}{2.3}3-\frac{n(n+1)(n+2)(n+3)}{2.3.4}4i+\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}5i^2-\dots$$

$$-\frac{n(n+1)(n+2)}{2.3}+\frac{n(n+1)(n+2)(n+3)}{2.3.4}i-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.3.4.5}i^2+\dots$$

Adding the neutral quantity,

$$\frac{n(n+1)}{2i}+\frac{n}{i^2}-\frac{n(n+1)}{i}+\frac{1}{i^3}-\frac{n}{i^2}+\frac{n(n+1)}{2i}-\frac{1}{i^3},$$

we obtain

$$\frac{n(n+1)}{2i}-\frac{n(n+1)(n+2)}{2}+\frac{n(n+1)(n+2)(n+3)}{2.2}i$$
$$-\frac{n(n+1)(n+2)(n+3)(n+4)}{2.2.3}i^2+\dots\dots$$
$$+\frac{n}{i^2}-\frac{n(n+1)}{i}+\frac{n(n+1)(n+2)}{2}-\frac{n(n+1)(n+2)(n+3)}{2.3}i+\dots\dots$$
$$+\frac{1}{i^3}-\frac{n}{i^2}+\frac{n(n+1)}{2i}-\frac{n(n+1)(n+2)}{2.3}i+\dots\dots-\frac{1}{i^3}$$

that is

$$\frac{n(n+1)}{2i}\left\{1-(n+2)i+\frac{(n+2)(n+3)}{2}i^2-\frac{(n+2)(n+3)(n+4)}{2.3}i^3+\dots\dots\right\}$$
$$+\frac{n}{i^2}\left\{1-(n+1)i+\frac{(n+1)(n+2)}{2}i^2-\frac{(n+1)(n+2)(n+3)}{2.3}i^3+\dots\dots\right\}$$
$$+\frac{1}{i^3}\left\{1-ni+\frac{n(n+1)}{2}i^2-\frac{n(n+1)(n+2)}{2.3}i^3+\dots\dots\right\}-\frac{1}{i^3}$$
$$=\frac{n(n+1)v^{n+2}}{2i}+\frac{nv^{n+1}}{i^2}+\frac{v^n}{i^3}-\frac{1}{i^3}$$
$$=\frac{1}{i^2}\left\{\frac{n(n+1)iv^{n+2}}{2}-(a'-nv^{n+1})\right\}.$$

The required expression for the difference between the values of the annuities, when the difference between the rates of interest involved is $\rho$, is thus

$$\frac{1}{i^2}\left\{\frac{n(n+1)iv^{n+2}}{2}-(a'-nv^{n+1})\right\}\rho^2+\frac{1}{i}(a'-nv^{n+1})\rho.$$

Equating this to $a'-a$, we have

$$\frac{1}{i^2}\left\{\frac{n(n+1)iv^{n+2}}{2}-(a'-nv^{n+1})\right\}\rho^2+\frac{1}{i}(a'-nv^{n+1})\rho-(a'-a)=0,$$

which is the result obtained by the former process.

Applying to this equation the method of solution applied to the former, we obtain as the required root

$$\frac{i(a'-a)}{a'-nv^{n+1}+\left\{\frac{n(n+1)i(a'-a)v^{n+2}}{2(a'-nv^{n+1})}-(a'-a)\right\}}$$
$$=\frac{i(a'-a)}{a-nv^{n+1}+\frac{n(n+1)}{2}\cdot\frac{i(a'-a)v^{n+2}}{a'-nv^{n+1}}}\quad\cdot\quad\cdot\quad\cdot\quad(2)$$

This, it will be observed, differs from Professor De Morgan's formula in the second term of the denominator, in which $a'-nv^{n+1}$ takes the place of $a-nv^{n+1}$.

If, following the example of Mr. Jones, we neglect all powers of $\rho$ above the first, we obtain the formula

$$\frac{i(a'-a)}{a'-nv^{n+1}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

in place of that given by him, namely

$$\frac{i(a'-a)}{a-nv^{n+1}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (4)$$

To ascertain the effect of these modifications, let us take the example given by Professor De Morgan. He takes 21·924788, the value of an annuity for 98 years at 4½ percent, and employs his formula to deduce the rate of interest involved in it from the value of a similar annuity at 4 percent; obtaining as the result ·0449808. The application of the formula (2) gives ·0449944, from which we see that the error resulting from its use is little more than ¼th of that which results from the use of Professor De Morgan's formula.

Mr. Jones's formula (4) can only be applied with advantage when the difference between the value of the given annuity and some one of the tabulated values is small. Given 19·474356, the value of an annuity for 30 years, the rate of interest involved when determined by (4) from the value of a similar annuity at 3 percent, comes out ·030506. The formula (3) gives ·030498, which is a considerably closer approximation to the true rate ·0305.

These alterations may seem inconsiderable. I am not, however, aware of the existence of any published notice of them, and I hope the growing importance of the problem to which the formulas are applicable will be held to justify the length of this communication.

I am, Sir,
Your most obedient servant,

J. J. McLAUCHLAN.

*26 St. Andrew Square, Edinburgh.*
*March* 1874.

P.S.—Since the above was in type, Mr. Makeham's paper "On the Solution of Problems connected with Loans repayable by Instalments" has appeared, in which he refers to the above mentioned improvement in formula (4), as due to Barrett; and points out that it is a particular case of a new and more general formula demonstrated by himself. Prof. De Morgan, who was the author of the "Account of a Correspondence between Barrett and Baily", referred to by Mr. Makeham, did not apparently think it necessary, in his subsequent paper, to investigate the principles upon which Barrett's suggestion depended, or their bearing upon formula (1), which latter has the great advantage of giving a practically correct result by one application; and the present must be considered as an attempt in that direction.

## ON THE EXPEDIENCY OF RECORDING LAW REPORTS IN THE *JOURNAL*.

*To the Editor of the Journal of the Institute of Actuaries.*

SIR,—It has often been a matter of regret that, with the other valuable matter in our *Journal*, no room has been found for some record of legal decisions, often establishing precedents which it is of much consequence we should keep in view.

As an illustration of my object, I enclose the report of a case having a most important bearing on the interests of fire insurance companies, where, as you will find, it is decided that these offices are not liable for the expenses of any fire brigade supported by local rates.

Other instances, in default of fuller reports, can always be selected and reprinted from the pages of the *Law Reporter*.

I am,
Your most obedient servant,
H. AMBROSE SMITH.

2 *King William Street, London,*
14 *May* 1874.

⁂ We shall at all times be ready to insert short accounts of legal decisions that are likely to interest our readers.—ED. *J.I.A.*

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### DRIGHLINGTON LOCAL BOARD *v.* BOWER.

On the 5th March 1872, a fire occurred in a mill insured, and the account sent in by the Drighlington Local Board for extinction expenses amounted to £42. 4*s*. 6*d*. In this account £10. 10*s*. was charged for water and £25. 4*s*. for apparatus; and as the insurance company refused to pay into Court beyond £5 for the use of such apparatus, an action was entered in the County Court of Bradford, and resulted in the judge declining to decide the question as to whether the Local Board had the right to charge for the use of the apparatus and for water supplied. The question at issue being of importance, it was determined to obtain the opinion of the Superior Court, and accordingly the case came before Lord Chief Baron Kelly, Mr. Baron Bramwell, Mr. Baron Pigott, and Mr. Baron Pollock, in the Court of Exchequer at Westminster, on the 24th November 1873, and without hearing Counsel for the company, the cause was decided in its favour.

In giving judgment the Lord Chief Baron said: "I am of opinion that the rule to enter the verdict for the defendants in this case must be made absolute." He then commented upon the Acts of Parliament empowering Local Boards to purchase apparatus for the extinguishing of fires, and to levy a rate for the maintenance of the same, and gave it as his opinion that if the Legislature had intended that Local Boards should be entitled to any remuneration for services rendered to any of the inhabitants who contributed to the rate, there would have been an express provision to that effect; but as no such provision exists, they cannot claim. He also stated that the County Court Judge ought to have directed a verdict for the defendants on the whole case.

Mr. Baron Bramwell said: "I think it about as plain a case as ever was," and considered that "the Local Board had a duty to perform towards the public, and that common property should be applied in extinguishing fires."

Mr. Baron Pigott said: "I also think it a very clear case, that no provision is made in the Acts for remuneration to be made for the extinction of fires within the districts of Local Boards, simply because it is to be done at the joint expense of the ratepayers, for the benefit of all. The Board had no power to charge."

Mr. Baron Pollock: "I agree that this action is not maintainable. It is admitted there is no express contract; and for us to say there is an implied one, would be contrary to the provisions of the Act and to principle."

## INSTITUTE OF ACTUARIES.

### PROCEEDINGS OF THE INSTITUTE.—SESSION 1873–4.

### *First Ordinary Meeting, Monday,* 24 *November* 1873.

The President in the Chair.

Read and confirmed the minutes of the anniversary meeting, held on 7 June 1873.

The following gentlemen were elected Associates, namely:—

Sidney Ashmore.
George Richard Browne.
George Kilson Edwards.
Uriah Woodard Gage.
Charles James Lucas.
Arthur Larke.
John Manley.
Frederic George McCutcheon.
William Perrin.
Charles Edward Miller.
William Fowden Simpson.
Francis William White.
William Joseph Hutchings Whittall.
Charles Windett.

Mr. R. P. Hardy read a paper by Mr. T. B. Sprague, M.A., "On the Apportionment (or Division by Mutual Consent) of a Fund between the Life Tenant and the Reversioner."

Mr. Bunyon also read a paper "On the Legal Incidents affecting the Duration of Base Fees in the Hands of Purchasers."

Thanks having been voted to Mr. Sprague and Mr. Bunyon, the meeting adjourned to Monday, 29 December 1873.

### *Second Ordinary Meeting, Monday,* 29 *December* 1873.

The President in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

The following gentlemen were elected, namely:—

*Fellow.*

(Upon the nomination of the Council) J. D. Lancaster.

*Associates.*

Benjamin Blenkinsop.
Robert John Block.
Gerald Inglis Boon.
Willis Browne.
Harry Pinnington Hallows.
James Lamb Napier Hay.
Harry Osborne Ince.
Malcolm Waterston.
Alexander Hunter Young.

Mr. William Sutton, B.A., read a paper "On the Method used by Dr. Price in the Construction of the Northampton Mortality Table."

Thanks having been voted to Mr. Sutton, the meeting adjourned to Monday, 26 January 1874.

---

### *Third Ordinary Meeting, Monday,* 26 *January* 1874.

The President in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

The following gentlemen were elected Associates, namely:—

Charles Louis Aubè.
John Smith Eland.
Edgar Kember.
Pieter Schleiger.

The following was announced to be the result of the Examinations for 1873:—

#### FIRST YEAR'S EXAMINATION.

Sixteen gentlemen presented themselves for this Examination, and ten passed in the following order of merit:—

1. F. W. White.
2. C. Windett.
3. A. D. Clarke.
4. J. Whitcher.
5. E. Woods.
6. A. Larke.
7. W. J. H. Whittall.

Eq. { D. Brockelbank.
Eq. { H. O. Ince.

10. R. J. Block.

#### SECOND YEAR'S EXAMINATION.

Twelve gentlemen presented themselves for this Examination, and seven passed in the following order of merit:—

1. W. Evans.
2. T. H. Cooke.

Eq. { A. B. Adlard.
Eq. { J. Sorley.

5. E. B. Trew.
6. R. C. Tucker.
7. S. Hunter.

#### THIRD YEAR'S EXAMINATION.

Two gentlemen presented themselves for this Examination, and both passed, being bracketed equal, namely:—

Eq. { C. D. Higham.
Eq. { G. W. King.

Mr. A. J. Finlaison read a paper "On the Rate of Mortality found to prevail among Residents in India, being Subscribers to the Uncovenanted Service Family Pension Fund, between the years 1837–1872."

Thanks having been voted to Mr. Finlaison, the meeting adjourned to Monday, 23 February 1874.

---

### *Fourth Ordinary Meeting, Monday,* 23 *February* 1874.

The President in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

The following gentlemen were elected Associates, namely:—

| | |
|---|---|
| Francis Ernest Colenso. | Edward Goad. |

Mr. R. P. Hardy read a paper by Mr. Peter Gray "On the Arithmometer of M. Thomas (de Colmar), and its application to the Construction of Life Contingency Tables."

Thanks having been voted to Mr. Gray, the meeting adjourned to Monday, 30 March 1874.

### *Fifth Ordinary Meeting, Monday,* 30 *March* 1874.

The President in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

The following gentlemen were elected, namely:—

*Fellow.*

Henry William Smith.

*Associate.*

John Whitewright Bell.

Mr. George Humphreys, M.A., read a paper "On the Practice of the Eagle Company with regard to the Assurance of Lives classed as Unsound, and on the Rates of Mortality prevailing amongst the Lives so classed, assured during the Sixty-three Years ending 30 June 1871."

Thanks having been voted to Mr. Humphreys, the meeting adjourned to Monday, 27 April 1874.

### *Sixth Ordinary Meeting, Monday,* 27 *April* 1874.

MR. ARTHUR H. BAILEY, Vice-President, in the Chair.

Read and confirmed the minutes of the last ordinary meeting.

The following gentleman was elected an Associate, namely:—

Joseph Kennerley Rumford.

Mr. James Valentine read a paper "On a Comparison of Reserves brought out by the use of different Data in the Valuation of the Liabilities of a Life Office."

Thanks having been voted to Mr. Valentine, the meeting adjourned to Monday, 30 November 1874.

### *The Twenty-Fifth Annual General Meeting, Saturday,* 6 *June* 1874.

ROBERT TUCKER, Esq., the President, in the Chair.

Mr. R. P. Hardy (Hon. Secretary) read the notice convening the meeting, the minutes of the last ordinary meeting, which were signed by the President, and the following Report and Statement of Accounts:—

"The Council have pleasure in reporting that the progress of the Institute has been steadily maintained during the past year.

"The total number of members on the books at their close on the 31st

March last was 323. The annual subscriptions amounted to £651. This sum shows an increase of £28. 7*s*. over the previous year. The new members elected during the year were 5 Fellows and 30 Associates. The accounts submitted are duly audited by two of the three Auditors, Mr. Young having resigned before the close of the year. The income from all sources was £735. 8*s*. 1*d*.; the total expenditure £483. 9*s*. 7*d*. The balance of £251. 18*s*. 6*d*. raises the total amount of funds from £1,880. 1*s*. 2*d*. at the beginning of the year to £2,131. 19*s*. 8*d*. at its close, subject to any outstanding accounts.

"The Council have been pleased to be able to arrange a modification in both the amount and mode of payment of the fees for the Institute Examinations. Formerly, a single fee of five guineas was required from a member on receiving his certificate of competency; but one guinea is now paid on his passing each of the three Examinations. The sum of £19. 19*s*. received in the past year approximates very closely to the actual expenses of the examinations, whilst the Institute pays thirty guineas by way of fee to the Tutor appointed to assist students of the Second Year's subjects. The Council continue of opinion that such outlay has been of great service to the members for whose benefit it was incurred, and they propose to continue the class for the present year.

"The Council regret to have to record the death of Mr. John Laurence, who for many years held the honourable office of Treasurer to the Institute. It is also with every feeling of regret that the Council have to refer to the death of Mons. Adolphe Quetelet, an honorary member of the Institute, whose attainments in branches of science closely allied to the pursuits of this Institute were of European reputation.

The following seven papers have been read during the past session:—

24 *Nov.* 1873.—'On the Apportionment (or Division by Mutual Consent) of a Fund between the Life Tenant and the Reversioner.' By Mr. Thomas Bond Sprague, M.A.

'On the Legal Incidents affecting the Duration of Base Fees in the Hands of Purchasers.' By Mr. Charles John Bunyon, M.A.

29 *Dec.* 1873.—'On the Method used by Dr. Price in the Construction of the Northampton Mortality Table.' By Mr. William Sutton, B.A.

26 *Jan.* 1874.—'On the Rate of Mortality found to prevail among Residents in India, being Subscribers to the Uncovenanted Service Family Pension Fund, between the years 1837–1872.' By Mr. A. J. Finlaison.

23 *Feb.* 1874.—'On the Arithmometer of M. Thomas (de Colmar), and its application to the Construction of Life Contingency Tables.' By Mr. Peter Gray.

30 *Mar.* 1874.—'On the Practice of the Eagle Company with regard to the Assurance of Lives classed as Unsound, and on the Rates of Mortality prevailing amongst the Lives so classed, assured during the Sixty-three Years ending 30 June 1871.' By Mr. George Humphreys, M.A.

27 *April* 1874.—'On a Comparison of Reserves brought out by the use of different Data in the Valuation of the Liabilities of a Life Office.' By Mr. James Valentine.

"The Council congratulate the members upon the valuable nature of these communications, four of which were by fresh contributors.

"The Council have to report that the tenancy of the rooms, at present occupied jointly by the Institute and the Statistical Society, expires at

Michaelmas, and that the rooms will then be required by the London Library for their own use. It will therefore be necessary to make fresh arrangements for the future. The Council are taking steps to secure suitable premises in a locality generally convenient to the members, and they hope to be in a position to announce a satisfactory arrangement before the commencement of the coming Session."

The PRESIDENT said—"Gentlemen, I have to move 'That the report of the Council, the abstract of income and expenditure, and the balance sheet, be adopted, entered on the minutes, and printed in the *Journal*.' You will have noticed that the income exceeds the outgo by £251. 18*s*. 6*d*., and that the total funds have risen from £1,880. 1*s*. 2*d*. to £2,131. 19*s*. 8*d*. In a financial point of view this is no doubt highly satisfactory; but it is no part of the objects of the Institute, nor indeed its duty, to accumulate funds beyond our requirements. The Council would have had to consider how the surplus income could have been applied for the benefit of the members, but for a circumstance that I shall presently advert to, and which is alluded to in the last paragraph of the report. During the last year we have had to regret the loss of an old friend in the person of our late Treasurer, Mr. John Laurence. He was one of the original members of the Institute; and although not a young man when the association was formed, he cheerfully cast in his lot with us, and was always a cordial and warm supporter of the Institute. By the death of M. Quetelet a name will disappear from the list of the honorary members that we must all have been proud to see there. In common with the rest of the world we regret the loss of a distinguished man and a charming writer. The papers read during the last Session add to the value of our transactions. The first, by Mr. Sprague, is probably more useful to actuaries in their professional capacity than for office purposes. So also is the second, by Mr. Sutton, on the Northampton Table, which is interesting as showing for the first time how this once famous mortality table was constructed. The paper by Mr. Finlaison, a grandson of our first President, will always be useful for reference when the mortality of India is under consideration. In Mr. Peter Gray we welcomed an old contributor, and listened with pleasure to his paper on a subject which he has made peculiarly his own. A machine for calculating life contingency tables, if recommended by Mr. Gray, will well deserve the study of all who are engaged, or are likely to be engaged, in such computations. The mortality of unsound lives, brought under our notice by Mr. Humphreys, will always be useful to those who are engaged in insurance business; while the last and by no means the least important paper, by Mr. Valentine, treats of a subject of very considerable interest, and I regret that absence from London on the evening when it was read prevented my being present. I shall, however, have the opportunity of referring to it in the pages of the *Journal*. The last paragraph in the report tells you that our tenancy of these premises will expire at Michaelmas next, as the London Library want the use of the rooms for their own purposes. Up to the present time we have not been able to find suitable accommodation. We desire to get placed in a more central situation, where the members can have easy access to the library; and we shall endeavour to make arrangements in that direction by the 29 September. Of course it is impossible for us to say whether we shall have to incur any additional expense. We hope that it may not be materially increased, but, as you are aware, everywhere rents do not get lower. It is for that reason that the Council are unable to submit for your consideration any suggestion as to the application of the surplus income of the Institute, but they hope to be in a position to do so next year. I will not detain the meeting with any further observations. If any gentleman will second this proposition, I shall then be happy to answer any question which the members may desire to ask with reference to the report and accounts."

Mr. A. H. Bailey seconded the motion.

Mr. Newbatt wished to inquire whether the *Journal of the Institute* had been abandoned; and, if not, why the numbers for January and April had not appeared as usual. He thought it extraordinary that the January number, which, for several reasons, was of especial interest to himself, should not have reached all the members before the meeting.

The President stated that the January number was out, and he hoped that the one for April would soon follow. After he had satisfactorily explained that the delay in the delivery to the members was quite unintentional, the matter dropped, and the motion for the adoption of the report was carried unanimously.

A ballot was then taken for the election of officers for the ensuing year; and the following gentlemen were elected:—

### *President.*

Robert Tucker.

### *Vice-Presidents.*

Charles John Bunyon, M.A.
Archibald Day.
Henry William Porter, B.A.
T. B. Sprague, M.A.

### *Council.*

Marcus N. Adler, M.A.
Andrew Baden.
Arthur H. Bailey.
George William Berridge.
Samuel Brown.
Charles John Bunyon, M.A.
*David Chisholm.
*Frank Allan Curtis.
George Cutcliffe.
Archibald Day.
Joseph John Dymond.
Alexander Pearson Fletcher.
*John Ralph Grimes.
Major-Gen. J. C. Hannyngton.
Ralph Price Hardy.
Stewart Helder.
Augustus Hendriks.
*George Humphreys, M.A.
William Barwick Hodge.
Charles Jellicoe.
James Meikle.
Edward A. Newton, M.A.
William P. Pattison.
Arthur Pearson.
Henry William Porter, B.A.
Henry Ambrose Smith.
Thomas Bond Sprague, M.A.
William Smith, LL.D.
Robert Tucker.
John Hill Williams.

### *Treasurer.*

George Cutcliffe.

### *Honorary Secretaries.*

Ralph Price Hardy.
Edward A. Newton, M.A.

Mr. Grimes proposed a cordial vote of thanks to the President and Council, which was carried unanimously.

The President having thanked the members for this mark of their confidence, the meeting was closed.

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* Those marked with an asterisk are new members.

## INSTITUTE OF ACTUARIES.

### Dr. *Income and Expenditure for the Year ending* 31 *March* 1874. Cr.

| 1873-4. | £ s. d. | £ s. d. |
|---|---|---|
| Amount of Funds on 31 March 1873, viz.:— | | |
| Messenger Legacy Fund | 251 9 7 | |
| Hardy Memorial ,, | 218 5 11 | |
| Brown Prize ,, | 224 12 6 | |
| General ,, | 1185 13 2 | |
| | | 1880 1 2 |
| Annual Subscriptions, viz.:— | | |
| Town Fellows | 226 16 0 | |
| Country ,, | 67 4 0 | |
| Town Associates | 268 16 0 | |
| Country ,, | 87 3 0 | |
| ,, ,, (in advance) | 1 1 0 | |
| | | 651 0 0 |
| Mortality Experience Sales | | 7 15 6 |
| Examination Fees | | 19 19 0 |
| Dividend on Messenger Legacy Fund (£211. 1*s.* 10*d.* Cons.) | 6 5 1 | |
| ,, Hardy Memorial ,, (£200 Consols) | 5 18 6 | |
| ,, Brown Prize ,, (£95. 15*s.* 5*d.* 10½ per-cent East India Stock) | 9 18 6 | |
| ,, £688. 18*s.* 2*d.* Consols | 20 8 3 | |
| Interest on Deposit Account | 14 3 3 | |
| | | 56 13 7 |
| | | £2615 9 3 |

| 1873-4. | £ s. d. | £ s. d. |
|---|---|---|
| Rent | 100 0 0 | |
| Salaries | 100 14 10 | |
| Journal | 110 9 10 | |
| Stationery and Printing | 35 17 2 | |
| Ordinary Meetings | 19 5 3 | |
| House Expenses | 35 0 3 | |
| Postage, &c. | 15 18 10 | |
| Miscellaneous Petty Expenses | 6 11 1 | |
| Library | 10 5 4 | |
| Tutor | 31 10 0 | |
| Examination Expenses | 15 15 0 | |
| Subscription (T. A.) irrecoverable | 2 2 0 | |
| | | 483 9 7 |
| Amount of Funds, 31 March 1874, viz.:— | | |
| Messenger Legacy Fund | 257 14 8 | |
| Hardy Memorial ,, | 224 4 5 | |
| Brown Prize ,, | 234 11 0 | |
| General ,, | 1415 9 7 | |
| | | 2131 19 8 |
| | | £2615 9 3 |

### *Balance Sheet,* 31 *March* 1874.

| | £ s. d. | £ s. d. |
|---|---|---|
| Messenger Legacy Fund, viz.:— | | |
| £211. 1*s.* 10*d.* Consols, cost | 203 17 8 | |
| Unappropriated Dividends | 53 17 0 | |
| | | 257 14 8 |
| Hardy Memorial Fund, viz.:— | | |
| £200 Consols, cost | 179 14 6 | |
| Unappropriated Dividends | 44 9 11 | |
| | | 224 4 5 |
| Brown Prize Fund, viz.:— | | |
| £95. 15*s.* 5*d.*, 10½ per-cent East India Stock, cost | 200 0 0 | |
| Unappropriated Dividends | 34 11 0 | |
| | | 234 11 0 |
| General Fund | | 1415 9 7 |
| | | £2131 19 8 |

| | £ s. d. | £ s. d. |
|---|---|---|
| Union Bank Deposit Account | | 500 0 0 |
| Three per-cent Consols (£1100) | | 1008 19 3 |
| East India 10½ per-cent Stock (£95. 15*s.* 5*d.*) | | 200 0 0 |
| Cash, viz.:— | | |
| London and Westminster Bank | 398 1 1 | |
| Petty Cash | 3 19 4 | |
| | | 402 0 5 |
| Arrears of Subscriptions | | 21 0 0 |
| | | £2131 19 8 |

*Examined and found correct.*

(Signed) { THOS. J. SEARLE. / CHAS. D. HIGHAM. } *Auditors.*

12 *May* 1874.

JOURNAL

OF THE

INSTITUTE OF ACTUARIES

AND

ASSURANCE MAGAZINE.

## *On Lubbock's Formula for Approximating to the Value ov a Life Annuity. By* T. B. Sprague, M.A., *Manager ov the Scottish Equitable Life Assurance Society, and a Vice-President ov the Institute ov Actuaries.*

THE method proposed by the late Sir J. W. Lubbock ov approximating to the value ov a life annuity, appears hitherto to hav been very littl notist by mathematicians and not at all by practical men. It was demonstrated by him in the *Cambridge Philosophical Transactions* for the year 1829, in a paper entitld "On the Comparison of various Tables of Annuities"; and a short account ov it is also givn in the articl *Probability,* publisht by the Useful Knowledge Society, and very offen bound up with David Jones's work on *Annuities*: see § 56, p. 36. The former paper was reprinted in the *Assurance Magazine* (vol. v, p. 277). The original is disfigurd by numerous misprints, which ar all faithfully reproduced in the reprint; and when we add that very littl numerical illustration ov the application ov the method is givn, we hav probably said enuf to account for its attracting so littl attention in any quarter. Considerd as a means ov approximating to the value ov an annuity, the formula has been superseded by the simpler one givn by Mr. Woolhouse; but I believ it may still be practically useful in som cases.

I will therefore first proov the formula, and then giv som numerical illustrations ov its application.

The following is Sir J. Lubbock's demonstration, with such amplification as seems desirabl to render it more easily intelligibl.

Let $y_0, y_i, y_{2i}, \ldots . y_{ni}, y_{(n+1)i}, \ldots . y_{mni}$, be successiv values ov any variabl $y$, where $ni=1$, so that $y_0, y_{ni}, y_{2ni}, \ldots .$ $y_{mni}$, may be otherwize ritten $y_0, y_1, y_2, \ldots . y_m$. Then by one ov the fundamental propositions ov the Calculus ov Finite Differences, we hav

$$y_i=y_0+i\Delta y_0+\frac{i.i-1}{1.2}\Delta^2 y_0+\frac{i.i-1.i-2}{1.2.3}\Delta^3 y_0+\ldots .,$$

$$y_{2i}=y_0+2i\Delta y_0+\frac{2i.2i-1}{1.2}\Delta^2 y_0+\frac{2i.2i-1.2i-2}{1.2.3}\Delta^3 y_0+\ldots .,$$

$$y_{3i}=y_0+3i\Delta y_0+\frac{3i.3i-1}{1.2}\Delta^2 y_0+\frac{3i.3i-1.3i-2}{1.2.3}\Delta^3 y_0+\ldots .,$$

$$\ldots\ldots\ldots\ldots\ldots\ldots$$

$$y_{(n-1)i}=y_0+(n-1)i\Delta y_0+\frac{\overline{n-1}.i(\overline{n-1}.i-1)}{1.2}\Delta^2 y_0+\ldots ..$$

Hence, by addition,

$$\begin{aligned}
y_0+y_i+y_{2i}+\ldots .+y_{(n-1)i}=ny_0 \\
+i\{1+2+3+\ldots .+\overline{n-1}\}\Delta y_0 \\
+\frac{i}{1.2}\{1.\overline{i-1}+2.\overline{2i-1}+3.\overline{3i-1}+\ldots .+(n-1)(\overline{n-1}.i-1)\}\Delta^2 y_0 \\
+\frac{i}{1.2.3}\{1.\overline{i-1}.\overline{i-2}+2.\overline{2i-1}.\overline{2i-2}+\ldots+\overline{n-1}(\overline{n-1}.i-1)(\overline{n-1}.i-2)\}\Delta^3 \\
+\ldots .
\end{aligned}$$

Similarly, we hav

$$y_{(n+1)i}=y_{ni}+i\Delta y_{ni}+\frac{i.i-1}{1.2}\Delta^2 y_{ni}+\ldots .$$

$$y_{(n+2)i}=y_{ni}+2i\Delta y_{ni}+\frac{2i.2i-1}{1.2}\Delta^2 y_{ni}+\ldots .$$

$$\ldots\ldots\ldots\ldots\ldots$$

and adding, as before,

$$\begin{aligned}
y_{ni}+y_{(n+1)i}+y_{(n+2)i}+\ldots .+y_{(2n-1)i}=ny_{ni} \\
+i\{1+2+3+\ldots .+\overline{n-1}\}\Delta y_{ni} \\
+\frac{i}{1.2}\{1.\overline{i-1}+2.\overline{2i-1}+\ldots .\}\Delta^2 y_{ni} \\
+\ldots .
\end{aligned}$$

Proceeding in this way for each adjacent $n$ terms, and adding all the results together, we get

$$
\begin{aligned}
&+y_i+y_{2i}+\ \ldots\ldots\ +y_{ni}+y_{(n+1)i}+\ \ldots\ldots\ +y_{mni-i}\\
&\quad=n\{y_0+y_{ni}+y_{2ni}+\ \ldots\ldots\ +y_{(m-1)ni}\}\\
&\quad+i\{1+2+3+\ \ldots\ldots\ +\overline{n-1}\}\{\Delta y_0+\Delta y_{ni}+\ \ldots\ldots\ +\Delta y_{(m-1)ni}\}\\
&\quad+\frac{i.\overline{i-1}+2i.\overline{2i-1}+\ \ldots\ldots\ +\overline{n-1}i(\overline{n-1}i-1)}{1.2}\\
&\qquad\qquad\qquad\times\{\Delta^2 y_0+\Delta^2 y_{ni}+\ \ldots\ldots\ +\Delta^2 y_{(m-1)ni}\}\\
&\quad+\frac{i.\overline{i-1}.\overline{i-2}+2i.\overline{2i-1}.\overline{2i-2}+\ \ldots\ldots\ +\overline{n-1}i(\overline{n-1}.i-1)(\overline{n-1}.i-2)}{1.2.3}\\
&\qquad\qquad\qquad\times\{\Delta^3 y_0+\Delta^3 y_{ni}+\ \ldots\ldots\ +\Delta^3 y_{(m-1)ni}\}\\
&\quad+\ \ldots\ldots
\end{aligned}
$$

Now

$$
\begin{aligned}
\Delta y_0 &= y_1-y_0,\\
\Delta y_{ni} &= \Delta y_1 = y_2-y_1,\\
\Delta y_{2ni} &= \Delta y_2 = y_3-y_2,\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
\Delta y_{(m-1)ni} &= \Delta y_{m-1} = y_m-y_{m-1};
\end{aligned}
$$

and adding,

$$\Delta y_0+\Delta y_{ni}+\Delta y_{2ni}+\ \ldots\ldots\ +\Delta y_{(m-1)ni}=y_m-y_0.$$

Similarly,

$$
\begin{aligned}
&\Delta^2 y_0+\Delta^2 y_{ni}+\ \ldots\ldots\ +\Delta^2 y_{(m-1)ni}=\Delta y_m-\Delta y_0,\\
&\Delta^3 y_0+\Delta^3 y_{ni}+\ \ldots\ldots\ +\Delta^3 y_{(m-1)ni}=\Delta^2 y_m-\Delta^2 y_0,\\
&\ldots\ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

so that we get by substitution,

$$
\begin{aligned}
&+y_i+y_{2i}+\ \ldots\ldots\ +y_{mni-i}=n\{y_0+y_{ni}+y_{2ni}+\ \ldots\ldots\ +y_{(m-1)ni}\}\\
&+i\{1+2+3+\ \ldots\ldots\ +\overline{n-1}\}(y_m-y_0)\\
&+\frac{i.\overline{i-1}+2i.\overline{2i-1}+\ \ldots\ldots\ +\overline{n-1}i(\overline{n-1}i-1)}{1.2}(\Delta y_m-\Delta y_0)\\
&+\frac{i.\overline{i-1}.\overline{i-2}+2i.\overline{2i-1}.\overline{2i-2}+\ldots\ldots+\overline{n-1}.i(\overline{n-1}.i-1)(\overline{n-1}.i-2)}{1.2.3}(\Delta^2 y_m-\Delta^2 y_0)\\
&+\ \ldots\ldots\ldots\ldots\ldots
\end{aligned}
$$

The coefficients ov the varios terms ar clearly those ov the powers ov $x$ in the expansion ov

$$1+(1+x)^i+(1+x)^{2i}+\ \ldots\ldots\ +(1+x)^{(n-1)i},$$

which is equal to $\dfrac{(1+x)^{ni}-1}{(1+x)^i-1}$, or to $\dfrac{(1+x)-1}{(1+x)^i-1}=\dfrac{x}{(1+x)^i-1}$, since $ni=1$. Now it is proovd in De Morgan's *Diff. Calc.* (see p. 314, § 184), that the expansion ov the last quantity is

$$\frac{1}{i}-\frac{i-1}{2i}x+\frac{i^2-1}{12i}x^2-\frac{i^2-1}{24i}x^3+\frac{(i^2-1)(19-i^2)}{720i}x^4$$
$$-\frac{(i^2-1)(9-i^2)}{480i}x^5+\frac{(i^2-1)(863-145i^2+2i^4)}{60480i}x^6-\ldots\ldots$$

Hence, substituting and putting $\frac{1}{n}$ for $i$, we get

$$y_0+y_i+y_{2i}+\ldots\ldots+y_{m-i}=n\{y_0+y_1+y_2+\ldots\ldots+y_{m-1}\}$$
$$+\frac{n-1}{2}(y_m-y_0)-\frac{n^2-1}{12n}(\Delta y_m-\Delta y_0)+\frac{n^2-1}{24n}(\Delta^2 y_m-\Delta^2 y_0)$$
$$-\frac{(n^2-1)(19n^2-1)}{720n^3}(\Delta^3 y_m-\Delta^3 y_0)+\frac{(n^2-1)(9n^2-1)}{480n^3}(\Delta^4 y_m-\Delta^4 y_0)$$
$$-\frac{(n^2-1)(863n^4-145n^2+2)}{60480n^5}(\Delta^5 y_m-\Delta^5 y_0)+\ldots\ldots \quad (1)$$

This is the formula givn by De Morgan (p. 316), Lubbock having stopt at the term $\frac{n^2-1}{24n}(\Delta^2 y_m-\Delta^2 y_0)$.

For brevity, we will rite this formula

$$n(y_0+y_1+y_2+\ldots\ldots+y_{m-1})+\frac{n-1}{2}(y_m-y_0)$$
$$-C_1(\Delta y_m-\Delta y_0)+C_2(\Delta^2 y_m-\Delta^2 y_0)-C_3(\Delta^3 y_m-\Delta^3 y_0)$$
$$+C_4(\Delta^4 y_m-\Delta^4 y_0)-C_5(\Delta^5 y_m-\Delta^5 y_0)+\ldots\ldots \quad (2)$$

putting $C_1$, $C_2$, $C_3$ .... for the coefficients ov the several terms after the two first. It will be seen that as far as we hav gon, each ov these quantities involvs $n^2-1$ as a factor.

The formula can be much more briefly proovd by the method ov Separation ov Symbols (or Calculus ov Operations). The problem is to find a formula for $y_0+y_1+y_{2i}+\ldots\ldots+y_{mni-i}$ in terms ov $y_0+y_{ni}+y_{2ni}+\ldots\ldots+y_{(m-1)ni}$, (or $y_0+y_1+y_2+\ldots+y_{m-1}$), and the differences ov these last quantities. We hav

$$y_0+y_i+y_{2i}+\ldots\ldots+y_{mni-i}=(1+D^i+D^{2i}+\ldots\ldots+D^{mni-i})y_0$$
$$=\frac{D^{mni}-1}{D^i-1}y_0=\frac{D^m-1}{D^i-1}y_0$$
$$=\frac{1}{D^i-1}(y_m-y_0).$$

But
$$\frac{1}{D^i-1}=\frac{1}{(1+\Delta)^i-1}=\frac{1}{\Delta}\cdot\frac{\Delta}{(1+\Delta)^i-1}$$
$$=\frac{1}{\Delta}\left\{\frac{1}{i}-\frac{i-1}{2i}\Delta+\frac{i^2-1}{12i}\Delta^2-\ldots\ldots\right\}.$$

Hence the abov sum is equal to

$$\left\{\frac{1}{i\Delta}-\frac{i-1}{2i}+\frac{i^2-1}{12i}\Delta-\ldots\right\}(y_m-y_0)$$

$$=\frac{1}{i\Delta}(y_m-y_0)-\frac{i-1}{2i}(y_m-y_0)+\frac{i^2-1}{12i}(\Delta y_m-\Delta y_0)-\ldots$$

It remains to evaluate the first term ov this series. The easiest method ov doing this is to make $i=n=1$ in the formula, which givs

$$y_0+y_1+y_2+\ldots+y_{m-1}=\frac{1}{\Delta}(y_m-y_0)$$

Substituting, we hav

$$y_0+y_i+y_{2i}+\ldots+y_{mni-1}=\frac{1}{i}(y_0+y_1+\ldots+y_{m-1})$$

$$-\frac{i-1}{2i}(y_m-y_0)+\frac{i^2-1}{12i}(\Delta y_m-\Delta y_0)-\ldots,$$

and finally, putting $\frac{1}{n}$ for $i$, we get the formula (1). Prof. De Morgan calls this (p. 315, § 185) "the most difficult instance which has yet occurd ov the separation ov the symbols ov operation and quantity".

If we hav calculated (or know otherwize) only the terms $y_0, y_1, y_2 \ldots y_m$, we shall not be directly abl to calculate $\Delta y_m$, $\Delta^2 y_m$, &c., which involv $y_{m+1}$, $y_{m+2}$, &c. But the formula can be transformd so as to involv only the givn quantities. For we hav

$$\frac{1}{{}^{i}-1}=\frac{1}{(D-\Delta)^{-i}D^i-1}=\frac{1}{\left(1-\frac{\Delta}{D}\right)^{-i}-1}=\frac{D}{\Delta}\cdot\frac{\frac{\Delta}{D}}{\left(1-\frac{\Delta}{D}\right)^{-i}-1}$$

$$=\frac{D}{\Delta}\left\{\frac{1}{i}-\frac{i+1}{2i}\cdot\frac{\Delta}{D}+\frac{i^2-1}{12i}\frac{\Delta^2}{D^2}+\frac{i^2-1}{24i}\frac{\Delta^3}{D^3}+\frac{(i^2-1)(19-i^2)}{720i}\frac{\Delta^4}{D^4}+\ldots\right\}$$

Hence, $\frac{1}{D^i-1}y_m=\left\{\frac{D}{i\Delta}-\frac{i+1}{2i}+\frac{i^2-1}{12i}\frac{\Delta}{D}+\ldots\right\}y_m$

$$=\frac{1}{i\Delta}y_{m+1}-\frac{i+1}{2i}y_m+\frac{i^2-1}{12i}\Delta y_{m-1}+\frac{i^2-1}{24i}\Delta^2 y_{m-2}+\ldots$$

But we hav seen abov that,

$$-\frac{1}{D^i-1}y_0=-\frac{1}{i\Delta}y_0+\frac{i-1}{2i}y_0-\frac{i^2-1}{12i}\Delta y_0+\frac{i^2-1}{24i}\Delta^2 y_0-\ldots$$

Hence

$$y_0+y_i+y_{2i}+\ldots+y_{mni-i}=\frac{1}{D^i-1}y_m-\frac{1}{D^i-1}y_0$$

$$=\frac{1}{i}\frac{1}{\Delta}(y_{m+1}-y_0)-\frac{i+1}{2i}y_m+\frac{i-1}{2i}y_0$$

$$+\frac{i^2-1}{12i}(\Delta y_{m-1}-\Delta y_0)+\frac{i^2-1}{24i}(\Delta^2 y_{m-2}+\Delta^2 y_0)-\ldots$$

Putting $i=n=1$, we get

$$y_0+y_1+y_2+\ \ldots\ldots\ +y_{m-1}=\frac{1}{\Delta}(y_{m+1}-y_0)-y_m;$$

whence, by substitution,

$$y_0+y_i+y_{2i}+\ \ldots\ldots\ +y_{mni-i}=\frac{1}{i}(y_0+y_1+\ \ldots\ldots\ +y_m)$$
$$-\frac{i+1}{2i}y_m+\frac{i-1}{2i}y_0+\frac{i^2-1}{12i}(\Delta y_{m-1}-\Delta y_0)+\ \ldots\ldots$$

(or, putting $\frac{1}{n}$ for $i$,)

$$=n(y_0+y_1+\ \ldots\ldots\ +y_{m-1})-\frac{n-1}{2}(y_m+y_0)-C_1(\Delta y_{m-1}-\Delta y_0)$$
$$-C_2(\Delta^2 y_{m-2}+\Delta^2 y_0)-C_3(\Delta^3 y_{m-3}-\Delta^3 y_0)-C_4(\Delta^4 y_{m-4}+\Delta^4 y_0)$$
$$-C_5(\Delta^5 y_{m-5}-\Delta^5 y_0)-\ \ldots\ldots\ldots\ldots\ldots \qquad (3)$$

This form ov the equation is givn by De Morgan (p. 318, § 191), but his method ov proof is very different from the abov. The differences here involvd can all be calculated by means ov the known quantities $y_0, y_1, \ldots\ldots\ y_m$. In fact, if these quantities ar completely differenst out, $\Delta y_0$, $\Delta^2 y_0$, $\Delta^3 y_0\ \ldots\ldots$ ar the leading differences, and $\Delta y_{m-1}$, $\Delta^2 y_{m-2}$, $\Delta^3 y_{m-3}, \ldots\ldots$ the concluding ones, as shoen in the folloing sceme.

$$\begin{array}{lllll}
y_0 & & & & \\
 & \Delta y_0 & & & \\
y_1 & & \Delta^2 y_0 & & \\
 & \Delta y_1 & & \Delta^3 y_0 & \\
y_2 & & \Delta^2 y_1 & & \\
 & \Delta y_2 & & & \\
y_3 & & \vdots & \vdots & \\
\vdots & \vdots & \vdots & \vdots & \\
y_{m-3} & & & & \\
 & \Delta y_{m-3} & & & \\
y_{m-2} & & \Delta^2 y_{m-3} & & \\
 & \Delta y_{m-2} & & \Delta^3 y_{m-3} & \\
y_{m-1} & & \Delta^2 y_{m-2} & & \\
 & \Delta y_{m-1} & & & \\
y_m & & & &
\end{array}$$

Mr. Woolhouse has obtaind by a process involving only elementary algebra, a formula which is equivalent to this, but in which the expansion is continued three terms further (*J.I.A.*, xi, 309). We lern from his result that the next three terms ov the abov series ar

$$-\frac{(n^2-1)(275n^4-61n^2+2)}{24192n^5}(\Delta^6 y_{m-6}+\Delta^6 y_0)$$
$$-\frac{(n^2-1)(33953n^6-9247n^4+497n^2-3)}{3628800n^7}(\Delta^7 y_{m-7}-\Delta^7 y_0)$$
$$-\frac{(n^2-1)(8183n^6-2617n^4+197n^2-3)}{1036800n^7}(\Delta^8 y_{m-8}+\Delta^8 y_0).$$

Mr. Woolhouse has also obtaind a simpler formula for the same sum, which is equivalent to

$$n(y_0+y_1+y_2+\dots\dots+y_{m-1})+\frac{n-1}{2}(y_m-y_0)$$

$$-\frac{n^2-1}{12n}\left\{\left(\frac{dy}{dx}\right)_m-\left(\frac{dy}{dx}\right)_0\right\}+\frac{n^4-1}{720n^3}\left\{\left(\frac{d^3y}{dx^3}\right)_m-\left(\frac{d^3y}{dx^3}\right)_0\right\}-\dots\dots \quad (4)$$

See vol. xi, pp. 317 and 331; but it must be borne in mind that the meanings ov $m$ and $n$ in these passages ar different from those in which they ar uzed abov, being, in fact, interchanged.

When the formulas ar applied to whole term annuities or assurances (including in this term all such benefits as ar not temporary, that is, to cease after a fixt term ov years), we hav

$$y_m=0,\ \Delta y_m=0,\ \Delta^2 y_m=0,\dots\dots$$

$$\left(\frac{dy}{dx}\right)_m=0,\ \left(\frac{d^2y}{dx^2}\right)_m=0,\dots\dots$$

and we get the simpler formulas,

$$(y_0+y_1+y_2+\dots\dots+y_{m-1})-\frac{n-1}{2}y_0+\frac{n^2-1}{12n}\Delta y_0-C_2\Delta^2 y_0+C_3\Delta^3 y_0-\dots\dots \quad (5)$$

nd

$$(y_0+y_1+y_2+\dots\dots+y_{m-1})-\frac{n-1}{2}y_0+\frac{n^2-1}{12n}\left(\frac{dy}{dx}\right)_0-\frac{n^4-1}{720n^3}\left(\frac{d^3y}{dx^3}\right)_0+\dots\dots \quad (6)$$

Mr. Woolhouse has shoen that in the case ov ordinary annuities on one or more lives the terms after the three first in (6) may be safely neglected as insignificant, and that the value ov $\left(\frac{dy}{dx}\right)_0$ is easily found, being equal to $-(\mu_0+\delta)$. But when we ar dealing with other functions, it may easily happen that the only means ov finding the value ov the diffco is to deduce it from those ov the differences; and in this case it will be better to employ Lubbock's formula.

The formula for the values ov the diffcos in terms ov the differences can be obtaind as follos.

Since $D=\epsilon^{\frac{d}{dx}}$, we hav $\frac{d}{dx}=\log_\epsilon D=\log_\epsilon(1+\Delta)$

$$=\Delta-\frac{\Delta^2}{2}+\frac{\Delta^3}{3}-\frac{\Delta^4}{4}+\dots\dots;$$

and in general

$$\frac{d^k}{dx^k}=\Delta^k\left(1-\frac{\Delta}{2}+\frac{\Delta^2}{3}-\dots\right)^k$$

$$=\Delta^k\left\{1-\frac{k}{2}\Delta+\frac{k(3k+5)}{24}\Delta^2\dots\right\}.$$

See De Morgan, *Diff. Calc.*, p. 264, § 65.

Hence

$$\frac{d^k}{dx^k}y_0=\Delta^k y_0-\frac{k}{2}\Delta^{k+1}y_0+\frac{k(3k+5)}{24}\Delta^{k+2}y_0-\dots$$

Substituting these values in Woolhouse's formulas, (4) and (6), we shoud arrive at Lubbock's, (1) and (5). Thus, taking only two terms ov the former, we hav

$$\frac{n^2-1}{12n^2}\cdot\frac{dy}{dx}-\frac{n^4-1}{720n^4}\cdot\frac{d^3y}{dx^3}=\frac{n^2-1}{12n^2}\left(\Delta y-\tfrac{1}{2}\Delta^2y+\tfrac{1}{3}\Delta^3y-\tfrac{1}{4}\Delta^4y+\dots\right)$$

$$-\frac{n^4-1}{720n^4}\left(\Delta^3y-\tfrac{3}{2}\Delta^4y+\dots\right)$$

$$=\frac{n^2-1}{12n^2}\Delta y-\frac{n^2-1}{24n^2}\Delta^2y+\frac{(n^2-1)(19n^2-1)}{720n^4}\Delta^3y-\frac{(n^2-1)(9n^2-1)}{480n^4}\Delta^4y+\dots$$

This process is interesting as shoing us how it arises that $n^2-1$ is a factor ov all the C's.

The preceding demonstrations ar, from the necessity ov the case, addrest exclusivly to those who may be calld fair mathematicians, ov whom I believ ther ar an increasing number to be found in the Institute; and I shall be pleasd if any ov them ar led hereby to giv more attention to the method ov separation ov symbols, which will offen be found useful in transforming formulas ov summation and interpolation. Instances ov its use will be found in the 15th vol., p. 307, where it has been applyd by Mr. Sutton to demonstrate Woolhouse's formula (6) directly, as well as to deduce it from Lubbock's, (5).

The following tabl has been calculated by Mr. J. J. McLauchlan, to facilitate the practical application ov Lubbock's formula. It contains the values ov $C_1$, $C_2$, $C_3$, $C_4$, $C_5$, $C_6$, with their logarithms, for values ov $n$ from 2 to 11, inclusiv.

*Tabl ov the Numerical Values ov the coefficients required in the application ov Lubbock's Formula for Summation.*

| $n$ | $\frac{n^2-1}{12n}$ | | | $\frac{n^2-1}{24n}$ | | | $\frac{(n^2-1)(19n^2-1)}{720n^3}$ | | $\frac{(n^2-1)(9n^2-1)}{480n^3}$ | | $\frac{(n^2-1)(863n^4-145n^2+2)}{60480n^5}$ | | $\frac{(n^2-1)(275n^4-61n^2+2)}{24192n^5}$ | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $C_1$ | Log $C_1$ | | $C_2$ | Log $C_2$ | $C_3$ | Log $C_3$ | $C_4$ | Log $C_4$ | $C_5$ | Log $C_5$ | $C_6$ | Log $C_6$ |
| 2 | $\frac{1}{8}$ | ·125000 | $\bar{1}$·096910 | $\frac{1}{16}$ | ·062500 | $\bar{2}$·795880 | ·039063 | $\bar{2}$·591760 | ·027344 | $\bar{2}$·436858 | ·020508 | $\bar{2}$·311919 | ·016113 | $\bar{2}$·207184 |
| 3 | $\frac{2}{9}$ | ·222222 | $\bar{1}$·346787 | $\frac{1}{9}$ | ·111111 | $\bar{1}$·045757 | ·069959 | $\bar{2}$·844843 | ·049383 | $\bar{2}$·693575 | ·037342 | $\bar{2}$·572196 | ·029569 | $\bar{2}$·470832 |
| 4 | $\frac{5}{16}$ | ·312500 | $\bar{1}$·494850 | $\frac{5}{32}$ | ·156250 | $\bar{1}$·193820 | ·098633 | $\bar{2}$·994021 | ·069824 | $\bar{2}$·844006 | ·052948 | $\bar{2}$·723849 | ·042038 | $\bar{2}$·623642 |
| 5 | $\frac{2}{5}$ | ·400000 | $\bar{1}$·602060 | $\frac{1}{5}$ | ·200000 | $\bar{1}$·301030 | ·126400 | $\bar{1}$·101747 | ·089600 | $\bar{2}$·952308 | ·068032 | $\bar{2}$·832713 | ·054080 | $\bar{2}$·733037 |
| 6 | $\frac{35}{72}$ | ·486111 | $\bar{1}$·686736 | $\frac{35}{144}$ | ·243056 | $\bar{1}$·385706 | ·153710 | $\bar{1}$·186703 | ·109037 | $\bar{1}$·037576 | ·082849 | $\bar{2}$·918285 | ·065902 | $\bar{2}$·818896 |
| 7 | $\frac{4}{7}$ | ·571429 | $\bar{1}$·756962 | $\frac{2}{7}$ | ·285714 | $\bar{1}$·455932 | ·180758 | $\bar{1}$·257098 | ·128280 | $\bar{1}$·108159 | ·097510 | $\bar{2}$·989051 | ·077595 | $\bar{2}$·889835 |
| 8 | $\frac{21}{32}$ | ·656250 | $\bar{1}$·817069 | $\frac{21}{64}$ | ·328125 | $\bar{1}$·516039 | ·207642 | $\bar{1}$·317314 | ·147400 | $\bar{1}$·168497 | ·112075 | $\bar{1}$·049508 | ·089208 | $\bar{2}$·950404 |
| 9 | $\frac{20}{27}$ | ·740741 | $\bar{1}$·869667 | $\frac{10}{27}$ | ·370370 | $\bar{1}$·568637 | ·234416 | $\bar{1}$·369986 | ·166438 | $\bar{1}$·221253 | ·126574 | $\bar{1}$·102345 | ·100767 | $\bar{1}$·003318 |
| 10 | $\frac{33}{40}$ | ·825000 | $\bar{1}$·916454 | $\frac{33}{80}$ | ·412500 | $\bar{1}$·615424 | ·261113 | $\bar{1}$·416828 | ·185419 | $\bar{1}$·268154 | ·141028 | $\bar{1}$·149304 | ·112288 | $\bar{1}$·050332 |
| 11 | $\frac{10}{11}$ | ·909091 | $\bar{1}$·958607 | $\frac{5}{11}$ | ·454545 | $\bar{1}$·657577 | ·287754 | $\bar{1}$·459021 | ·204358 | $\bar{1}$·310391 | ·155448 | $\bar{1}$·191584 | ·123781 | $\bar{1}$·092653 |

As an exampl I will now apply the formula (5) to calculate the value ov an annuity on a nominee ov 40, according to the English Tabl, No. III, Males, at 3 per-cent interest.

It will first, however, be desirabl to put the formula into a more convenient working shape. Thus, let $x$ be the age ov the nominee, and let $V_k$ denote the value ov the annuity payment to be receivd at the age $x+k$. Also, let $\Delta_1$, $\Delta_2$, $\Delta_3$, .... be the leading differences ov $V_0$, $V_n$, $V_{2n}$, ..... Then $V_0=1$, and the value ov the annuity on $x$ becoms

$$a_x=n(V_0+V_n+V_{2n}+\ldots\ldots)-\frac{n+1}{2}+C_1\Delta_1-C_2\Delta_2+C_3\Delta_3-\ldots\ldots \quad (7)$$

where unity has been subtracted, becaus (5) woud giv the value ov an annuity-due.

First taking $n=7$, we find

$$\begin{array}{lllll}
V_0=1\cdot0000 & & & & \\
 & -\cdot2654 & & & \\
V_7=\ \cdot7346 & & +\cdot0521 & & \\
 & \ \ \cdot2133 & & -\cdot0115 & \\
V_{14}=\ \cdot5213 & & \ \ \cdot0406 & & +\cdot0029 \\
 & \ \ \cdot1727 & & \ \ \cdot0086 & \\
V_{21}=\ \cdot3486 & & \ \ \cdot0320 & & \\
 & \ \ \cdot1407 & & & \\
V_{28}=\ \cdot2079 & & & & \\
V_{35}=\ \cdot0990 & & & & \\
V_{42}=\ \cdot0318 & & & & \\
V_{49}=\ \cdot0055 & & & & \\
V_{56}=\ \cdot0004 & & & & \\
\hline
\text{Sum}=2\cdot9491 & & & &
\end{array}$$

Hence $\Delta_1=-\cdot2654$, $\Delta_2=\cdot0521$, $\Delta_3=-\cdot0115$, $\Delta_4=\cdot0029$; and taking the values ov the coefficients from the abov tabl, the value ov the annuity is approximatly

$$\begin{aligned}
7\times2\cdot9491-4-\frac{4}{7}\times\cdot2654-\frac{2}{7}\times\cdot0521-\cdot1808\times\cdot0115-\cdot1283\times\cdot0029 & \\
=20\cdot6437-4\cdot0000 & \\
-\ \ \cdot1517 & \\
-\ \ \cdot0149 & \\
-\ \ \cdot0021 & \\
-\ \ \cdot0004 & \\
=20\cdot6437-4\cdot1691 & \\
=16\cdot4746. &
\end{aligned}$$

Next, taking $n=11$, we hav

$$\begin{array}{lllll}
V_0=1\cdot0000 & & & & \\
 & -\cdot3924 & & & \\
V_{11}=\ \cdot6076 & & +\cdot1115 & & \\
 & \ \ \cdot2809 & & -\cdot0310 & \\
V_{22}=\ \cdot3267 & & \ \ \cdot0805 & & +\cdot0453 \\
 & \ \ \cdot2004 & & +\cdot0143 & \\
V_{33}=\ \cdot1263 & & \ \ \cdot0948 & & \\
 & \ \ \cdot1056 & & & \\
V_{44}=\ \cdot0207 & & & & \\
V_{55}=\ \cdot0006 & & & & \\
\hline
\quad\ 2\cdot0819 & & & &
\end{array}$$

Hence the value ov the annuity is approximatly

$$11 \times 2{\cdot}0819 - 6 - \frac{10}{11} \times {\cdot}3924 - \frac{5}{11} \times {\cdot}1115 - {\cdot}2878 \times {\cdot}0310 - {\cdot}2044 \times {\cdot}0453$$

$$\begin{aligned} &= 22{\cdot}9009 - 6{\cdot}0000 \\ &\qquad\qquad - {\cdot}3567 \\ &\qquad\qquad - {\cdot}0507 \\ &\qquad\qquad - {\cdot}0089 \\ &\qquad\qquad - {\cdot}0093 \\ &= 22{\cdot}9009 - 6{\cdot}4256 \\ &= 16{\cdot}4753. \end{aligned}$$

The value ov the annuity calculated in the ordinary way is 16·4744.

Woolhouse's formula (6), being similarly delt with, becoms (see *Jurnl,* xi, 320)

$$n(V_0 + V_n + V_{2n} + \ldots.) - \frac{n+1}{2} - \frac{n^2-1}{12}(\mu + \delta) \quad . \quad . \quad . \quad (8)$$

Here $\mu_{40} = \frac{3465 + 3529}{544146} = {\cdot}01285$

Also $\delta = {\cdot}02956$

therefore $\mu_{40} + \delta = {\cdot}04241$

Making $n = 7$, we hav the value ov the annuity

$$\begin{aligned} &= 16{\cdot}6437 - 4 \times {\cdot}04241 \\ &= 16{\cdot}6437 - {\cdot}1696 \\ &= 16{\cdot}4741. \end{aligned}$$

Making $n = 11$, we hav the value

$$\begin{aligned} &= 16{\cdot}9009 - 10 \times {\cdot}04241 \\ &= 16{\cdot}9009 - {\cdot}4241 \\ &= 16{\cdot}4768. \end{aligned}$$

Comparing the two processes, we see that when we hav the values ov $\mu$ and $\delta$ alreddy computed, Woolhouse's is decidedly the shorter. On the other hand, it is easy to see that Lubbock's formula applies, not only to annuities, but to other benefits; and that it will be applicabl to find the values ov such quantities as contingent annuities, the value ov which cannot be found exactly except by a very long series ov calculations (see Davies, p. 354).

I will next sho how the formula (3) may be applyd to find the value ov a temporary annuity—say $a_{x.\overline{mn}|}$. Making the same substitutions as before, and observing that, since (3) would giv the value ov an annuity-due, we must subtract $V_0$ (or 1) and add $V_{mn}$;

also, denoting the concluding differences ov $V_0, V_n, V_{2n}, \ldots . V_{mn}$, by $\Delta'_1, \Delta'_2, \Delta'_3 \ldots .$, we get

$$a_{x.\overline{mn}|} = n(V_0+V_n+V_{2n}+ \ldots . +V_{mn}) - 1 - \frac{n-1}{2}(V_0+V_{mn})$$
$$+C_1(\Delta_1-\Delta'_1)-C_2(\Delta_2+\Delta'_2)+C_3(\Delta_3-\Delta'_3)-C_4(\Delta_4+\Delta'_4)+ \ldots . \quad (9)$$

If in the calculation we stop at an even order ov differences, arriving, for instance, at a singl difference $\Delta_{2k}$ ov the order $2k$, we must consider that $\Delta'_{2k}=\Delta_{2k}$. In illustration ov this point I will first apply the formula to calculate the sum ov the 4th powers ov the natural numbers from 1 to 29. We hav, by processes which need no explanation, the folloing values and differences:—

| | | | | | |
|---|---|---|---|---|---|
| $1^4$ | 1 | | | | |
| | | 4,095 | | | |
| $8^4=$ | 4,096 | | 42,434 | | |
| | | 46,529 | | 94,668 | |
| $15^4=$ | 50,625 | | 137,102 | | 57,624 |
| | | 183,631 | | 152,292 | |
| $22^4=$ | 234,256 | | 289,394 | | |
| | | 473,025 | | | |
| $29^4=$ | 707,281 | | | | |
| Sum= | 996,259 | | | | |

Here $n=7$, $\Delta_1=4{,}095$, $\Delta'_1=473{,}025$, &c.; and the sum ov the series $1^4+2^4+3^4+ \ldots . +29^4$, got by substitution in (9), omitting the term $-1$, is,

$$\begin{aligned}
&7\times 996259-3(1+707281)+\frac{4}{7}(4095-473025)\\
&-\frac{2}{7}(42434-289394)+\frac{62}{343}(94668-152{,}292)\\
&-\frac{44}{343}(57624+57624)\\
&=6{,}973{,}813-2{,}121{,}846\\
&\qquad\qquad\;\;-\;\;267{,}960\\
&\qquad\qquad\;\;-\;\;\;\;94{,}808\\
&\qquad\qquad\;\;-\;\;\;\;10{,}416\\
&\qquad\qquad\;\;-\;\;\;\;14{,}784\\
&=6{,}973{,}813-2{,}509{,}814\\
&=4{,}463{,}999.
\end{aligned}$$

Mr. Woolhouse has givn (*J. I. A.* xi, 303) a formula by means ov which the sum ov the series can be found directly. It is equal to $\frac{6\times 29^5+15\times 29^4+10\times 29^3-29}{30}$, which givs 4,463,999, as abov.

If, now, we wish to apply the formula (9) to find the value ov the temporary annuity, $a_{40\,\overline{28}|}$, by the same tabl as before, turning to the figurs on page 314, we hav $V_0+V_7+V_{14}+V_{21}+V_{28}=2{\cdot}8124$; $n=7$, $\Delta_1=-{\cdot}2654$, $\Delta'_1=-{\cdot}1407$, &c., and the value ov the annuity is approximatly

$$\begin{aligned}7\times 2{\cdot}8124-1-3(1+{\cdot}2079)-\frac{4}{7}({\cdot}2654-{\cdot}1407)&\\ -\frac{2}{7}({\cdot}0521+{\cdot}0320)-{\cdot}1808({\cdot}0115-{\cdot}0086)-{\cdot}1283({\cdot}0029+{\cdot}0029)&\\ =19{\cdot}6868-4{\cdot}6237&\\ -\ {\cdot}0713&\\ -\ {\cdot}0240&\\ -\ {\cdot}0005&\\ -\ {\cdot}0007&\\ =19{\cdot}6868-4{\cdot}7202&\\ =14{\cdot}9666.&\end{aligned}$$

The value calculated in the ordinary way is 14·9659.

Lastly, if we apply Woolhouse's formula to find the value ov the same annuity, we hav

$$a_{x.\overline{mn}|}=n(V_0+V_n+V_{2n}+\ \ldots\ldots\ +V_{mn})-1-\frac{n-1}{2}(V_0+V_{mn})$$
$$-\frac{n^2-1}{12}\{\mu_x+\delta-V_{mn}(\mu_{x+mn}+\delta)\}.$$

Taking the same exampl as before, we get,

$$\begin{aligned}a_{40\,\overline{28}|}&=7\times 2{\cdot}8124-1-3(1+{\cdot}2079)\\ &\quad -4\{{\cdot}04241-{\cdot}2079({\cdot}05700+{\cdot}02956)\}\\ &=14{\cdot}9655.\end{aligned}$$

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## *On an application of the Theory of the Composition of Decremental Forces. By* W. M. Makeham, F.I.A.

THE determination of the probable effect, upon the increase of population, of the extinction of small pox, is a problem which has been discussed at some length by three of the great continental mathematicians of the last century, namely, D. Bernouilli, D'Alembert, and Laplace. I propose, first, to solve the problem by what may be termed the theory of the composition of decremental forces, and then to reproduce (by way of comparison) from Mr. Todhunter's *History of the Theory of Probabilities,* the solutions given by the three eminent mathematicians in question.

The law of the composition of decremental forces may be thus stated:—

1. Let $L_x$ denote the successive values of any given function

of the variable $x$, then the decremental force of the series will be represented by the expression $-\frac{1}{L_x}\cdot\frac{dL_x}{dx}$.

2. If $F'_x$ denote the decremental force of a given series $L'_x$, and $F''_x$ that of any other given series $L''_x$, then $F_x$, the decremental force of a series, $L_x$, subject to the combined simultaneous action of the two decremental forces $F'_x$ and $F''_x$, is equal to the sum of the two last mentioned decremental forces. That is to say, $F_x = F'_x + F''_x$.*

3. From the last equation it follows that $L_x = L'_x L''_x$, assuming that to the radix of each series (namely, $L_0$, $L'_0$, and $L''_0$,) may be given any value we please, by multiplying each term of the series by a constant.*

Let $L'_x$ denote the number of persons surviving at age $x$ out of a given number, $L'_0$, born, taking mortality caused by small pox only into account; and similarly let $L''_x$ denote the same with respect to all other causes. Then $L_x$, the number surviving in a body subject to *all* causes of mortality, is equal to $L'_x\ L''_x$. Hence if we form, from hypothesis or observation, the two series denoted by $L_x$ and $L'_x$ respectively, we may determine the series $L''_x$ (which represents the effect of the extinction of small pox) by the equation $L''_x = \frac{L_x}{L'_x}$.

D'Alembert's statement and solution of the problem above referred to are as follows:—Suppose a large number of infants born nearly at the same epoch; let $y$ represent the number alive at a certain time; let $u$ represent the number who have died (during the period) of small pox; let $z$ represent the number who would have been alive if small pox did not exist: required $z$ in terms of $y$ and $u$.

Let $dz$ denote the decrement of $z$ in a small time, $dy$ the decrement of $y$ in the same time. If we supposed the $z$ individuals to be subject to small pox, we should have,

$$dz = \frac{z}{y}\cdot dy.$$

But we must subtract from this value of $dz$ the decrement arising from small pox, to which the $z$ individuals are by hypothesis not liable; this is $\frac{z}{y}\cdot du$. Hence

$$dz = \frac{z}{y}dy + \frac{z}{y}du.$$

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* See *Journal*, vol. xiii, p. 350.

We put $+\frac{z}{y}du$ and not $-\frac{z}{y}du$, because $z$ and $y$ diminish while $u$ increases. Then

$$\frac{dz}{z}=\frac{dy}{y}+\frac{du}{y};$$

therefore

$$\log z=\log y+\int\frac{du}{y},$$

$$\text{and } z=y\epsilon^{\int\frac{du}{y}}.$$

This expression may be easily reduced to that previously obtained; for $y$ and $z$ correspond to $L_x$ and $L''_x$ respectively, while $\frac{du}{y}$ is equivalent to $F'_x dx$, where $F'_x$ is the force of mortality (or decremental force) of the series $L'_x$. Hence $\epsilon^{\int\frac{du}{y}}=\epsilon^{\int F'_x dx}=\frac{1}{L'_x}$.*

D'Alembert appears to have been but little satisfied with the solution which he had arrived at. In a later publication, where he again touches upon the subject, he says:—"S'il est quelqu'un à qui la solution de ce problème soit réservée, ce ne sera sûrement pas à ceux qui la croiront facile." It appears however from Mr. Todhunter's work, that Laplace, in one of his investigations of the problem, gives the result in the same form as that deduced by D'Alembert.

The equation $L''_x=\frac{L_x}{L'_x}$ is the complete solution of the general problem; for the total force of mortality is thereby resolved into its two elementary component parts, namely, that arising from the specified cause alone, and that arising from all other causes together. In the particular instance under consideration however, our knowledge of one of the characteristics of the specified disease, namely, the immunity from a repetition of the attack enjoyed by those who recover, suggests the possibility of resolving the function $L'_x$ into yet simpler elements.

In the preceding investigation $F'_x$ denoted the force of mortality from small pox measured upon $L'_x$, the *whole body* of survivors at age $x$. Let $\overline{F}'_x$ denote the force of mortality measured upon that portion only who are still liable to it, namely, those who have not already suffered from the disease. And let $\overline{L}_x$ represent the body in which the annual decrement consists exclusively of the number *attacked* during the year by small pox. Then the number of deaths in the time $x$ in such a body will be $\int_0^x \overline{L}_x.\overline{F}'_x dx$; and subtracting this from $\overline{L}_0$, we shall evidently get the number of survivors at age $x$ in a body of individuals subject to mortality from small pox only. That is to say, $L'_x = \overline{L}_0 - \int_0^x \overline{L}_x.\overline{F}'_x dx$; and the equation just obtained becomes

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* If $F_x$ denote the decremental force of any series $L_x$, the relation between them is universally represented by the equation $\epsilon^{-\int F_x dx}=L_x$.

$$L''_x = \frac{L_x}{\overline{L}_0 - \int_0^x \overline{L}_x \overline{F}'_x dx}.$$

D. Bernouilli first gave a numerical solution of the problem in this form by making hypothetical assumptions of the values of $\overline{L}_x$ and $\overline{F}'_x$; and Laplace, following in his steps, gave the following general solution of the problem.

Let $x$ denote the age expressed in years; let $\xi$ denote the number who survive at that age out of a given number who were born; let $s$ denote the number of these survivors who have not had the small pox. Further, let $si.dx$ represent the number attacked, and $sir.dx$ the number of deaths, from small pox, in the element $dx$ of a year, $i$ and $r$ being each a function of $x$. The number who die of other diseases in the same time is therefore $-d\xi - sir.dx$. But this last number must be diminished in the ratio of $s$ to $\xi$, because we only want the diminution of those who have not yet had the small pox, of whom the number is $s$. Thus

$$-ds = si.dx - \frac{s}{\xi}(d\xi + sir.dx).$$

This equation is to be integrated. We have

$$s\frac{d\xi}{\xi} - ds = si.dx - \frac{s^2 ir.dx}{\xi}.$$

Therefore

$$\frac{s.d\xi - \xi.ds}{s^2} = \frac{\xi i.dx}{s} - ir.dx.$$

Put $q$ for $\frac{\xi}{s}$; thus $\frac{dq}{dx} = iq - ir$.

Let $v$ denote $e^{-\int i dx}$; thus $\frac{d}{dx} qv = -irv$;

therefore $qv = C - \int irv.dx$. The constant is unity if we suppose the lower limit of the integral to be 0; for $q$ and $v$ are both equal to unity when $x=0$; thus

$$qv = 1 - \int irv.dx \quad . \quad . \quad . \quad . \quad . \quad [1]$$

Again, if $z$ represent the number who would have been alive (in lieu of $\xi$) if there had been no small pox, we shall have for the mortality among that number $-d\xi - sir.dx$ reduced in the ratio of $z$ to $\xi$. Hence

$$-dz = -\frac{z}{\xi}(d\xi + sir.dx) \quad . \quad . \quad . \quad . \quad [2]$$

Therefore

$$\frac{dz}{z} = \frac{d\xi}{\xi} + \frac{s}{\xi} ir.dx$$

$$\frac{1}{z}\cdot\frac{dz}{dx} - \frac{1}{\xi}\cdot\frac{d\xi}{dx} = \frac{s}{\xi}ir = \frac{ir}{q}$$

$$= \frac{irv}{1 - \int irv.dx} \quad . \quad . \quad \text{from [1]}$$

Therefore, by integration,

$$\frac{z}{\xi} = \frac{\mathrm{C}}{1 - \int irv.dx}.$$

As before, the constant is unity; thus

$$z = \frac{\xi}{1 - \int irv.dx}.$$

This is the general solution of the problem obtained by Laplace. Bernouilli's solution may be deduced from it by supposing $i$ and $r$ constant. The formula then becomes,

$$z = \frac{\xi}{1 - r(1 - \epsilon^{-ix})}.$$

Laplace's result may be shown to be identical with that previously obtained by the method of decremental forces, namely,

$$\mathrm{L}''_x = \frac{\mathrm{L}_x}{\overline{\mathrm{L}}_0 - \int_0^x \overline{\mathrm{L}}_x \overline{\mathrm{F}}'_x dx}.$$

For $v = \epsilon^{-\int i dx} = \overline{\mathrm{L}}_x$, $i$ being by definition the decremental force of $\overline{\mathrm{L}}_x$. And $ir = \overline{\mathrm{F}}'_x$. Therefore, taking $\overline{\mathrm{L}}_0 =$ unity, the two become exactly the same.

If we wish to determine $\mathrm{S}_x$, the number of persons who have not had small pox among the $\mathrm{L}_x$ survivors at age $x$, it may at once be found by the equation

$$\mathrm{S}_x = \overline{\mathrm{L}}_x \mathrm{L}''_x = \frac{\mathrm{L}_x . \overline{\mathrm{L}}_x}{\mathrm{L}_0 - \int_0^x \mathrm{L}_x \overline{\mathrm{F}}'_x dx}.^*$$

This problem was also solved by D. Bernouilli upon the same assumptions as the last.

The facility and certainty with which problems of this nature are resolved by the aid of the method of decremental forces are in striking contrast with the intricacy—and consequent liability to error—involved in the more direct processes of Bernouilli and Laplace. D'Alembert, as appears from the remark already quoted, evidently felt himself upon treacherous ground, and not without reason, as the following will show.

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* $\mathrm{S}_x$ is the function denoted by $s$ in Laplace's investigation.

Let it be required to determine the number in the mortality table of those who will die by the small pox at age $x$ and upwards. From the principles of the theory of the composition of decremental forces we see at once that the number in question is represented by the definite integral $\int_x^\infty S_x\overline{F}'_x dx$. D'Alembert proposed to solve this problem, and proceeded thus. Let $w$ denote the number of those who do *not* die of small pox. Hence, out of this number $w$ during the time $dx$ none will die of small pox, and the number of those who die of other diseases will be $-(d\xi+sir.dx)\frac{w}{\xi}$. Therefore $-dw=-\frac{w}{\xi}(d\xi+sir.dx)$. Now it will be seen from [2], in Laplace's investigation, that this is precisely the form of the differential equation for determining $z$, the number who would be alive at age $x$ (or the number who would die at age $x$ and upwards) if small pox were exterminated—which is, of course, a very different thing from the number who do not die of small pox in a body in which that disease is one of the forces of mortality. D'Alembert's error consists in reducing the mortality from other diseases (namely, $-d\xi-sir.dx$) in the ratio of $w$ to $\xi$. For it is evident that if $w$ represent the number of those who do *not* die of small pox, then the entire mortality from other diseases must be distributed among them exclusively; and therefore the true decrement, $-dw$, is $-(d\xi+sir.dx)$. This will give $\frac{dw}{dx}=\frac{d\xi}{dx}+sir$; and, by integration, $w=\xi+\int sir.dx$. Hence $\xi-w$, or the number dying of small pox, $=-\int sir.dx$; or taking the limits, $x$ and $\infty$,

$$\xi-w=\int_x^\infty sir.dx,$$

which agrees with the expression $\int_x^\infty S_x\overline{F}'_x dx$ previously found.

It will be observed that these solutions all proceed upon the assumption that the extermination of small pox does not affect the mortality arising from other causes. This must be proved before any reliance can be placed upon the conclusions arrived at. I give the investigation merely as an illustration of the comparative advantages of the different methods of solution.

# *On the Interpretation of the Statements required by the "Life Assurance Companies Act,* 1870", *with special reference to the question of Expenses. By* DAVID DEUCHAR, F.I.A., F.F.A., *Secretary and Joint Actuary of the Edinburgh Life Assurance Company.*

[Read before the Actuarial Society of Edinburgh, 1 April 1874.]

SINCE the passing of the Life Assurance Companies Act, 1870, the accounts and valuation statements lodged by the various companies with the Board of Trade, in accordance with the requirements of the Act, have been printed and published annually. The valuation returns afford, in the majority of cases, sufficient information to enable an actuary to form a tolerably correct opinion of the financial strength or weakness of the companies to which they refer. In some cases, however, owing to the special character of the facts brought out in the schedules, it is important that the actuary should have before him, when testing the sufficiency of the reserve, some facts, relating to the growth of the business, and the purposes to which the expenditure has been applied, which are not elicited by the Act. For instance, he should know the annual amount of new business which has been transacted, and the annual amount of assurances surrendered and dropped, as well as the claims by death, in order to enable him to form an opinion as to the average duration of the existing policies. He should also have a division of the expenses, showing those applicable to new business and those applicable to the management of business already acquired. An approximation to these particulars may generally be made by the actuary after a study of the reports and other voluntary publications of the companies; so that, on the whole, it may be said that the Act affords to the public the means of judging, through the medium of the actuarial profession, of the relative positions of the various offices as regards stability and profitableness.

In the interpretation of the results exhibited by the Valuation Statements, serious errors are not likely to be made, as it is admitted that this portion of the Government Returns is capable of being thoroughly understood by actuaries alone. It is quite otherwise with the Revenue Accounts and Balance Sheets, which present no appearance of difficulty to the non-actuarial mind, and which are carefully scrutinized and commented on by persons altogether outside of insurance circles. The Revenue Accounts especially come in for a large share of public notice, and on no point is there a greater concentration of attention than on expenses. Comparisons are instituted by policyholders, by the public press, and by some of the offices themselves, between the respective rates of expenditure of the various life assurance companies; and tabular statements are published showing the rates of expenditure of all the companies. In thus devoting special attention to the subject of "expenses", the public are following a true instinct, as the subject undoubtedly merits all the attention which it is receiving; but I think it can be shown that the matter is not so entirely devoid of complexity as would at first seem to be the case.

The rate of expenses is generally reckoned on the total premium income, including both first premiums and renewals. Where the expenses incurred at the commencement of the assurance do not differ to any great extent from the annual expenses thereafter, this mode of reckoning the expenses is a suitable one. This is probably the case, to some extent, with a few of the existing life assurance offices. Some of the legal offices, for instance, do not require to go to any expense in forming agencies or obtaining business. Their shareholders are members of the various legal bodies, and many of these shareholders act as agents for their respective companies, receiving a moderate commission. The business of these offices consists, in great part, of life assurances obtained in connection with loans on life interests, or on other forms of security in which a life assurance is required in order to protect the lender. Business of this kind is not influenced by advertising or canvassing. The borrower asks his solicitor to find him a sum of money on a certain security, and the solicitor, as agent and as shareholder, has a double interest in sending his client to his own company. The assurances required in connection with loans are frequently of very large amount, and in consequence of this, the offices transacting this class of business frequently require to share their risks with other companies. Reassurances are obtained from these other companies on different lives, in return for the sums thus given off; and a very considerable proportion of the business of the legal life offices must consist of these reassurances. They must also occasionally receive proposals for assurance on the lives of their own shareholders and their clients for family purposes. Probably, however, the bulk of the business of such offices generally consists of assurances in connection with loans and reassurances. More especially when the commission on the first premiums is only 5 per-cent, the medical fees being comparatively relatively unimportant, in consequence of the average sum assured on each life being large, it is not surprising that business of this class involves very little preliminary outlay. It is also scarcely necessary to say that business of this class cannot fail to prove extremely profitable, if the mortality is found to be favourable. In addition to the advantages derived from a very moderate rate of expenditure, there are considerable gains from the high rate of interest realized on the class of transactions cultivated by such offices,—not because the securities are in any way inferior to others of a simpler character, but merely because these securities are unsuitable to any ordinary lender.

The case of a low rate of expenditure on the acquisition of business is, however, by no means a common one. When the business is of the ordinary class, and does not come to the office of itself, but has to be sought after, the proportion of the first premium which is swallowed up in expenses is necessarily high. The cultivation of an ordinary agency for a life assurance company requires great perseverance and tact. The agent who trusts to an announcement on a brass plate or in a window-blind, to bring to him "Persons desirous of effecting assurances on their lives", will in most cases very soon come to the conclusion that there are no such persons in his part of the country. In order to be successful, the agent must give time and attention to the subject. His difficulties do not arise mainly from the competition of other companies' agents—although this is not to be despised,—but rather from the apathy and indifference of the majority of the public. If, therefore, life assurance business is to be carried on extensively, it is absolutely necessary that active and energetic agents should be secured.

The usual inducements offered to agents with the view of securing new business, are (1st) a higher commission on the first premium; (2nd) occasional advertisements in the local papers; and (3rd) a liberal supply of circulars and other documents to be issued to the public. But these inducements are not as a rule very successful in getting the agent to devote time to the work. Some agents never issue the circulars which are sent to them from time to time for distribution. Others, again, issue them, but take no further trouble in the matter. The majority of the agents make some efforts, but not in a systematic way; and the consequence is that they do not secure nearly the amount of business which they might obtain by working diligently and systematically. It is rare that an agent becomes successful without having been visited by an officer of the company, and the repetition of such visits from time to time is in many cases almost a necessity. The establishment of a personal relationship between the agent and the office is generally attended with great success, but this necessarily involves a considerable expenditure. Even though much attention be given to the agencies, it is impossible to secure the success of all the agents, and in individual cases expenditure must frequently be made without any adequate return.

Let us, in the first place, consider the minimum expenses likely to be incurred in connection with business obtained through ordinary agencies, exclusive of share of the head office expenses. Assuming that the policies effected through ordinary agencies are, on the average, for £500 each, and that ten such policies are introduced by the agent during each year, producing an annual premium income of £150, the following will be the actual outlay thereon:—

| | £ | s. | d. |
|---|---|---|---|
| Cost of prospectuses, circulars, policy forms, proposal forms, &c. | 0 | 10 | 0 |
| Carriage of do. from head office, and postages of circulars at agency | 0 | 10 | 0 |
| Medical fees | 10 | 10 | 0 |
| Ordinary postages and receipt stamps | 0 | 9 | 0 |
| Bank charges on remittances | 0 | 2 | 0 |
| Policy stamps | 2 | 10 | 0 |
| Commission of 10 per-cent on the first premiums | 15 | 0 | 0 |
| | £29 | 11 | 0 |

This sum is equal to nearly 20 per-cent on the gross amount of the premiums received for the first year.

An annual business consisting of ten policies for a total sum of £5,000 may appear small; but it must be higher than the average amount of business done through a single agency. One hundred such agencies would produce an annual new business of £500,000, which is about the average amount transacted by the Scottish offices, while the average number of agencies in the case of these offices is probably at least 500 instead of 100. It is evident, therefore, that an agency such as that described must be charged with a considerable share of the general expenses of management, in addition to the items mentioned above; and if this share were added, the rate of expenditure on the amount of the first premiums would be very largely increased, as I shall have occasion to show in a subsequent portion of this paper.

If, then, it can be shown that the total expenses of procuring assurances are rarely less in an ordinary company than 50 per-cent on the amount of the first year's premiums, while the expenses of managing the business after it has been acquired, rarely reach 10 per-cent on the amount of the second and subsequent years' premiums, it follows that the ordinary mode of reckoning the expenses on the total premium income, without discriminating between old and new business, is incorrect, and does not afford sufficient means of judging as to the cost at which a company is worked. In the case of all recently established offices, the proportion of new premiums to renewal premiums must in ordinary circumstances be very much greater than that which exists in the case of old companies; and on this account, the rate of expenditure of the first mentioned class of offices, when reckoned on the total income, must be higher than that of old offices, even though the new business is acquired by the young offices at a lower rate of initial expenditure than that which the older companies are paying.

Let us consider the case of two offices, A and B, each possessing a premium income of £100,000, but whose rates of expenditure, as ordinarily reckoned, differ widely—A's being only 12 per-cent, while B's is 18 per-cent. In such a case the public are apt to consider that A must be a very much better office for the insured than B, where the rate of expenses is 1½ times that of A. It is quite possible, however, that a more discriminating comparison will reveal that office B is very much more economically managed than office A. For instance, A's premium income may be made up of £2,222 of new premiums obtained at an expenditure of 100 per-cent, and £97,778 of renewal premiums, on which the expenses are 10 per-cent, making up the 12 per-cent on the total premium income already mentioned. On the other hand, B's premium income may consist of £24,712 of new premiums obtained at a cost of 50 per-cent, and £75,288 of renewal premiums on which the rate of expenditure is 7½ per-cent, making together a rate of 18 per-cent on the total premium income. Thus B's initial expenditure may be only one-half of that of A, and its renewal expenditure may be only three-fourths of that of A; and yet, in the eyes of the public, and in the tabular statements showing the rates of expenses of the various offices, B appears as if managed very extravagantly as compared with A.

Enough has been said, I think, to show that, in the first place, the expenses of acquiring business are generally quite of a different character from those of managing it after it has been gathered in, and in the second place, the revenue account in its present form is not an infallible guide to the economy with which an office is managed. The next question which arises is:—How can the form of the Revenue Account be altered, so as to render it serviceable to persons who are desirous of investigating the expenditure of assurance companies?

The subdivision of the headings, *Premiums, Expenses of Management,* and *Commission,* might, I think, effect this in a simple and satisfactory way.

The heading *Premiums* should, I think, be divided thus:—

1. New annual premiums (for assurance of £ ) . . . . . £
   *Less* reassured (£ ) . . .
   £
2. New single premiums (for assurance of £ ) . . . £
   *Less* reassured (£ ) .
3. Renewal premiums . . . £
   *Less* reassured . . .

£

In the accounts of one or two offices, the premiums are subdivided as in the preceding example; but the amounts assured, though given in the Report, are not given in the Schedule, and therefore do not appear in the Returns printed by the Board of Trade. It is important, I think, to state the amount of new assurances effected during the year in connection with the new premiums; as without information of this kind an outside actuary cannot judge of the description of assurance which is being transacted. Now that the plan has become popular of granting whole-life assurances in respect of which only 10, 15, 20, or 25 annual payments are exigible, it is to be expected that the rate of premiums on new assurances must be higher than the average rate in the case of offices which do not cultivate this class of business. Where a large portion of the new business of a company consists of assurances on the 15-*payment Scale of Premiums,* the rate of first premium may be raised to £4 per-cent. As the aggregate value of the first and subsequent payments on this scale is not greater than that of the payments on the ordinary scale, it is evident that the total sum which may with propriety be expended on the acquisition of the business should not be much greater in the case of this description of policy, than in the case of a policy on the ordinary scale of premium; that is to say, if it be prudent to incur an initial expenditure of £2, *and no more,* in the acquisition of a life policy for £100, on which the annual premium payable during the whole of life is £3, it will not be prudent to incur a much larger initial expenditure than £2 if the assurance is arranged on the 15-payment scale at a premium of £4 instead of £3.

The item *Commission,* on the other side of the account, should, I think, be subdivided thus:—

1. First year's commission, new business, *less commission received* . . . . £
2. Commission on single payments and commuted commission, *less received* . . £
3. Renewal commission, *less received* . . £

The *Expenses of Management* should, I think, be stated as much in detail as possible; and it might be desirable to exhibit the proportions applicable to new business and renewals thus:—

*Expenses of Management.*

Directors' fees . . . . . . . £
Medical officers' fees . . . . . .
Solicitors' fees . . . . . . .
Salaries at head office . . . . . .
Salaries at branches . . . . . .
Travelling expenses . . . . . .
Rents . . . . . . . .. .
Stationery and printing . . . . .
Advertising . . . . . . .
Postages . . . . . . . .
Taxes . . . . . . . .
Total . . . . . £

Proportion applicable to new business . . . . £
Proportion applicable to renewals

These proportions can be fixed with far greater accuracy by persons inside the office, who are acquainted with its whole system of management, than by outside actuaries who have no other guide than the publications of the offices. There would be no temptation to the officers of the companies to state other than the true proportions, if they were required to publish at same time a detailed statement of the expenses in the above form. If we had these details of information in regard to premiums, commission, and expenses of management, it would be much easier than at present to form a well-grounded opinion as to the direction in which any company is going—whether it is becoming better or worse—between valuation periods.

While it is *desirable* to have the accounts of the offices published in a form which admits of the expenses being analyzed and tabulated in connection with the results attained by the outlay, it may almost be said to be *essential* that every office which spends money with the view of getting business, should adopt a similar plan of testing the actual cost at which the new business is obtained. When no test of this kind is applied, the expenditure must be to a certain extent an unintelligent expenditure. In the case of an office having Branches or District Agencies, and where the bulk of the business is subject to a renewal commission of 5 per-cent, the mode of testing the relative success of each branch would be by deducting 5 per-cent on the amount of renewals at the branch from the total of the branch expenses and commission, and applying the balance of the expenses and commission to the new sums assured. In reckoning the total initial expenditure, a share of the *Head Office* charges must of course be added.

This plan conveys a far clearer idea of what is going on than the ordinary method of reckoning the expenses in proportion to the total premium income of the branch. Where the ordinary method is adopted, it may appear that a great improvement has been effected in a district agency, the expenses on the premium income of the district having fallen, say, from 15 to 13 per-cent since the end of the previous year. But when more carefully looked at, it may be found that this apparent improvement is due simply to a great falling off in the amount of new business, with its attendant expenses of medical fees, extra commission, &c.

Before passing from the subject of *Expenses,* it may be instructive to consider the following statement, made up on the plan indicated, from which the total expenses of obtaining new business in the case of the Scottish offices can be deduced. My reason for including only the Scottish offices in this statement, is because they resemble each other more closely than the English offices do. They all transact business mainly with the provident classes, and they all employ some machinery for extending business. The resulting figures may therefore be of use as indicating the average cost at which a business of this class can be carried on. The figures are taken from the latest published accounts.

*Statement relating to the following Companies, all having their Chief Offices in Scotland, taken from the latest Reports and Statements published up to this date (1st April 1874).*

| Name of Company. | Date of commencing Life Business. | Period to which the Figures apply. | Life Assurance and Annuity Funds at end of Year. | New Life Assurances effected during Year. | Estimated New Annual Premiums. | Premiums received during Year. | Commission. | Expenses of Management. |
|---|---|---|---|---|---|---|---|---|
| | | Year ending | £ | £ | | £ | £ | £ |
| Scottish Widows' Fund . | 1815 | 31 Dec. 1872 | 5,439,201 | 1,213,800 | Some of the offices do not publish their new premiums. Others publish these, but do not distinguish between new annual premiums and single payments. None of the offices make any distinction between ordinary annual premiums and premiums on a higher or lower scale from the ordinary whole-term rates. On this account it has been thought better to estimate the first premiums at an average rate of £3 per-cent on the sums assured. | 466,813 | 11,589 | 40,972 |
| North British and Mercantile | 1823 | 31 Dec. 1872 | 2,464,027 | 732,875 | | 278,555 | 10,323 | 29,156 |
| Edinburgh . . . . | 1823 | 31 Mch. 1873 | 1,059,401 | 467,215 | | 115,830 | 5,631 | 13,152 |
| Scottish Union . . . | 1824 | 31 July 1873 | 953,864 | 466,876 | | 149,091 | 9,401 | 13,492 |
| Standard . . . . | 1825 | 15 Nov. 1872 | 4,100,310 | 1,190,453 | | 531,007 | 21,421 | 50,592 |
| Scottish Provincial . . | 1825 | 31 Jan. 1873 | 686,837 | 308,274 | | 108,819 | 5,334 | 11,977 |
| Scottish Amicable . . | 1826 | 31 Dec. 1872 | 1,461,729 | 425,139 | | 157,607 | 5,277 | 12,647 |
| Scottish Equitable . . | 1831 | 1 Mch. 1873 | 2,033,722 | 316,900 | | 183,689 | 6,226 | 11,582 |
| Caledonian . . . . | 1833 | 14 May 1873 | 452,545 | 114,600 | | 47,150 | 1,880 | 6,006 |
| Scottish Provident . . | 1837 | 31 Dec. 1872 | 2,004,165 | 757,779 | | 219,656 | 7,168 | 20,347 |
| Life Association of Scotland | 1838 | 5 Apl. 1873 | 1,452,709 | 794,477 | | 259,294 | 12,980 | 25,502 |
| City of Glasgow . . . | 1838 | 20 Jan. 1873 | 634,111 | 352,880 | | 115,348 | 4,714 | 11,054 |
| Scottish National . . | 1843 | 14 May 1873 | 495,263 | 346,950 | | 74,546 | 3,526 | 8,739 |
| Scottish Imperial . . | 1866 | 31 Dec. 1872 | 17,025 | 82,445 | | 11,104 | 721 | 705 |
| Scottish Commercial . . | 1867 | 31 Jan. 1874 }<br>Period of 13 months. } | 18,534 | 44,100 | | 4,539 | 218 | 825 |
| 15 Companies . . | | | £23,273,443 | £7,614,763 | £228,433 | £2,723,048 | £106,409 | £256,248 |
| Averages . . . | . . | . . . . | £1,551,560 | £507,650 | £15,230 | £181,535 | £7,094 | £17,083 |

It will be seen that the commission is at the rate of nearly 4 per-cent on the premium income, and that the commission and expenses of management amount together to $13\frac{1}{4}$ per-cent on the premium income. Taking the average results of the 15 offices, we obtain figures which may perhaps be more useful than the large totals in the statement. We have then:—

| | |
|---|---|
| Average Funds . . . . . | £1,551,560 |
| „ New Assurances . . . . | 507,650 |
| „ Premium Income . . . . | 181,535 |
| „ Expenses and Commission, 13·24 per-cent . . . . | £24,035 |

Now let us consider what is the rate of expenditure at which a company of this size could be managed, irrespective entirely of new business.

| | |
|---|---|
| The renewal commission may be estimated at . . . . . . . . | £6,500 |
| The expenses of managing the loans and investments—say $\frac{1}{8}$th per-cent . . | 2,000 |
| Share of rent, salaries, directors' fees, office expenses, postages, bank charges, taxes | 3,500 |
| | £12,000 |

This is equal to about $7\frac{1}{4}$ per-cent on the renewal premium income. Deducting £12,000 from £24,177 (the total amount of expenses and commission), there remains the sum of £12,177, which is applicable to the new business of £507,650. This yields a rate of £2. 7*s.* 11*d.* on each £100 assured. The average age at entry being about 40, and the average premium on a whole-life assurance being therefore about £3, this represents an expenditure in acquiring the business of 80 per-cent of the first year's premium. Add to this the risk of the first year, which by the Institute experience, $H^M$ *first year of assurance,* is, at the average age at entry, about 10*s.*, and we find a sum of £2. 17*s.* 11*d.* per £100 assured, or 96 per-cent of the first year's premium expended on the year's risk and expenses. We see, therefore, that in the case of the Scottish offices the whole of the first year's premium is swallowed up in claims and expenses, and thereafter the expenditure is $7\frac{1}{4}$ per-cent on the premium income. This may not be the most desirable state of matters. It would of course be preferable if business could be obtained at a lower rate of expenditure; but the state of matters *exists,* and it cannot be made worse by being faced.

In the case of offices whose expenditure resembles that described above, the question necessarily arises as to the mode in which this expenditure is to be provided for in the construction of the premiums. It is usual to consider the premium as consisting of two unequal parts, the larger being the *mathematical, pure,* or *net premium,* and the smaller being the *loading,* or addition for expenses, contingencies, and profit. In the case of non-participating assurances, the loading is generally an equal percentage on the mathematical premium at all ages; occasionally, however, the addition is made in the form of a *constant* and a percentage. In the case of assurances intended to participate in the profits, there is a larger addition, usually regulated in some measure by the bonus system of the office and the prospects of profit which it holds out to entrants at the various ages. Though the amount of the annual loading may differ at each age at entry, it is, I believe, invariably treated in this country in practice as of the same amount during the first year as during the subsequent years of the existence of the policy. Why should this be so, if it does not correspond with the manner in which the expenses are incurred?

Leaving out of account the additions for profit and for contingencies, let us assume that an addition of about 15 per-cent on the mathematical premium is usually added for expenses, equal perhaps to about 13 per-cent on the gross non-profit rates, or 11 per-cent on the gross participating rates. This is probably not far from the mark; and the resulting average rate on the whole premiums (participating and non-participating) will probably be about 11½ per-cent. It may be the case that taking an average duration of policy, the 11½ per-cent per annum is exactly equal in value at the outset to the rate of 79 per-cent on the first premium, with 7¼ per-cent on renewals, which we have seen represents something like the average rate of expenditure in the case of a class of offices employing agents and district managers; but this fact does not show that it is a matter of indifference whether the provision for expenses is made in the one form or the other. Assuming that at the outset the value of the expenses, according to fact and according to hypothesis, are equal; at the end of the first year, after all the preliminary expenses have been paid, the values of the future expenses differ widely by the two methods.

The actual future expenses are 7¼ per-cent, while the proportion reserved by the ordinary method is 11½ per-cent, or nearly 50 per-cent too much; while in the case of past expenses the proportion allotted by the hypothesis is only 11½ per-cent, as compared with 80 per-cent of actual expenditure.

The unsuitableness, theoretically, of the ordinary net premium method of valuation to an office which transacts its business in the ordinary manner, becomes evident when we consider that by this method at the end of one year a reserve is required of nearly 50 per-cent, on the average, of the amount of the first year's premium. If 79 per-cent be spent in acquiring the business, and 21 per-cent in meeting the first year's risk, whence is the additional 50 per-cent to be derived? If the company is a proprietary one, and has been only one year in existence, the deficiency must be made up from the capital or from the interest on the capital. In the case, however, of a mutual office, there is no fund at the outset from which this deficiency can be made up: and in such a case, if the ordinary net premium system be correct, the office must be unable to meet its liabilities! And yet the present value of the gross future premiums, under deduction of 7¼ per-cent, is probably far more than equal to the present value of the sums assured. In such a case it would be absurd to say that the office is insolvent. The first year's contract has been fulfilled; and all expenses have been paid, without in any way encroaching on the provision for future expenses and claims, which will be amply met by the future premiums. We are therefore driven to disregard the result of the net premium valuation, and to seek for some other method less inconsistent with fact.

The method which seems most suitable to such circumstances is that proposed by Mr. Sprague in his paper "On the Proper Method of Estimating the Liability of a Life Insurance Company under its Policies", which appeared in the *Journal of the Institute of Actuaries* for July 1870. This is—*To reserve for policies in their first year only sufficient to meet the unexpired current risk, and to consider that all policies of more than one year's standing have been effected at the next higher age,* the expression for the value of a policy on this plan being

$$A_{x+n}-(1+a_{x+n})\varpi_{x+1}.$$

By this plan every year pays its way, and the principal advantages of both the net and gross premium systems are retained. As compared with the ordinary gross premium system of valuation, this plan is greatly to be preferred, as it does not create, in the case of the more recent policies, an apparent asset, which may at any time be rendered valueless by the policies being dropped,—the future margin, which was wholly or partially reckoned on, being lost to the company.

This must conclude the present paper. I had intended to give illustrations showing the importance of testing, not only the amounts of new business produced by the various means employed, but also the permanency of the business thus obtained; and I had also proposed to myself to show how, owing to the *mortality profit* derived from the introduction of a large new business, and from other causes, the unsuitableness of the ordinary net-premium methods of valuation is partially neutralized, or at all events concealed, in the case of offices which have already a large existing business. These points, however, must be left to a future paper.

## *Does a Large New Business benefit the Policyholders of a Life Company? By* JAMES R. MACFADYEN, *of the Legal and General Life Assurance Society.*

[Extracted from the *Insurance Times* of New York.]

THE question with which I have headed this paper is one that may seem startling enough. It has been so widely taken for granted that a large new business must be an unmixed good to all concerned in a life company, that to debate whether it be so or no, will seem in the eyes of many to be a very idle thing. And yet, though I have been considering the subject for a long time, I cannot answer the question unhesitatingly in the affirmative. In saying this, it ought to be pointed out, that the matter is regarded from a purely practical point of view. The problem is not *ought* a large new business to benefit policyholders? but, as a matter of fact, *does* it? Even if the question were answered in the negative, it would not follow that no new business, or an insufficient quantity of it, would better suit the interests of the policyholders. Waste must be supplied, and a certain degree of magnitude maintained in life companies.

Though attempting to analyze the matter from a practical point of view, I do not propose to take the bonuses already declared by the various societies into consideration. No doubt to a "with-profit" policyholder the bonus is in this matter the sole measure of the benefit he may get in one office more than in another. And no doubt, also, it is the "with-profit" policyholder alone that is concerned with the answer to the question heading this paper; because, to make comparison between life companies possible, we must assume that each is solvent, and that each offers conditions of assurance alike liberal. But still, while admitting all this, I do not think it necessary or safe to make the declared bonus of offices one of the elements to be taken into account in discussing the matter. It is not necessary, because the subject can be dealt with thoroughly at the source and fountain head of all bonus, namely, the premiums payable. It is not safe, because bonus is partially dependent on the actuarial basis; and this new factor in our calculations will interfere sadly with our comparisons. In an assurance office, the past bonus is no index of the future. Even were the actuarial basis of each company under examination the same, comparisons of the bonus, as the measure of profit to policyholders in the various societies, would still be hazardous. The system of division of surplus employed may be of such a character as to render declared bonus an unsafe ground of comparison of the individual office with other companies, or even with itself at former periods of its existence. If realized profit is to form an element in the discussion at all, the bonus-producing power of the various companies should be the standard of measurement. But, as I have already said, it seems unnecessary to bring the bonus into the discussion of the matter at all.

In treating as to whether a very large or a somewhat smaller new business is the more remunerative to those connected with a life company, I shall do so purely from the policyholder's point of view. There is, I think, no question but that a large business, if not absolutely unprofitable, is, to the shareholders and officials of a life office, emphatically a good thing; but it is not so clear that a policyholder is benefited by it. So far as he is concerned, the fact that the larger societies make a greater amount of profit, wón't settle the question in their favor; for it has to be divided among a correspondingly greater number. I say "correspondingly", because we cannot assume that in its larger new business the greater office has a disproportionate excess of non-participating assurances.

There are only two sources from which a policyholder can reap any advantage in a large over what he could obtain in a smaller company. The *one* is, there will be less violent fluctuations in the first class of offices, in consequence of their greater number of transactions. The *other* is, such expenses as are common to all life societies, whatever their new business may be, will weigh less heavily on the larger office, there being in it more shoulders to bear the burden.

With reference to the first of these advantages of great over small offices, not much can be said. In my last two articles,* I have fully discussed fluctuations, so far at least as their most important form is concerned. As I have shown, they diminish in violence in proportion to the increase of the number of risks. So it must be admitted that in this point the larger have an advantage over the smaller offices. And even if expedients are adopted in order to artificially swell the number of risks, this advantage of the great companies is only lessened, but not destroyed. Though the pull the larger offices have in this respect effectually disposes in the comparison of very diminutive competitors for public favor, there will still remain many societies with sufficient risk numbers to make this argument by no means conclusive against them. The drawback in this respect may be outweighed by greater benefits in another direction. The discussion of the second advantage enjoyed by large offices may throw some light on this possibility.

It is clear that, so far as the expenses not absolutely dependent on the size of a life company are concerned, the position of the great offices is the stronger of the two. These expenses form, however, but a fraction of a life company's outlay. If, then, it be found on examination that the remaining portion of the expenditure is proportionately higher in the larger than in the smaller offices—that, in fact, the very bulk of the one class is due to such greater outlay—the slight advantage on the unavoidable expenses may be more than swallowed up. Irrespective of the mortality fluctuations, the answer to the question heading this paper will be found in seeing whether the ratio of expenses to premium revenue is less in the larger than in the smaller companies. If the ratio be not less, then, excepting the possible reduction in fluctuations, the former can offer no inducement to the would-be policyholder that the latter cannot give. I have therefore prepared the following table, in order to test the comparative positions of larger and smaller offices in this particular. So far as the expenditure and premium revenue are concerned, it is compiled from the 1874 Blue-book, containing the accounts deposited with the Board of Trade by the

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* We hope at a future time to lay before our readers the articles here referred to.—Ed. *J. I. A.*

British life companies.* The amount of new business is taken from White's Statistical Register. Only societies that have been in existence prior to the year 1864 have been included, as I consider that most of the younger offices will be still struggling to establish themselves, and therefore not fairly comparable with companies that have already taken root. I have not thought it necessary to tabulate the experience of offices whose annual new business is less than £100,000 assured. The average premium income of this class is so small, as to put them out of the competition altogether. It has, of course, been requisite to exclude also from the table, societies of which the amount of new business was unknown. The operation of the preceding rule would however have kept the majority of those offices out in any case. Companies whose business, in whole or part, is of an industrial nature, are obviously unsuitable for comparison with societies transacting the ordinary life business. These have therefore been omitted from the table, as have also been offices which having various kinds of business conduct their life branch without charging cost of management:

| Amount of New Assurance in last completed year. | No. of Offices. | Average Premium Revenue. | Average of Percentage of Expenses to Premium Revenue in individual Offices. |
|---|---|---|---|
| Over £500,000 | 8 | £323,000 | 14·5 |
| Between £400,000 and £500,000 | 6 | 162,000 | 14·0 |
| „ £300,000 and £400,000 | 18 | 156,000 | 13·4 |
| „ £200,000 and £300,000 | 14 | 103,000 | 13·1 |
| „ £100,000 and £200,000 | 9 | 72,000 | 15·6 |
| | 55 | | |

A glance at the preceding table shows that the amount of fresh assurances increases with the average premium revenue. Speaking, then, of the average office in the classification here made, we can say that the greater its new business, the larger its premium revenue.

Looking next at the column where the expense percentage is shown, there seem to be two forces acting on the proportion of outlay to premium income. The first approaches its maximum intensity as the company diminishes in size. It is obviously the result on a small revenue of the expenses incidental to a *going* business. It is a force sufficiently strong to make the outlay of this class of office the highest of any under examination. And, as already mentioned, had the table been continued below £100,000 of fresh assurances, its weight would have become more and more crushing to the profit-producing power. As the new business augments, this influence loses strength, and reaches its minimum in the largest companies on our list. Simultaneously with its decay, however, another force grows in strength. The expense curve, which had been gradually falling, begins again to ascend, and goes on increasing till it reaches another maximum, at the point in the table where the influence first spoken of is at its minimum.

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* *Note.*—Commission, extension cost, and, in short, everything spent in the conduct of the business, unless it could be fairly said to be non-recurrent outlay, have been taken into account in calculating the proportion of expenditure to premium revenue in the individual offices.

The only explanation of the effect of this new power on the expense curve seems to be, that as the new business augments in a life office, the cost of obtaining it increases still more, and very soon neutralizes the advantage of the additional premium revenue. The life assurance field seems capable, by ordinary culture, of giving only a certain crop of new business; if a larger return is wished from the soil than the amount it thus naturally yields, it can only be had at a cost to the policyholder in manure, at least as great as the return is worth to him. The life offices are selling a commodity that, however salutary, is not palatable. A certain number of persons will, without excessive outlay on the part of the societies, be self-denying enough to join the offices. If a life company, not content with this number, go "out into the highways and hedges", to compel new entrants to come in, it may have done these entrants, and the State also, a service; but to the policyholders already assured, the consciousness of having thus done good, must be the sole reward; for they will reap no other.

I am not now giving any opinion as to whether the greater outlay of the offices getting the greatest number of new risks is, or is not, a desirable thing. I am simply asserting that the increased business brings no additional *profit* to the policyholders. In thus arguing, it must be remembered that I am not talking of individual companies. There are societies getting a large new business at a small cost; and, on the other hand, there are offices lavish in their outlay, with but little to show for it. Discussing life companies in the mass, I say that I can find no evidence that a great number of new entrants has a beneficial effect on the springs of profit; and I appeal to the preceding table in proof of the statement. It is quite true that, in that table, there are individual societies with an expenditure so great, as to raise materially the average over the whole section to which they belong. But no single class has a monopoly of such instances; and if we reject from the comparison offices of this stamp, our result will still be practically the same as at present. It is true also that even under a form of schedule settled by act of parliament, there are in the various companies, discrepancies which the returns offer no means of correcting, in the manner of regarding several items in the accounts. This, however, is too small a matter to affect our conclusions.

If, instead of taking the figures of the last Blue-book, we take the average of the three years during which Parliament has compelled the registration of accounts, the following is, for the same companies as before, the result,

| Amount of New Business in *last* completed year. | Average of three years' percentage of expenses to premium revenue in individual offices. |
|---|---|
| Over £500,000 | 14·4 |
| Between £400,000 and 500,000 | 14·0 |
| „ 300,000 and 400,000 | 13·7 |
| „ 200,000 and 300,000 | 14·4 |
| „ 100,000 and 200,000 | 15·7 |

It will be seen that this result is virtually in accordance with that of the previously given table.

It may be noted that in the last three of the sections, the average outlay of the triennial period is somewhat in excess of that of the latest of the three years. This, however, happens solely from the very lavish expenditure of two or three offices, (far beyond all their neighbors in this respect,) having been recently diminished. Whether this be owing to the wholesome publicity secured by the Act, or to other causes, it is unnecessary for our purpose to enquire. It is sufficient to know that no generalization can be drawn from this diminution. The expenditure percentage of almost all the other companies is either stationary or increasing, and it is so even if there has been considerable augmentation to the premium revenues. I do not give the figures for the three years, because the period is too short to eliminate temporary perturbations in the receipts and expenditure. I may say generally, however, that scarcely a single office shows any diminution in the ratio of outlay to premium revenue; and of the very few that do, those acquainted with the societies in question, will have no difficulty in tracing the result to causes peculiar to themselves.

In fact, when the expenditure proportion goes up in a life company, it very rarely comes down again.

I have now shown, from the individual policyholder's view point, that though offices with a large number of new assurances, may have, speaking generally, greater freedom from fluctuations than smaller societies, they have no more profit-producing power than these latter. I will complete my analysis by calling attention to the comparative values of the business obtained by each of the two classes of companies. It is certain that the policies obtained by severe pressure on the provident public, are not so valuable to an office as those secured by less extreme means. And, speaking generally, and not stating it as universally true, it may be affirmed that the larger the number of new assurances, the greater the proportion in it of this less valuable business will be. The ratio of discontinuances to total numbers augments as the striving after fresh proposals becomes more intense. Even if, through this cause, the increase of the office's premium revenue be not like that of the water in the Danaides' jars, these lapses are seldom beneficial to a life company. They are almost all risks of very short duration. It is doubtful whether policies discontinued so soon after entry, are remunerative to any going office, however sparing in outlay it may be. It is more than doubtful, when this plentiful crop of risks has been obtained at great cost. And it must be remembered that direct outlay is not the only element in the matter. The selection of discontinuers has an influence on the mortality of the company.

Summing up then the comparative advantages to an assurer, of societies transacting a larger or a smaller new business, the first offer greater freedom from fluctuation, and somewhat less profit-producing power, than the latter. It can hardly be too often repeated, however, that these conclusions are the result of an analysis of the experience of the *class*, and would mislead if applied indiscriminately to every individual office of that class. Though it is not a rule, then, without exception, it is at least something gained, to have shown, speaking generally, as I believe I have done, that the excessive striving after new business, now so prevalent among life companies, is a very doubtful gain to the policyholders in them. I am far from desirous that the assurance world should be a sort of "Sleepy Hollow", but the danger in our day lies rather in the opposite extreme. Offices with a large amount of new assurances are praised on all hands for their achievements, without any count being taken of their cost. In not a few instances, the policyholders were better off when the company was less ambitious. I am not urging a plea for laziness or want of energy. I am deprecating no one of the forms in which life proposals are nowadays angled for. By all means let assurance societies get as large as possible, and let every nerve be strained by their officials to make them so; but in thus acting, let not the cost of the operation be ignored, or even put into the background.

The question discussed above by Mr. Macfadyen is one of great practical importance, and the result he has arrived at is a remarkable one, well deserving the attention of the managers of life insurance companies. It will be noticed, however, that Mr. Macfadyen makes no allowance for the heavier ratio of expense that necessarily attaches to the new business as compared with the old in every company, whether large or small, whether long established or of recent origin. Without expressing any opinion on the present occasion as to whether Mr. Macfadyen's remarkable figures fully justify the conclusions he deduces from them, we have thought it would be interesting to many of our readers to see how the figures would come out, if allowance were made in the manner proposed by Mr. Deuchar (in the paper preceding Mr. Macfadyen's), for the greater expense attaching to the new business. Mr. Macfadyen having kindly furnished us with the names of the 55 offices whose accounts have been examined by him, we have recomputed the average percentages of expense, taking the figures—not from the bluebook, as he has done, but from White's *Insurance Register;* and the results are shown in column (2) of the following table. The figures, it will be seen, substantially agree with Mr. Macfadyen's; not only as regards their absolute magnitude, but what is of more importance, as regards their comparative magnitude. We have next computed the average percentages of the renewal premiums that are expended, on the suppositions, first, that one-half the new premiums, and secondly, the whole of the new premiums in each company, may be considered as a set-off against the expenses; and the results are shown in columns (3) and (4) of the table.

| Amount of New Assurances. (1) | Average Percentage of Expenses to Premium Income. (2) | Average Percentage of Renewal Premiums spent in addition to | |
|---|---|---|---|
| | | One-half the New Premiums. (3) | The whole of the New Premiums. (4) |
| Over £500,000 | 14·23 | 10·66 | 5·62 |
| Between £400,000 and £500,000 | 13·90 | 9·99 | 4·50 |
| " £300,000 " £400,000 | 13·18 | 9·92 | 5·15 |
| " £200,000 " £300,000 | 12·89 | 9·03 | 3·33 |
| " £100,000 " £200,000 | 15·98 | 12·34 | 7·23 |

The results, it will be seen, support Mr. Macfadyen's statements as to the greater expense attending the management of companies doing a very large new business; for they show that even when the fullest possible allowance is made on account of the expense attaching to a large new business, the average rate of expense attaching to the management of the old business is greater in the companies doing a very large new business than in those doing a more moderate amount.—ED. *J. I. A.*

# *On the Mortality of the Clergy. By* HERR HEINRICH STÜSSI, *of Zürich. Translated and Abridged by* D. A. BUMSTED, *Fellow of the Institute of Actuaries.*

## INTRODUCTION.

I HAVE somewhere read that the experience of life assurance companies furnishes the only certain means of ascertaining the law of mortality, since by this means only is it possible to keep in view through a lengthened period and trace up to death, each of a selected number of persons. As far as the possibility is concerned, we may admit that the law of mortality of the members may be deduced in this manner by proper methods. It is important, however, to bear in mind that, in the case of assurances, we have to deal with selected lives; and the influence of this selection, though it may be disregarded in the later years of assurance, is of considerable importance in the earlier years, so that, unless special precautions be taken, the mortality of the general population cannot be correctly obtained. The mortality may be more nearly determined if an account be kept of the rejected proposals, as is done in England,* and the deaths noted which happen among those cases. This, I certainly think, should be done universally, as it must be of practical use to the companies to know how far the rules by which they are guided in accepting or rejecting proposals are justified. No such account is kept, as far as I know, either in Germany or Austria. We may otherwise eliminate the influence of selection, by omitting from the calculation the earlier years of assurance, during which this influence is felt.

With regard to the mortality of children, correct results may be obtained from endowments; as there is scarcely any selection of lives, and the assurances are almost always effected on condition that the premiums shall be returned in the event of the early death of the child.

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* The author is of course mistaken in supposing that any investigation of this kind is commonly made in England, altho' it was certainly done for a time by one company.—ED. *J. I. A.*

It is further to be borne in mind, that the general death rates of the country can never be obtained from the observations of a single company. One company does business principally in one district, another elsewhere. One may acquire more insurances from the upper, another more from the lower classes. One company may be appropriated to a particular section of the community, such as teachers, clergymen, railway officials, and be consequently not accessible to the others. It is only when the different companies bring together their observations and experience that a true representation of the general death rates of the country can be obtained. Unfortunately, only a small number of the continental companies publish any information as to their mortality; scarcely any state definitely and clearly the number of lives assured and the number of deaths at each age. The information usually given—that there were so many deaths more or less than should have happened according to the tables of mortality—is entirely worthless, when we do not know by what mortality-table the company calculates; and this is never stated. I am convinced that the great majority of companies are perfectly indifferent about such trifles as statistics. They conduct their business in a shopkeeping spirit. The chief point is to get plenty of new business, and so increase the income. What is the use of calculating whether few or many have died, unless indeed to fill up a gap in the Report? It will not bring the dead to life again, or increase the profits. To this indifference contributes also the childish mutual jealousy which is felt by all the companies without exception, as is clearly indicated by the suppression of facts in their reports. The English offices, by united action, constructed the Seventeen Offices' Table, which is generally adopted even on the continent; and to this they have recently added the $H^M$ Table: to such splendid results is the Institute of Actuaries already able to point! In Germany, on the other hand, the Life Assurance Collegium was wrecked* upon those petty jealousies and trade mysteries, almost as soon as founded, and yet it consisted of managers and actuaries who still wish to be looked upon as white ravens. Truly, "Something is rotten in the State of Denmark!"

It seems to me that we should be more likely to obtain good materials for the construction of a general table of mortality from those who, living in the midst of the population, have been appointed to watch over particular sections of it. Among such may be mentioned the overseers of the poor, who are compelled to keep in sight every citizen, however far he may wander, so that they may not lose his contribution to the rates: also the superintendent-registrars of births, marriages, and deaths, under whose observation, notwithstanding the many changes of residence which now take place, the bulk of the population of their districts still pass from birth to death. Again, observations by schoolmasters would afford valuable statistics as to the rate of sickness as well as death of children. It would be a task well worthy of the Statistical Society to organize a system of statistical observations. I am convinced that, as in meteorology a great number of men willingly undertook the laborious work of collecting observations, so on a proper representation of the importance of statistics, men would be found who would devote themselves to this work; and my countrymen would show that, in this respect, they are able by self-sacrifice to do as much good as other States with an army of paid agents.

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* The Collegium seems to be still in existence, but instead of comprising all Germany and Switzerland, it has dwindled down to a mere Berlin Club. Its Journal has long since disappeared.

Similar difficulties to those above described have prevented me from deducing complete tables of observations from the materials of an assurance company at my disposal, and I had therefore to content myself with a few fragments, of which the following is one: this fragment is, however, defective in some points; for instance, in a case where a clergyman was assured jointly with his child, it was impossible to discover from the materials furnished me whether, in the case of a claim, it was the father or the child who died. Again, no mention was made in any case of the cause of death.

## GERMAN OBSERVATIONS.

The German observations hitherto published upon the mortality of different classes of society are very scanty, and, partly from the small number of facts observed, partly from the mode of treating them, untrustworthy.

The first and best arrangement of the kind is given by Casper, in his well known work on *The probable Duration of Human Life*, published in Berlin in 1835. In his classification, according to occupation, of the individuals observed, he describes clergymen as "men who lead a very regular and temperate life, exempt from pressing anxieties; and as the great majority have country cures, their life is almost equally divided between home-study and exercise in the open air."

The average duration of the life of clergymen, is, according to Casper, 65·1 years, and consequently far exceeds that of all other classes. Of 100 clergymen, 42 reach an age over 70, 27 over 75, 14 over 80, 5 over 85, and 1 over 90 years; so that, in respect of reaching the highest ages, the clergy stand above all others. According to Casper, the nature of a clergyman's life, as given above, fully explains this remarkable longevity. He adds a clergy mortality table from observations giving 657 deaths between the ages 24 and 93; from which it appears that the average yearly mortality per-cent is ·58 for the ages 25 to 45, 2·00 for the ages 45 to 65, and 1·18 for the ages 25 to 65.

In Kolb's Statistics it is stated that, according to observations of De Neufville, at Frankfort, from 1820 to 1852, the average age at death of the clergy was 65 years and 4 months, which is certainly higher than that of all other classes.

Escherich states, that among the protestant clergy are found the greatest number of old men, but the mortality is higher than the average at the ages between 50 and 60. With the Romish clergy it is higher at the ages 45 to 65.

In Haushofer's Statistics, according to observations of Lombard, in 1835, the average duration of life of the protestant clergy is given as 63·8 years.

## ENGLISH OBSERVATIONS.

The influence of occupation upon the rate of mortality was treated very fully by Mr. Francis G. P. Neison, in the *Journal of the Institute of Actuaries* for July 1872.

According to him the annual mortality per-cent of the clergy was

| | For the Ages | | |
|---|---|---|---|
| | 25–45. | 45–65. | 25–65. |
| Clergy in general | ·52 | 1·72 | 1·04 |
| Protestant Ministers | ·54 | 1·58 | 1·01 |
| Clergy of Church of England | ·48 | 1·72 | 1·02 |
| Roman Catholic Priests | ·97 | 2·69 | 1·57 |
| General population | 1·05 | 2·30 | 1·50 |

He thinks that the higher mortality of the Roman catholic clergy is to be explained principally by the risk to which they are exposed in visiting and attending upon the members of their flocks.

## AUSTRIAN OBSERVATIONS.

The observations which were at my disposal embrace 1364 ecclesiastics, namely, 2 rabbis, 13 protestant clergymen, 209 Roman catholic clergymen, and 1140 Greek catholic (Ruthenian) ecclesiastics of both communities.

In the case of several clergymen, the communion to which they belonged was not stated; but, in a country almost entirely catholic, it is very improbable that protestant clergymen would have omitted to describe themselves as such; and we may be equally sure that the Greek catholic clergy described themselves correctly, as they enjoy certain privileges. Therefore all the clergy who are not otherwise described may be safely ranked as Roman catholics.

The privilege mentioned above, which was granted to the Greek catholic ecclesiastics, consisted in this, that they were admitted without medical certificates, and in their case, therefore, there was no "selection"; although it is true that the clergy form a select class of the population, because, as a rule, only healthy individuals undertake these duties. The facts that no previous observations exist upon the rate of mortality among the Ruthenian clergy, and that no others can be expected for a long time, lead me to hope that the following remarks will not be entirely devoid of value.

The observations extend over 50 years, ending with 1871. The average age at entry of the 209 Roman catholic clergy was 38·3 years: the total number of years of assurance 798, of which 462 belonged to ages under 45, and 336 over 45. Seven died; one under 45, and six over 45 years of age: so that the mortality per-cent under 45 years was ·22; over 45, 1·8; and at all ages, ·9. These numbers, however, are far too small to give trustworthy results. The average age at death was 55¼: the extremes, 44 and 72 years.

The total number of years of assurance of the 1140 Greek catholic ecclesiastics was 5754, of which 3101 years belonged to ages under 45, and 2653 to ages over 45: 93 of them died, 27 under 45, and 66 above that age. The percentage of mortality, accordingly, under 45 years of age, is ·87; over 45, 2·53; and at all ages, 1·62.

Classifying the clergy without distinction of creed, we have the following results:—

The number of years of assurance of the 1364 clergy was 6624, and 101 died, which gives an average yearly mortality of 1·52 per-cent. Under 45, the years of assurance were 3607, with 29 deaths, which represents an average percentage of mortality of ·80, while above 45 there were 3018 years of observation and 72 deaths, giving an average mortality of 2·38 per-cent. It may be further remarked, that the average age at entry of all the lives was 40 years and 7 months; the average age at death was 51 years and 5 months; the average duration of assurance of those who died, 3 years and nine months; and therefore the average age at entry of those who died, was 47 years and 8 months.

The appended table (see p. 352) gives the rates of mortality for clergymen according to Casper, and as derived from the present observations,—in the first instance without any adjustment. Having found the probability of living a year at each age, the sums of every five adjacent probabilities were taken, and the fifth of each sum inserted as the probability at the middle age. The values thus found, with the exception of those at the first four ages, were similarly treated, and the results of this process are taken as the adjusted probabilities. Lastly, tables were constructed in the usual form, to a radix of 1000 living at the age of 25. These tables could not safely be carried beyond age 65, for want of more extended observations.

## CONCLUSION.

Casper's observations refer most probably to protestant clergymen, and it must be admitted that his description of the life of a country parson gives an exact representation of the Roman catholic clergy also at the time he wrote, and in many localities at the the present time. Our own recollections of that time will convince us that no other class contained proportionately so many old men as that of the clergy. Modern society has, however, already made its influence felt upon them, and it is to be feared that still greater changes will yet take place. In proof of this the following, among many reasons, may be given. Greater demands are now made upon the clergy; they have to pass more strict examinations, requiring greater application during the time of student life: their sermons must be more full of matter, not something copied out of the best collection at hand, or a string of empty common-places. Increased demands are made upon them for school instruction, partly on account of the increase of schools, and also because the teachers are no longer contented to do the work of the clergy in imparting religious instruction; constantly increasing duties in connection with the poor relief, also with the registration of births, &c., and preparation of certificates, and multiplied applications for assistance in useful efforts of all kinds. The clergy also impose heavier duties upon themselves: their higher scientific cultivation incites them to greater diligence in ecclesiastical studies, or in the domains of art and science, while others devote themselves with equal diligence to the accumulation of wealth. In short, the protestant clergy, as a body, have abandoned that narrow contemplative life which Casper so strikingly pictures, though we may still find in secluded places specimens of the earlier type. Impulses from within and from without have spurred them on to an increased activity, which, in some degree, may tend to improve health, but probably, in a much greater degree, to shorten life. It is very doubtful whether, at the present time, 42 per-cent of the ecclesiastics attain ages over 70 years.

The following table compares the percentage of yearly mortality according to the various observations.

I have added for the purpose of comparison, the percentage of mortality according to the table constructed by Brune from the experience of the Prussian Widows' Fund, from 1776 to 1834. This appeared very suitable, for two reasons; first, because it distinguishes between male and female lives, and secondly, because it is probably derived from nearly the same class of society as that from which the clergy in Casper's Tables were taken.

| | Ages. | | |
|---|---|---|---|
| | 25–45 | 45–65 | 25–65 |
| Clergy in general— | | | |
| In England | ·52 | 1·72 | 1·04 |
| In Austria | ·80 | 2·18 | 1·52 |
| Protestant Ministers— | | | |
| In England | ·54 | 1·58 | 1·01 |
| In Prussia (Casper) | ·58 | 2·00 | 1·18 |
| Church of England Clergy | ·48 | 1·72 | 1·02 |
| Catholic Priests— | | | |
| Roman Catholic, in England | ·97 | 2·69 | 1·57 |
| Greek Catholic, in Austria | ·87 | 2·21 | 1·51 |
| General Male Population— | | | |
| In England and Wales, 1863 to 1871 | 1·15 | 2·52 | 1·84 |
| In Prussia (Brune) | ·97 | 2·59 | 1·68 |
| In Austria | ? | ? | ? |

According to this table, the mortality of the clergy of all denominations is less than the average mortality of the male population, which may partly be attributed to the circumstance that those who devote themselves to the clerical profession are almost exclusively vigorous healthy men of sound constitutions. The rate is most favourable for the English protestant ministers.

In the first period (25 to 45) the mortality of the Church of England clergy is the lowest, and that of the protestant ministers is only slightly higher. The mortality of the catholic priests of the Roman and Greek Churches is very much higher, and is nearly equal to the average of the general male population.

In the second period (45 to 65) the rates remain substantially the same. We notice, however, that the mortality of the Roman catholic clergy is rather less in proportion, and is not nearly so much in excess of the rate indicated by Casper's Table, which latter still much exceeds the rate of the English protestant ministers.

In inquiring into the causes of the heavy mortality of the catholic priests, the first thing that occurs to us is that celibates show, on the whole, a greater mortality than married men; and we might be inclined rashly to reason thus:—the catholic clergy are all celibates—the protestants are all married—celibates, however, are known to suffer a greater mortality than married men—*ergo*, a greater mortality must be expected for the catholic priests. This conclusion is not quite correct. We must not forget that the high mortality of celibates arises partly from the fact that the weaker and more defective, and consequently shortlived, individuals do not marry, and further, that celibates usually lead irregular and often disorderly lives, and thus shorten their days. Neither of these reasons, however, applies to the catholic priests. The requirement that the individuals devoting themselves to this profession should be healthy and free from blemish, is more strictly enforced than in the case of protestant ministers, and all the priests have their own homes, and thus enjoy the same regularity of life as the protestant ministers. Further, the Greek catholic clergy are, mostly, married, and yet experience, at all ages, a much greater mortality than the protestants.

We are, therefore, compelled to accept Mr. Neison's hypothesis, that this high mortality is caused by the cares, hardships, and even dangers, which surround the catholic priest much more than the protestant minister in attending upon his flock. In this respect the catholic priest may perhaps be compared to the physician. Like the latter, if duty call him out, he must go, regardless of wind and weather; neither personal indisposition, nor the danger of infection, may deter him from bringing the last consolations to the dying.*

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* This explanation seems hardly conclusive, especially as there are only 209 lives observed, and no record has been made of the diseases which caused the deaths. Part of the mortality may perhaps be attributed to the diet, frequent fastings, &c.—D. A. B.

The circumstance that, for the ages 45 to 65, the mortality of the Greek catholic clergy is lower than that of the Roman catholic, might perhaps be explained by the fact that the former live in families, while the latter do not.

Let us notice, in conclusion, the tables and curves which are appended, showing the probabilities of dying in a year, and the numbers living at each age. The curve representing the probability of dying in a year by Casper's Table is, to age 31, nearly parallel with the curve for the general male population (Brune) but is slightly in favour of the clergy. While, however, the probabilities of dying by Brune's Table increase continuously, they fluctuate a good deal in Casper's Table, but are always less than those shown by Brune's curve, until age 63 and upwards, when they are greater. The curve of the Austrian (Ruthenian) observations proceeds more irregularly. Ranging above both the others at age 30, it sinks below both at age 36; then rises more quickly, and in the 50th year again overtakes the others. Then for 5 years it remains nearly stationary, as is the case in Casper's curve for the previous 5 years. From age 55 the curve begins quickly to mount, and from age 60 and upwards the probabilities of dying in a year, for the Ruthenian clergy, are considerably greater than those for the male population by Brune's Table.

The heavy mortality at the beginning of the table is, indeed, sufficiently explained by the fact, that in these years the clergyman, in entering upon his duties, undergoes a physical examination, in which his constitution is exposed to all possible attacks of weather and infection; and in which trial, as the curve shows, many a one fails. Observations of the last illnesses of those who die would demonstrate this still more plainly.

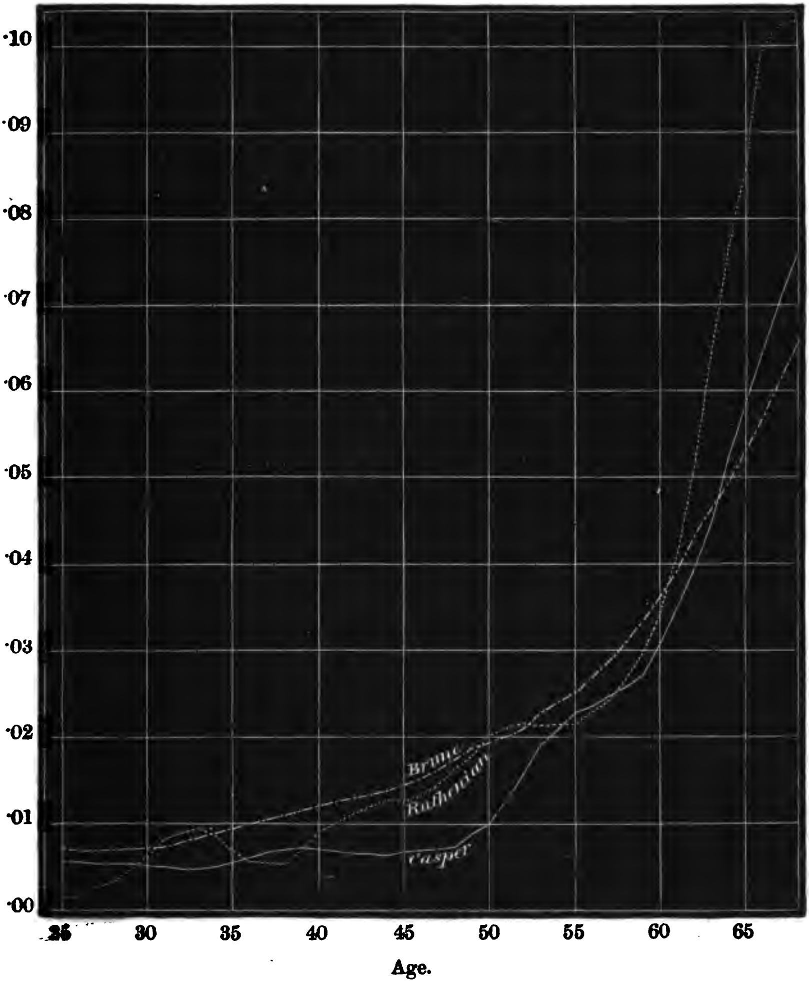
The number surviving at the end of each year, up to 40, out of 1000 clergymen aged 25 at entry, is, according to Casper's Table, not very different from that given by the present observations. From this point, however, there is a continually increasing difference, always in favour of Casper's clergy; so that, at age 68, there are 465 of the latter surviving, but only 358 of the Greek catholics. The number living, according to Brune's Table (Males), is always relatively less than the number of the clergy of both denominations at the same age. Only from age 66 downwards is the proportion unfavourable to the Greek catholic ministers: against 358 Greek ministers living at age 68, there are nearly 400 living, according to Brune's Table (Males).

## *Clergy Mortality Tables.*

| Age. | Casper's Observations. | | | | | | | Austrian (Ruthenian) Observations. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Unadjusted Number of | | Probable further Duration of Life in Years. | Probability of Dying in a Year per-cent. | | Adjusted Number of | | Unadjusted Number of | | Probability of Dying in a Year per-cent. | | Adjusted Number of | |
| | Living. | Dying. | | Unadjusted. | Adjusted. | Living. | Dying. | Living. | Dying. | Unadjusted. | Adjusted. | Living. | Dyi |
| 25 | 653 | 4 | 42 | ·61 | ·55 | 1000 | 5 | 42 | ... | ... | [illegible] | 1000 | 2 |
| 26 | 649 | 4 | 41 | ·61 | ·55 | 995 | 5 | 63 | ... | ... | [illegible] | 998 | 2 |
| 27 | 645 | 4 | 40 | ·61 | ·54 | 990 | 5 | 96 | ... | ... | [illegible] | 996 | 2 |
| 28 | 641 | 4 | 39 | ·62 | ·54 | 985 | 5 | 117 | ... | ... | [illegible] | 994 | 3 |
| 29 | 637 | 2 | 38 | ·31 | ·53 | 980 | 5 | 131 | 1 | ·76 | [illegible] | 991 | 4 |
| 30 | 635 | 4 | 37 | ·63 | ·52 | 975 | 5 | 150 | ... | ... | [illegible] | 987 | 6 |
| 31 | 631 | 2 | 36 | ·32 | ·50 | 970 | 5 | 154 | 2 | 1·30 | [illegible] | 981 | 8 |
| 32 | 629 | 5 | 35 | ·80 | ·49 | 965 | 5 | 174 | 1 | ·58 | [illegible] | 973 | 9 |
| 33 | 624 | 2 | 34 | ·32 | ·49 | 960 | 5 | 191 | 4 | 2·09 | [illegible] | 964 | 9 |
| 34 | 622 | 3 | 33 | ·48 | ·51 | 955 | 5 | 186 | 2 | 1·08 | [illegible] | 955 | 7 |
| 35 | 619 | 3 | 32 | ·48 | ·55 | 950 | 5 | 207 | 1 | ·49 | [illegible] | 948 | 7 |
| 36 | 616 | 1 | 32 | ·16 | ·60 | 945 | 6 | 209 | ... | ... | [illegible] | 941 | 6 |
| 37 | 615 | 7 | 31 | 1·14 | ·67 | 939 | 6 | 219 | 1 | ·46 | [illegible] | 935 | 5 |
| 38 | 608 | 4 | 30 | ·66 | ·70 | 933 | 7 | 231 | ... | ... | [illegible] | 930 | 5 |
| 39 | 604 | 7 | 29 | 1·16 | ·73 | 926 | 7 | 241 | 2 | ·81 | [illegible] | 925 | 6 |
| 40 | 597 | 3 | 28 | ·50 | ·69 | 919 | 6 | 235 | 3 | 1·28 | [illegible] | 919 | 8 |
| 41 | 594 | 3 | 27 | ·50 | ·66 | 913 | 6 | 225 | 2 | ·89 | [illegible] | 911 | 9 |
| 42 | 591 | 4 | 26 | ·68 | ·64 | 907 | 6 | 228 | 2 | ·88 | [illegible] | 902 | 10 |
| 43 | 587 | 4 | 25 | ·68 | ·63 | 901 | 6 | 245 | 4 | 1·63 | [illegible] | 892 | 11 |
| 44 | 583 | 2 | 24 | ·34 | ·61 | 895 | 5 | 229 | 4 | 1·74 | [illegible] | 881 | 11 |
| 45 | 581 | 4 | 23 | ·69 | ·66 | 890 | 6 | 226 | 1 | ·45 | [illegible] | 870 | 11 |
| 46 | 577 | 6 | 22 | 1·04 | ·68 | 884 | 6 | 218 | 3 | 1·38 | [illegible] | 859 | 11 |
| 47 | 571 | 2 | 21 | ·35 | ·70 | 878 | 6 | 201 | 2 | 1·00 | [illegible] | 848 | 12 |
| 48 | 569 | 4 | 20 | ·70 | ·75 | 872 | 7 | 214 | 4 | 1·87 | [illegible] | 836 | 14 |
| 49 | 565 | 6 | 19 | 1·06 | ·89 | 865 | 8 | 220 | 3 | 1·36 | [illegible] | 822 | 15 |
| 50 | 559 | 2 | 19 | ·36 | 1·00 | 857 | 9 | 203 | 5 | 2·46 | [illegible] | 807 | 16 |
| 51 | 557 | 8 | 18 | 1·40 | 1·32 | 848 | 11 | 186 | 5 | 2·69 | [illegible] | 791 | 17 |
| 52 | 549 | 5 | 17 | ·91 | 1·58 | 837 | 13 | 189 | 4 | 2·12 | [illegible] | 774 | 17 |
| 53 | 544 | 16 | 16 | 2·94 | 1·89 | 824 | 16 | 186 | 4 | 2·15 | [illegible] | 757 | 16 |
| 54 | 528 | 13 | 16 | 2·46 | 2·09 | 808 | 17 | 165 | 3 | 1·82 | [illegible] | 741 | 16 |
| 55 | 515 | 12 | 15 | 2·33 | 2·24 | 791 | 18 | 146 | 3 | 2·06 | [illegible] | 725 | 15 |
| 56 | 503 | 8 | 14 | 1·59 | 2·30 | 773 | 18 | 132 | 3 | 2·27 | [illegible] | 710 | 16 |
| 57 | 495 | 14 | 14 | 2·83 | 2·45 | 755 | 19 | 119 | 2 | 1·68 | [illegible] | 694 | 17 |
| 58 | 481 | 11 | 13 | 2·70 | 2·59 | 736 | 19 | 107 | 3 | 2·80 | [illegible] | 677 | 18 |
| 59 | 470 | 10 | 12 | 2·13 | 2·74 | 717 | 20 | 107 | 3 | 2·80 | [illegible] | 659 | 19 |
| 60 | 460 | 11 | 11 | 2·39 | 3·04 | 697 | 21 | 90 | 4 | 4·44 | [illegible] | 640 | 23 |
| 61 | 449 | 18 | 10 | 4·01 | 3·50 | 676 | 24 | 70 | 2 | 2·86 | 4·11 | 617 | 25 |
| 62 | 431 | 19 | 10 | 4·41 | 3·96 | 652 | 25 | 60 | 1 | 1·67 | 5·15 | 592 | 30 |
| 63 | 412 | 11 | 9 | 2·67 | 4·49 | 627 | 28 | 55 | 4 | 7·27 | 6·49 | 562 | 36 |
| 64 | 401 | 22 | 9 | 5·48 | 5·26 | 599 | 31 | 43 | 5 | 11·37 | 7·76 | 526 | 41 |
| 65 | 379 | 25 | 8 | 6·60 | 5·88 | 568 | 33 | 26 | 1 | 3·70 | 8·63 | 485 | 42 |
| 66 | 354 | 23 | 8 | 6·50 | 6·45 | 535 | 35 | 17 | 3 | 16·67 | 9·94 | 443 | 44 |
| 67 | 331 | 25 | 7 | 7·56 | 7·06 | 500 | 35 | 10 | 1 | 9·09 | 10·22 | 399 | 41 |
| 68 | 306 | 27 | 7 | 8·82 | 7·59 | 465 | 35 | 9 | 1 | 10·00 | 10·30 | 358 | 37 |
| 69 | 279 | 14 | 7 | 5·02 | ... | 430 | ... | 5 | | | | | |
| 70 | 265 | 25 | 6 | 9·44 | ... | ... | ... | 4 | | | | | |
| 71 | 240 | 24 | 5 | 10·00 | | | | | | | | | |
| 72 | 216 | 19 | 5 | 8·80 | | | | | | | | | |
| 73 | 197 | 21 | 5 | 10·66 | | | | | | | | | |
| 74 | 176 | 28 | 5 | 15·80 | | | | | | | | | |
| 75 | 148 | 14 | 5 | ... | | | | | | | | | |
| 76 | 134 | 18 | 5 | ... | | | | | | | | | |
| 77 | 116 | 14 | 4 | | | | | | | | | | |
| 78 | 102 | 11 | 4 | | | | | | | | | | |

*Clergy Mortality.*

**The ordinate represents the probability of dying in a year.**



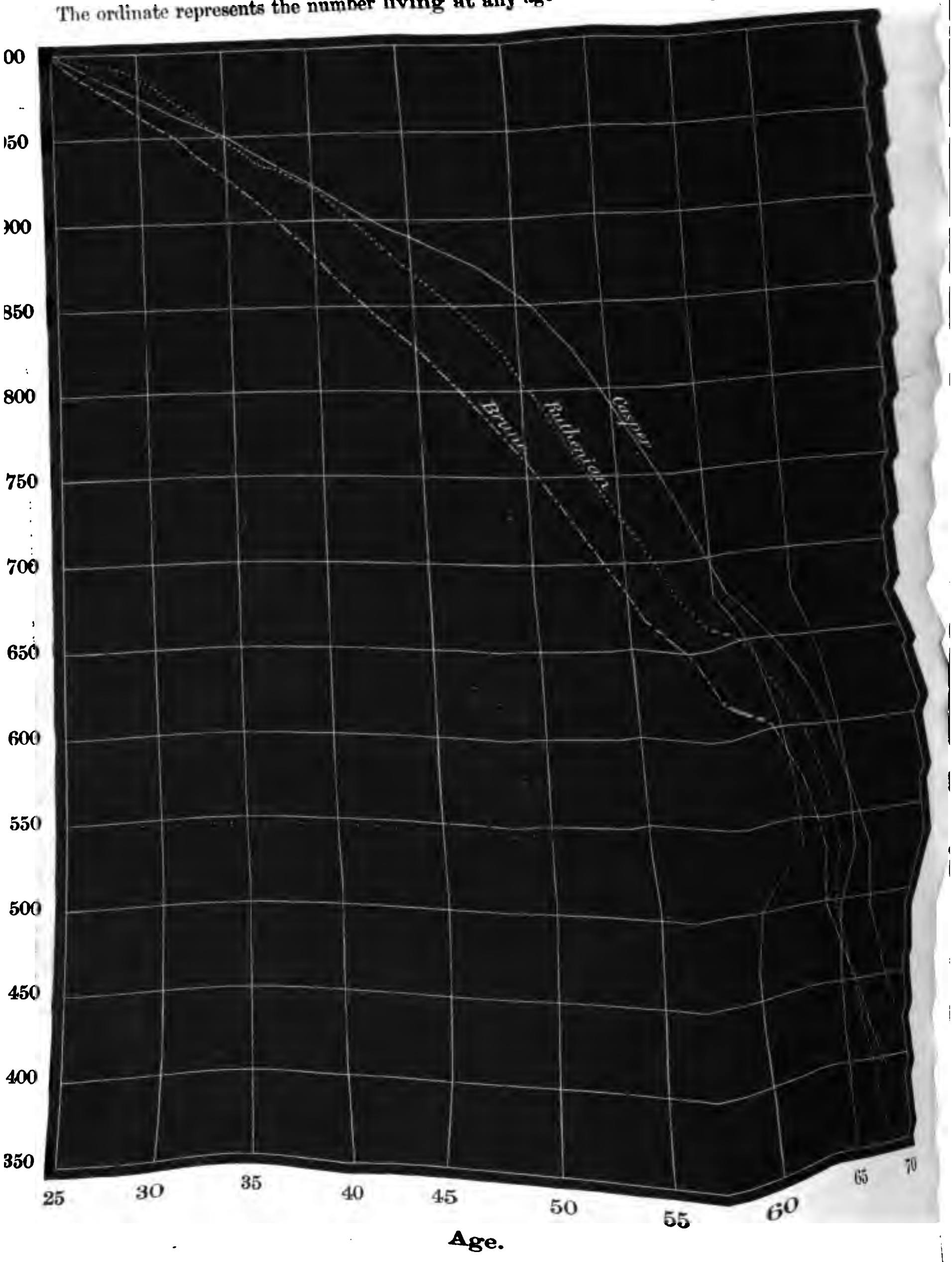
……………………… **Stüssi's observations on the Ruthenian clergy.**

——————— **Casper's „ „ Prussian „**

—·—·—·— **Brune's Table from experience of the Prussian Widows' Fund (Male Lives).**

## *Clergy Mortality.*

The ordinate represents the number living at any age out of 1000 living at 25.



## *The Growth of Life Companies.*

[From the *Spectator* of New York and Chicago.]

THE history of life insurance in England shows that it is to the degree of activity with which new business is sought, and not to superiority in age, that companies owe their ability to show large accumulations of assets. We reprint from the London *Review* the names and assets of the ten leading English life companies, in round figures.

| Founded. | Name. | Life Fund. |
|---|---|---|
| 1825 | Law Life | £5,500,000 |
| 1815 | Scottish Widows' | 5,400,000 |
| 1825 | Standard | 4,200,000 |
| 1762 | Equitable | 4,000,000 |
| 1835 | National Provident | 3,200,000 |
| 1806 | London Life | 3,100,000 |
| 1806 | Rock | 3,000,000 |
| 1807 | Eagle | 3,000,000 |
| 1823 | Economic | 2,700,000 |
| 1836 | Liverpool and London and Globe | 2,500,000 |

It will be seen that length of duration is a secondary agent in developing the assets of an English company.

A comparison with the ten American companies having the largest assets shows the same result;—that their relative rank in the list is the consequence of activity in the pursuit of new business and not of prior organizing. The following are the American companies with their assets, 31 December 1873, in round figures:

| Founded. | Name. | Assets. |
|---|---|---|
| 1843 | Mutual of New York | £13,120,000 |
| 1846 | Connecticut Mutual | 7,520,000 |
| 1845 | Mutual Benefit, of Newark | 5,700,000 |
| 1845 | New York Life | 4,960,000 |
| 1859 | Equitable, of New York | 4,560,000 |
| 1850 | Ætna, of Hartford | 3,840,000 |
| 1858 | Northwestern, of Milwaukee | 2,800,000 |
| 1837 | New England Mutual | 2,520,000 |
| 1850 | Charter Oak, of Hartford | 2,360,000 |
| 1851 | Phœnix, of Hartford | 1,800,000 |

These tables also illustrate the more rapid growth of the business of life insurance in America than in England. The total ages of the ten English companies enumerated above are exactly 600 years, and during those years they have accumulated assets to the amount of £36,600,000. The total years of the American companies are 256, and in that time they have accumulated assets of £49,180,000.

# HOME AND FOREIGN INTELLIGENCE.

## THE SCOTTISH EQUITABLE LIFE ASSURANCE SOCIETY.

*Established* 1831.

### Extract from Report by the Directors.

### *Position and Progress of the Business.*

The Society's position at the end of its forty-second year is as follows:—

| | |
|---|---|
| Number of Policies in Force (on the Lives of 9399 Members) | 10,972 |

Total Amount of Existing Assurances—

| | £ | s. | d. |
|---|---|---|---|
| Original Sums Assured | 6,109,867 | 0 | 0 |
| Vested Bonus Additions up to and including those of 1868 | 860,547 | 0 | 0 |
| Additions now declared | 306,818 | 0 | 0 |
| | £7,277,232 | 0 | 0 |

| | £ | s. | d. |
|---|---|---|---|
| Accumulated Funds | 2,033,721 | 0 | 0 |

Annual Income, as per "Revenue Account", consisting of—

| | | | |
|---|---|---|---|
| Premiums | £183,689 | 0 | 0 |
| Interest | 81,873 | 0 | 0 |
| Assignment Fees | 66 | 0 | 0 |
| | £265,628 | 0 | 0 |

The progressive increase of the business during the last thirty years is exhibited in the following table:—

| | Existing Assurances. | Annual Revenue. | Accumulated Fund. |
|---|---|---|---|
| | £ | £ | £ |
| At 1st March 1843 | 1,707,716 | 64,000 | 227,755 |
| Do. 1853 | 4,058,140 | 144,085 | 763,871 |
| Do. 1863 | 5,885,674 | 212,298 | 1,463,421 |
| Do. 1873 | 7,277,232 | 265,628 | 2,033,721 |

The Directors have now to report the results of the Eleventh Investigation into the affairs of the Society, with the view to the declaration of a bonus. While the present investigation is the *second* which has been conducted on the principles of the altered constitution adopted in 1867, it is the *first* which has taken place after an interval of five years as therein provided.

*Report of the Committee of Directors appointed to make the Eleventh Investigation into the Affairs of the Scottish Equitable Life Assurance Society, as at 1st March* 1873.

* * * * *

The valuations of the assets and liabilities under the whole policies in force at 1st March 1873, have been conducted on principles similar to those adopted in 1868, and the whole plan of the investigation was arranged, and many of its details were completed, under the immediate superintendence of their lamented Manager, before his last illness incapacitated him from attending to the Society's affairs. The investigation was completed by Mr. Finlay.

In submitting to the Court of Directors the results of the investigation, the Committee have to explain that the valuations of the assets and liabilities, so far as they depend on life contingencies, have been made on the basis of the Carlisle table of mortality, the rate of interest assumed in all the calculations being 3 per-cent, excepting in the valuations of the vested bonus additions to existing assurances, in which, as provided in the altered Constitution, the rate of 3½ per-cent interest has been assumed. In the participating assurances (being nineteen-twentieths of the whole business) the loading reserved is 26½ per-cent of the value of the net premiums. In non-participating assurances the loading reserved is 13½ per-cent of the net premiums. The annual sum thus reserved in making the valuation of the premiums is £35,299, and the present value thereof is £516,684.

The Committee now submit the following Valuation Balance-Sheet:—

| | | | | £ | s. | d. |
|---|---|---|---|---|---|---|
| 1. Gross Funds of the Society and Arrears . . . . | | | | 2,120,784 | 16 | 7 |
| 2. Liabilities— | | | | | | |
| (1.) Claims by Death under Policies which had emerged before 1st March 1873, but not matured . . . . | 82,088 | 1 | 1 | | | |
| (2.) Commission and other Charges applicable to the year to 1st March 1873 . . . | 4,975 | 0 | 4 | | | |
| | £87,063 | 1 | 5 | | | |
| (3.) Net Liability under the Society's Policies, as per Schedules to be recorded with the Board of Trade, conformably to Act of Parliament, 33 and 34 Vict., cap. 61. . | 1,855,033 | 0 | 0 | | | |
| | | | | 1,942,096 | 1 | 5 |
| There remains, after reserving the loading as above, a Surplus for Division of . . . . | | | | £178,688 | 15 | 2 |

In estimating the surplus which has arisen during the period since 1st March 1868, the contingent prospective additions which have been paid on policies becoming claims during that period must also be taken into account. These amount to £13,548, making, with the above sum now available for division, the total estimated surplus of the Quinquennium £192,236.

The whole of the above surplus of £178,688. 15*s.* 2*d.* now falls to be allocated in retrospective additions to all the participating policies in force at the date of investigation, and it will yield a reversionary bonus of £1 per centum per annum, amounting to an addition of £306,818 to the sums assured under all the policies of the Society, and payable at the death of the parties entitled thereto. This addition, as required by the resolutions of 1867, will be allocated in the case of policies to which additions have previously been made, not only on the sums originally assured, but also on the previously vested additions (not surrendered at the date of this investigation), and that for the five years since last investigation, at 1st March 1868, and in the case of policies which have not previously received additions,—that is, all policies issued since 1st March 1868, on the amount originally assured; and that for the number of years such policies shall have existed previous to 1st March 1873. These additions, in terms of the said resolutions, became vested at 1st March last in the case of all policies which were then of five years' standing and upwards; but in the case of each policy which was then of less than five years' standing, the additions will not become vested until such policy shall have been in force for five complete years.

The Committee also recommend, in accordance with the said resolutions, that besides the above retrospective additions, *contingent prospective additions* be made to all participating policies which may become claims between 1st March 1873 and 1st March 1878 (the date at which the next investigation falls to be made), and which, at the date of their so becoming claims, shall be of not less than five years' standing, and that at the rate of 15*s.* per-cent, being *three-fourths* of the retrospective additions now recommended; and that these contingent prospective additions be in proportion to the sums assured (including previously vested additions), and to the duration of the policies since 1st March 1873, and be charged upon, and paid out of, the surplus fund to be realized up to the period at which these contingent prospective additions shall become payable.

The Committee further recommend that vested additions may be surrendered by members, in whole or in part, for a present sum in money, or that these additions may be applied in extinction or reduction of the premiums payable for a period of five years, or for the remainder of the duration of the life assured; and that at the rates contained in the table already sanctioned by the Society.

The foregoing Report having been taken into consideration at a Court of Ordinary Directors held on 26th June, was unanimously approved of, and the Board of Directors allocated the vested additions and contingent prospective additions recommended by the Committee.

It was also resolved that these vested additions, as well as those formerly declared, may be at any time surrendered for their cash value, or may be applied to reduce the annual premiums payable by policyholders, either for a period of five years, or for the remainder of the lives assured at the rates contained in the table already sanctioned by the Society.

*The bonus now declared is nominally at the rate of* £1 *per-cent, but it will be kept in view that being allocated on the original sums assured, accumulated with the previously vested additions, it is equivalent to a bonus at the rate of* £1. 16*s.* 10*d. per annum on each* £100 *assured in the First year of the Society, of* £1. 14*s.* 4*d. on each* £100 *of the Fifth year, and of* £1. 11*s.* 2*d. on each policy of the Tenth year of the Society, and proportionally on policies of subsequent years.*

The Directors may further state that the additions declared at this and the previous investigations amount to £2,012,982, and a policy of £1000 effected in the first year of the Society is now increased to £1932. 18*s*. 8*d*., a policy of the fifth year to £1804. 1*s*. 5*d*., of the tenth year to £1634. 5*s*. 10*d*., and so on according to the number of years the policy has subsisted.

*The following is taken from the Chairman's address:—*

I now come to what may be called the most important branch of this year's Report, and that is the result of the quinquennial investigation into the affairs of the Society. You are aware that this is the first investigation under the new quinquennial system. From the institution of the Society down to 1868, the investigations were triennial. That was found to be in some respects not a very satisfactory method of proceeding, partly because the period of three years is too short to allow full scope for the system of averages, and partly because, owing to our system of accumulating the bonuses—that is to say, declaring at each investigation an addition to a policy, not only upon the amount originally assured, but also upon all the previously vested additions—the accumulation was found to increase too rapidly, and to give, perhaps, an undue advantage to some of the older policies. Another reason for making the change was, that, as you are aware, the original constitution of the Society had been framed upon calculations based upon the Northampton tables of mortality, and upon the assumption of interest being at 4 per-cent. For reasons, with a repetition of which I need not again trouble you, as they were fully explained at the time, that was considered to be not such a satisfactory basis for a life assurance society as one founded upon tables which more nearly approached the reality of the thing. The Northampton tables showed a greatly higher mortality than really existed, and the rate of interest assumed was higher than could be properly calculated upon as certainly to be maintained. We therefore, with your entire approbation, adopted a new system of valuation, taking the Carlisle tables of mortality, and making all our calculations on the assumption that we should not get more for our investments than 3 per-cent interest. Accordingly, in 1867 the new tables were adopted, and the investigation at 1868 was made on the basis of these tables. The period was then changed from a triennial to a quinquennial one, and we are now at the first quinquennial investigation, of which you have heard the results.

I should explain to you here that the valuation on the present occasion has, in order to make the security of the assured doubly sure, been conducted on even more rigid and stringent principles than ever has been the case hitherto. Not only have the ages of the members been taken with greater minuteness than was formerly considered necessary, but we have reserved a larger portion of the premiums than has hitherto been the case. We have reserved 26·513 of the value of the net premiums, or a trifle over 26½ per-cent, in valuing the participating policies, which constitute nineteen-twentieths of the business, and we have reserved 13½ per-cent of the premiums in valuing the non-participating policies, the loading on which is considerably smaller than in the case of the participating policies. The result is that we have, during the last five years, realized a profit of £192,236. Of that sum, £13,548 have been already paid to the holders of those policies which have become claims since 1st March 1868, by way of a contingent prospective bonus at the rate of 15*s.* per-cent, leaving a surplus of £178,688, which now falls to be divided. We found that, after making all the calculations, we were enabled to declare a bonus of one pound per-cent for each of the five years of the quinquennium upon each policy now in force, not merely upon the sum originally assured, but upon the sum originally assured *plus* all prior vested additions which have been made at prior investigations; and we have also resolved that a contingent prospective bonus of three-fourths of that amount—viz., 15*s.* per-cent—shall be added to each policy which shall be in force at present, and which shall become a claim before the next investigation on 1st March 1878; provided, of course, that at the death of the assured, the policy shall have been in force for at least five years. This bonus of £1 per-cent is nominally precisely the same as that which we declared at the previous investigation in 1868; but is in reality higher. You know the result of our system of allocating bonus upon bonus in the way which has been already explained. It gives, as it ought to do, some advantage to the older policies; that is to say, the more premiums a man pays,—the more the risk undertaken by the Society diminishes,—and the more profit the Society makes by retaining his shares of the past profits,—the greater ought to be his share of subsequent profits: at all events it appears to be only right and fair that he should obtain some additional benefit; and accordingly that is secured to him by allocating to him a bonus, not only upon the sums which he has originally insured, but also upon the additions which have been previously declared.

*Tabular view of Additions to each Policy of £1000.*

| Year of Society. | Effected on or before 1st March | Original Benefit. | Additions at 1st March 1873. | | | Accumulated Sum at 1st March 1873. | | | † Amount payable, if the Assured die after payment of the Premium in the years | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | Ending 1st March 1874. | | | Ending 1st March 1875. | | | Ending 1st March 1876. | | | Ending 1st March 1877. | | | Ending 1st March 1878. | | |
| | | | £ | *s.* | *d.* | £ | *s.* | *d.* | £ | *s.* | *d.* | £ | *s.* | *d.* | £ | *s.* | *d.* | £ | *s.* | *d.* | £ | *s.* | *d.* |
| 1 | 1832 | 1000 | 932 | 18 | 8 | 1932 | 18 | 8 | 1947 | 8 | 8 | 1961 | 18 | 7 | 1976 | 8 | 6 | 1990 | 18 | 5 | 2005 | 8 | 5 |
| 5 | 1836 | 1000 | 804 | 1 | 5 | 1804 | 1 | 5 | 1817 | 12 | 1 | 1831 | 2 | 8 | 1844 | 13 | 3 | 1858 | 3 | 10 | 1871 | 14 | 6 |
| 10 | 1841 | 1000 | 634 | 5 | 10 | 1634 | 5 | 10 | 1646 | 10 | 11 | 1658 | 16 | 1 | 1671 | 1 | 3 | 1683 | 6 | 5 | 1695 | 11 | 6 |
| 15 | 1846 | 1000 | 449 | 10 | 2 | 1449 | 10 | 2 | 1460 | 7 | 7 | 1471 | 5 | 0 | 1482 | 2 | 5 | 1492 | 19 | 10 | 1503 | 17 | 3 |
| 20 | 1851 | 1000 | 358 | 15 | 2 | 1358 | 15 | 2 | 1368 | 19 | 0 | 1379 | 2 | 10 | 1389 | 6 | 8 | 1399 | 10 | 5 | 1409 | 14 | 3 |
| 25 | 1856 | 1000 | 235 | 7 | 10 | 1235 | 7 | 10 | 1244 | 13 | 1 | 1253 | 18 | 5 | 1263 | 3 | 9 | 1272 | 9 | 0 | 1281 | 14 | 4 |
| 30 | 1861 | 1000 | 134 | 0 | 0 | 1134 | 0 | 0 | 1142 | 10 | 1 | 1151 | 0 | 2 | 1159 | 10 | 4 | 1168 | 0 | 5 | 1176 | 10 | 6 |
| 35 | 1866 | 1000 | 81 | 10 | 0 | 1081 | 10 | 0 | 1089 | 12 | 3 | 1097 | 14 | 5 | 1105 | 16 | 8 | 1113 | 18 | 11 | 1122 | 1 | 1 |
| 40 | 1871 | 1000 | *30 | 0 | 0 | 1030 | 0 | 0 | ... | | | ... | | | 1050 | 3 | 6 | 1060 | 18 | 0 | 1068 | 12 | 6 |
| 41 | 1872 | 1000 | *20 | 0 | 0 | 1020 | 0 | 0 | ... | | | ... | | | ... | | | 1050 | 12 | 0 | 1058 | 5 | 0 |
| 42 | 1873 | 1000 | *10 | 0 | 0 | 1010 | 0 | 0 | ... | | | ... | | | ... | | | ... | | | 1047 | 17 | 6 |

* The additions to policies effected after 1st March 1868 do not vest until the policies shall have been in force for five years.

† By a law of the Society passed on 14th November 1867, it is enacted that contingent prospective additions to policies shall be calculated at three-fourths of the vested additions declared at the Investigation immediately preceding the period at which the policies become claims. Contingent prospective additions are therefore entered in the preceding table at 15*s.* per-cent per annum.

TABLE *showing the Terms on which the vested Additions to Policies may be surrendered for a* PRESENT CASH PAYMENT, *or applied to* REDUCE THE ANNUAL PREMIUMS.

| Age next Birth-day. | Cash Value for each £10 of Bonus. | Deduction from Annual Premium for each £10 of Bonus. | | Age next Birth-day. | Cash Value for each £10 of Bonus. | Deduction from Annual Premium for each £10 of Bonus. | |
|---|---|---|---|---|---|---|---|
| | | For Five Years. | For Life. | | | For Five Years. | For Life. |
| | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* |
| 20 | 2 18 7 | 0 12 9 | 0 2 10 | 55 | 5 1 10 | 1 2 8 | 0 7 11 |
| 25 | 3 1 2 | 0 13 3 | 0 3 1 | 60 | 5 11 5 | 1 5 6 | 0 10 1 |
| 30 | 3 5 5 | 0 14 3 | 0 3 5 | 65 | 5 18 10 | 1 7 7 | 0 12 5 |
| 35 | 3 9 5 | 0 15 2 | 0 3 10 | 70 | 6 6 8 | 1 10 5 | 0 16 0 |
| 40 | 3 16 2 | 0 16 9 | 0 4 6 | 75 | 6 13 5 | 1 14 8 | 1 1 0 |
| 45 | 4 3 0 | 0 18 3 | 0 5 2 | 80 | 6 19 8 | 1 18 6 | 1 6 6 |
| 50 | 4 11 5 | 1 0 1 | 0 6 3 | | | | |

NOTE.—According to the preceding table, a member aged 50 next birthday may obtain, for each £10 of vested bonus surrendered, a *present cash payment* of £4. 11*s.* 5*d.*, or a *deduction from his annual premium* of £1. 0*s.* 1*d.* for five years, or of 6*s.* 3*d.* for the remainder of his life.

*The following additional particulars are taken from the returns under the Fifth and Sixth Schedules of the Life Assurance Companies Act,* 1870:—

## BALANCE SHEET *on the* 1*st March* 1873.

### LIABILITIES.

| | £ | *s.* | *d.* |
|---|---|---|---|
| Assurance Fund,—being the Total Funds, as per First Schedule . . . . . . . . . . | 2,033,721 | 15 | 2 |
| Claims admitted but not paid . . . . . . | 82,088 | 1 | 1 |
| Commission and other charges in connexion with Premiums in Agents' hands outstanding . . | 4,239 | 11 | 5 |
| Accounts for Advertising, Printing, &c., due but not paid . . . . . . . . . . . | 735 | 8 | 11 |
| | £2,120,784 | 16 | 7 |

### ASSETS.

| | £ | *s.* | *d.* |
|---|---|---|---|
| Mortgages on Property within the United Kingdom . | 1,088,915 | 16 | 10 |
| Mortgages on Property out of the United Kingdom (*none*) . . . . . . . . . . . | ......... | | |
| Loans on Statutory Assignments of County and other Rates* . . . . . . . . . | 226,003 | 18 | 11 |
| Loans on the Society's Policies (within their Surrender Value) . . . . . . . . | 217,270 | 8 | 11 |
| Loans on Reversionary and Life Interests . . . | 21,238 | 2 | 6 |
| INVESTMENTS:— | | | |
| In British Government Securities (*none*) . . | ......... | | |
| In Indian, Colonial, and Foreign Government Securities (*none*) . . . . . . . . | ......... | | |
| In Railway and other Debentures . . . | 284,500 | 0 | 0 |
| In Railway Shares (*none*) . . . . . . | ......... | | |
| Carried forward . . . . . | £1,837,928 | 7 | 2 |

* These were included in last year's Account under "Mortgages on Property".

| | £ | s. | d. |
|---|---|---|---|
| Assets brought forward | 1,837,928 | 7 | 2 |
| In House Property (Offices in Edinburgh, London, and Dublin) | 16,500 | 0 | 0 |
| In Bank of England Stock | 88,572 | 3 | 8 |
| In Government and other Life Annuities | 2,346 | 18 | 5 |
| In Feu-Duties (Ground-Rents) | 20,300 | 13 | 1 |
| Loans upon Personal Security (*none*) | ......... | | |
| Agents' Balances (since received) | 84,178 | 2 | 9 |
| Outstanding Premiums (do.) | 5,733 | 8 | 7 |
| Outstanding Interest (including £23,766. 17*s*. 9*d*. proportions of Interest from last terms of payment till 1st March 1873) | 24,477 | 11 | 10 |
| Cash in Bank on current and deposit Accounts | 40,722 | 1 | 8 |
| Cash on hand | 25 | 9 | 5 |
| | £2,120,784 | 16 | 7 |

The principles upon which the valuation and distribution of profits among the policyholders are made (as set forth in this answer and in Nos. 3 and 4) are determined by the Deed of Constitution, as amended by subsequent resolutions at general meetings of the Society. This provides that—

"*Primo*, An account shall be taken of the Gross Funds, Assets, and Property of the Society.

"*Secundo*, An Account shall be taken of the Whole Obligations of the Society and Charges on the Gross Fund.

"From these accounts of the Gross Fund, and of the Obligations of the Society, the surplus at each period of investigation shall be ascertained and declared. . . . The Directors shall have the power to allocate the whole surplus which shall then be ascertained, in retrospective additions to all the policies of participating members, issued up to the date of such investigation, and then in force; and the said additions shall be allocated, in the case of policies to which additions have previously been made, not only on the sums originally assured, but also on the previously *vested* additions (not surrendered at the date of investigation), and that for the number of years which shall have elapsed since the immediately preceding investigation; and in the case of policies which shall not have previously received additions on the amount originally assured, and that for the number of years such policies shall have existed previous to the investigation in question. . . . Besides the foresaid retrospective additions, contingent prospective additions shall be made on all participating policies which may become claims between one period of investigation and another, and which shall, at the date of their so becoming claims, be of not less than five years' standing, and that at the rate of three-fourths of the retrospective additions allocated at the immediately preceding investigation; which contingent prospective additions shall be in proportion to the sums assured (including all previously *vested* additions), and to the duration of the policies since the said immediately preceding investigation, and shall be charged upon and paid out of the surplus fund to be realized during the period for which such contingent prospective additions become payable."

The proportion of the annual premium income reserved as a provision for future expenses and profits is 20·398 per-cent in the case of participating policies, and 12·931 per-cent in the case of non-participating.

Every participating policy in force at the date of division receives an allocation of bonus; but in the case of a policy then of less than five years' endurance, the bonus does not vest until it shall have been in force for five years complete.

The total amount of profit made by the Society during the quinquennium was £192,236. 15*s.* 2*d.*

The contingent bonus payable on policies becoming claims between the previous and the present divisions, for each year's premium paid during that term, is chargeable on the profits of this quinquennium, and on this account £13,548 has been paid since 1st March 1868.

The balance of £178,688. 15*s.* 2*d.* was allocated in additions of £306,818 to the sums assured by the whole existing policies effected on the participation scheme, being 10,677 in number, the sums thereby assured and previous bonuses amounting to £6,674,958. 3*s.* 5*d.*

The vested bonus is allocated to policies which have not participated in previous divisions at an equal rate per-cent on the sum assured, irrespective of age, according to the number of premiums paid; and to policies which have previously participated, at the same rate per-cent for five years on the sums assured, increased by previous bonuses which have not been surrendered.

*Specimens of Additions to Policies of £100, provided previous Bonuses have not been surrendered.*

DURATION OF POLICY.

| FOR ALL AGES AT ENTRY. | 5 Years. | 10 Years. | 15 Years. | 20 Years. | 25 Years. | 30 Years. | 35 Years. | 40 Years. |
|---|---|---|---|---|---|---|---|---|
| | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* |
| | 5 0 0 | 5 5 0 | 5 11 1 | 6 3 0 | 6 13 6 | 7 6 10 | 8 2 1 | 8 17 11 |

A policyholder is entitled at any time to receive the value of the vested additions as a present payment, or to commute them into deductions from the premiums either for five years or for life, at rates varying according to age. A table of these values is printed on the Bonus Notice, from which the following are extracts:—

*Value of £10 of vested Bonus Additions.*

| AGE NEXT BIRTHDAY. | Present Payment. | Deduction from Annual Premium. | |
|---|---|---|---|
| | | For Five Years. | For Life. |
| | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* |
| 20 | 2 18 7 | 0 12 9 | 0 2 10 |
| 30 | 3 5 5 | 0 14 3 | 0 3 5 |
| 40 | 3 16 2 | 0 16 9 | 0 4 6 |
| 50 | 4 11 5 | 1 0 1 | 0 6 3 |
| 60 | 5 11 5 | 1 5 6 | 0 10 1 |
| 70 | 6 6 8 | 1 10 5 | 0 16 0 |
| 80 | 6 19 8 | 1 18 6 | 1 6 6 |

### Consolidated Revenue Account *for five years, commencing 2nd March* 1868, *and ending* 1*st March* 1873.

| | £ | s. | d. |
|---|---|---|---|
| Amount of Funds on 1st March 1868, the beginning of the quinquennial period | 1,777,650 | 12 | 11 |
| Premiums (after deduction of Reassurance Premiums) | 887,002 | 10 | 8 |
| Consideration for Annuities granted (*none*) | ......... | | |
| Interest and Dividends (less Income Tax) | 390,989 | 16 | 4 |
| Other Receipts— | | | |
| Assignment Fees | 330 | 7 | 6 |
| | £3,055,973 | 7 | 5 |

| | £ | s. | d. |
|---|---|---|---|
| Claims under Policies (after deduction of Sums Reassured) | 884,739 | 6 | 10 |
| Surrenders | 45,649 | 18 | 10 |
| Annuities (*none*) | ......... | | |
| Commission | 31,433 | 3 | 3 |
| Expenses of Management | 57,155 | 12 | 2 |
| Dividends and Bonuses to Shareholders (*none*) | ......... | | |
| Other Payments— | | | |
| Reduction in market price of Bank of England Stock, and Valuation of House Property, &c., at 1st March 1873 | 3,273 | 11 | 2 |
| Amount of Funds on 1st March 1873,—the end of the period | 2,033,721 | 15 | 2 |
| | £3,055,973 | 7 | 5 |

### Summary and Valuation of the Policies, (see p. 365).

### Valuation Balance Sheet, *as at* 1*st March* 1873.

| Dr. | £ | s. | d. |
|---|---|---|---|
| To Net Liability under Assurance transactions (as per Summary Statement) | 1,855,033 | 0 | 0 |
| To Surplus | 178,688 | 15 | 2 |
| | £2,033,721 | 15 | 2 |

| Cr. | £ | s. | d. |
|---|---|---|---|
| By Life Assurance Funds (as per Balance-Sheet) | 2,033,721 | 15 | 2 |
| | £2,033,721 | 15 | 2 |

The average rate at which the whole realized funds of the Society were invested at the close of each year during the period since the last investigation was at follows:—

| | £ | s. | d. |
|---|---|---|---|
| Year ending 1st March 1869 | 4 | 5 | 8 |
| " " 1870 | 4 | 4 | 11 |
| " " 1871 | 4 | 5 | 4 |
| " " 1872 | 4 | 5 | 0 |
| " " 1873 | 4 | 5 | 11 |

The Society has no table of minimum values for the surrender of policies. For participation policies effected at the ordinary tabular

| DESCRIPTION OF TRANSACTIONS. | Number of Policies. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | | | VALUATION. | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Value by the Carlisle table, Interest 3 per-cent, except Bonus 3½ per-cent. | | | |
| | | Sums Assured and Bonuses. | Office Yearly Premiums. | | | Net Yearly Premiums as Valued. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Yearly Premiums. | Net Liability. |
| ASSURANCES. | | | | | | | | | | |
| I.—With Participation in Profits. | | £ | £ | s. | d. | £ | £ | £ | £ | £ |
| Single Lives—Whole Term of Life . . | 10,470 | 6,566,989 | 176,196 | 4 | 4 | 140,257 | 3,747,620 | 2,410,000 | 1,918,420 | 1,829,200 |
| Do. do., Limited No. of Premiums | 104 | 42,554 | 1,768 | 1 | 2 | 1,407 | 19,260 | 14,957 | 11,906 | 7,354 |
| Two Joint Lives do. . . . . . . | 17 | 9,730 | 430 | 11 | 10 | 343 | 6,497 | 5,000 | 3,980 | 2,517 |
| Two Joint Lives and Survivor . . . | 86 | 55,685 | 594 | 13 | 7 | 473 | 24,496 | 10,555 | 8,402 | 16,094 |
| Extra Premiums payable . . . . | ... | ... | 1,165 | 15 | 6 | ... | ... | ... | ... | ... |
| Total Assurances with Profits . . | 10,677 | 6,674,958 | 180,155 | 6 | 5 | 142,480 | 3,797,873 | 2,440,512 | 1,942,708 | 1,855,165 |
| II.—Without Participation in Profits. | | | | | | | | | | |
| Single Lives—Whole Term of Life . . | 136 | 159,453 | 5,212 | 3 | 6 | 4,688 | 83,252 | 77,698 | 69,772 | 13,480 |
| Do. Endowment Assurances . | 89 | 26,550 | 1,098 | 2 | 3 | 986 | 15,872 | 14,188 | 12,756 | 3,116 |
| Do. Short Term Assurances . | 31 | 44,910 | 611 | 11 | 11 | 402 | 1,828 | 2,371 | 1,645 | 183 |
| Do. Survivorships . . . . | 16 | 27,793 | 416 | 0 | 0 | 332 | 3,771 | 4,111 | 3,162 | 608 |
| Two Joint Lives—Whole Term of Life . | 1 | 100 | 4 | 5 | 5 | 4 | 57 | 63 | 54 | 3 |
| Two Joint Lives and Life of Survivor . | 11 | 12,450 | 253 | 2 | 2 | 220 | 5,579 | 4,475 | 3,880 | 1,699 |
| Single Lives—Ascending Scale of Premiums | 11 | 24,200 | 335 | 10 | 0 | 273 | 10,211 | 12,087 | 9,927 | 284 |
| Extra Premiums payable . . . . | ... | ... | 29 | 15 | 0 | ... | ... | ... | ... | ... |
| Total Assurances without Profits . | 295 | 295,456 | 7,960 | 10 | 3 | 6,905 | 120,570 | 114,993 | 101,196 | 19,373 |
| Total Assurances . . . . . | 10,972 | 6,970,414 | 188,115 | 16 | 8 | 149,385 | 3,918,444 | 2,555,505 | 2,043,904 | 1,874,539 |
| Deduct Reassurances . . . . . | ... | 205,140 | 6,672 | 5 | 11 | 5,280 | 100,052 | 99,014 | 80,546 | 19,506 |
| Total of the Results . . | 10,972 | 6,765,274 | 181,443 | 10 | 9 | 144,105 | 3,818,391 | 2,456,491 | 1,963,358 | 1,855,032 |

rates on which three years' premiums have been paid, the minimum surrender value at present allowed is thirty-three per-cent of the premiums paid, and in addition the published value of the vested bonus.

For non-participation policies (whole life) and for endowment assurances after payment of three premiums, the minimum value allowed is twenty-five per-cent of the premiums paid.

The extra premiums charged for foreign residence have been thrown off before making the valuations. The additions to the premiums on under-average lives (which are few in number) have also been thrown off in the valuations, the annual extra being held to cover the additional risk.

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## LIFE ASSOCIATION OF SCOTLAND.

*Established* 1838.

### VALUATION AS AT 5 APRIL 1871.

### Valuations of Liabilities.

The Abstracts of the Valuations of the Association's liabilities have been made up, like the Statements of the Accounts, closely in accordance with the forms in which they must be deposited with the Board of Trade. They will be published along with the Statements of every Life Office in the kingdom in such full detail as that any actuary, if so disposed, will be able to check the calculations for himself. Notwithstanding the publicity thus to be given to details which have hitherto been only placed on the table at the annual meetings, the Directors have found no reason to make any change in the tables or principles on which the valuations have hitherto been conducted, and they have the fullest conviction that the Statements will bear the strictest scrutiny, whether as regards the valuations themselves, or the amount of surplus arisen since the preceding investigation.

The tables of mortality used by the Association were furnished at almost the commencement of its business, by the late Mr. Griffith Davies, and stated at the time to have been deduced from the experience of some of the principal London Life Offices. They show, in fact, a mortality somewhat less than that of the Equitable Society's experience published by Mr. Davies in 1825, the last age of life being 97. The rate of interest used by the Association has been always 3½ per-cent for Life Assurances. The Annuities have been valued by the same mortality tables at 4 per-cent interest. The calculations were made in the first place, as usual, by the Manager and his assistants; and not only have they been checked and verified by Professor Kelland of the Edinburgh University, one of the Directors, who has taken an interest in the valuations from the commencement of the Institution,—but they have been submitted to and certified also by the Auditor of the Association's accounts—Mr. William Wood, Chartered Accountant and Actuary. The results are fully exhibited by the annexed "Summary of the Valuations" and "Valuation Balance Sheet". As on former occasions, the whole margin over and above the net premiums has been left unvalued. This margin is shown by the "Summary of the Valuations" to be equivalent in present money to so much as £656,300, and is reserved to meet the expenses and form part of the surplus in future years.

### ALLOCATION OF SURPLUS.

As stated in the "Valuation Balance Sheet", the surplus on hand at the date of balance was £131,530. 9*s.* 11*d.*

| | | |
|---|---|---|
| The surplus arisen during the four years since 1867, the date of the last formal valuations, together with £80,503. 8*s.* 11*d.* of undivided surplus then in the funds, has been . . . . . . . . | | £292,393 7 7 |
| Of which, there has been paid during the four years in cash bonuses to the policy-holders in Class A. . . . | £142,862 17 8 | |
| and dividends to shareholders | 18,000 0 0 | |
| | | 160,862 17 8 |
| Leaving on hand at 5th April 1871, as above | | £131,530 9 11 |

*Policies in Class B.*—Of the surplus, the proportion arising from the assurances in Class B, enables the Directors to declare to the participating policyholders in that class as at 5th December 1870, contingent reversionary bonus additions to their sums assured, at the same rate as was allocated at the declaration of bonus in 1865, namely, £4 per £100 assured for every year's premium paid since the last division of surplus. This rate of bonus applies to all ordinary assurance policies effected by annual premiums payable during life, or during a limited number of years. Proportionate bonuses will be added to the other participating policies. These new bonus additions amount to £163,521. 3*s.* 2*d.*, requiring £28,022. 5*s.* 10*d.* in present cash, and that sum has been set aside to meet it. The next allocation of bonus in this class will fall to be made at the end of another period of five years.

*Policies in Class A.*—As above stated, the policyholders in Class A have received out of the balance of surplus in hand in 1867 and the surplus that has since accrued, cash bonuses to the amount of £142,862. 17*s.* 8*d.* The Directors now allocate to these policyholders, cash bonuses for the year ending 5th April 1872, at the rates they now enjoy, namely:—37½ per-cent of the premiums for the current year on the first series of policies, effected previously to 6th April 1855,—30 per-cent of the premiums on policies effected in the year ending 5th April 1856,—and 25 per-cent of the premiums on all subsequent participating policies, forming the second series, and dated on or before 5th April 1866. These cash bonuses amount to £38,811, and will be applied in part payment of the current year's premiums.

The Directors are gratified in being able thus to maintain the same rates of bonus to both classes of policyholders as were previously declared, while many other Life Offices have of late years declared smaller bonuses than formerly. Including £38,811 now allocated in

Class A, for the current year, not less than £483,907. 8*s*. 9*d*. in cash bonuses will have been paid up to this time to the policyholders in that class alone. From the past progress of the Institution, and the results of the frequent investigations that have been made into the funds, the Directors have good reason to anticipate that the surplus which will arise in future, will be fully as ample as in the past. Formal and precise valuations of the liabilities, as required by the Act of Parliament, will be made every five years, the next in 1876; but intermediate valuations will be submitted to the Directors, in order to show the progress of the funds as compared with the relative liabilities, and observations will be made from time to time, of the mortality that may be experienced.

The Directors have further resolved to carry a sum of £12,500 to the paid-up capital or guarantee fund, which will be thus increased to £87,500. The remainder of the surplus, amounting to £52,197, will be left unallocated and carried back into the general funds, subject to the deduction of a small sum to meet part of the current year's dividend.

The dividend for the current year will be at the same rate as formerly, namely 11 per-cent on the capital of £87,500. A portion of this was paid on 15th May last, and the remainder will be paid on 11th November proximo, per dividend warrants, which will be forwarded by post to each shareholder.

### MORTALITY.

The following table shows the number of deaths occasioned by the several classes of disease during the four years since 1867. The Directors regret to state that, although they have continued to exclude with the utmost vigilance all persons not perfectly temperate, the fatal diseases in not a few cases, and without distinction of country, originated in or were aggravated by intemperance.

*Abstract of the Causes of Death during Four Years.*

| | | | |
|---|---|---|---|
| 162 | of diseases of brain and nervous system. | 23 | of cancer and other malignant diseases. |
| 231 | „ respiratory organs. | 6 | „ gout and rheumatism. |
| 103 | „ heart and great vessels. | 54 | „ fevers and contagious diseases. |
| 115 | „ liver, stomach, and bowels. | 13 | „ other diseases. |
| 35 | „ kidneys and urinary organs. | 43 | „ accidents and such like. |
| 25 | of exhaustion, debility, and decay. | 810 | total from all causes. |

*The following further particulars are taken from the returns under the Fifth and Sixth Schedules of the Life Assurance Companies Act.*

### STATEMENT respecting the VALUATION of the LIABILITIES under the Policies and Annuities of the Life Association of Scotland.

The values are calculated precisely, and according to the after-mentioned table of mortality. The policies on lives assured at higher premiums than the ordinary rates are dealt with as i of correspondingly higher ages.

The principles on which profits are distributed am holders, and the proportion of the profits so distributed, by the Directors from time to time. In Class A the prof in cash in proportion to the annual life premium payal sponding to each policy, and are applied in part paym due premiums, or paid in cash to the holders of paid-u the next anniversary of the commencement of the ris the profits are applied in providing reversionary b payable along with the sums assured at death if the liv their respective "Expectation of Life" according t table. These bonus additions for whole of life policie rate for every year's premium paid or year elapsed sin distribution.

The rate of interest used in the calculations is 3½ assurance policies and bonus additions, and 4 per-cent

The proportion of the annual premiums reserved a future expenses and profits, and therefore left unval margin between these premiums and the relative deduced from the above mentioned table at 3½ per-cen margin ranges from 25 to 32½ per-cent on the pu participating policies, and from 10 to 20 per-cent for policies.

The Consolidated Revenue Account for the four y 6th April 1867 and ending 5th April 1871, is as foll

| | | |
|---|---|---|
| Amount of Funds at 6th April 1867 . . . . . | | 1 |
| Being as then reported . . . | £1,106,053 7 5 | |
| *Less*—Claims on Policies then intimated, but not payable and since paid; also outstanding sums then receivable but not since paid, and cross Reassurances . . . | 69,164 18 2 | |
| Premiums less Reassurances . . . . . . | | |
| Consideration for Annuities granted . . . . . | | |
| Interest less Income Tax . . . . . . . | | |
| Increase in Value of Investment . . . . . | | |
| | | £2 |
| Claims under Policies (after deduction of sums Reassured) . . . . . . . . . . | | |
| Surrenders . . . . . . . . . . . | | |
| Annuities . . . . . . . . . . . | | |
| Commission . . . . . . . . . . | | |
| Expenses of Management and all other Charges . . | | |
| Interest on Paid-up Capital and Dividends . . . . | | |
| Cash Bonuses paid to Policyholders in abatements from Premiums and otherwise . . . . . . | | |
| Amount of Funds at 5th April 1871 . . . . . | | |
| | | £ |

### *Summary and Valuation of the Policies as at 5th April 1871.*

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | | VALUATION. |
|---|---|---|---|---|---|
| | Number of | Sums Assured and | Office Yearly | Net Yearly | VALUE by the Mortality Tables† of the Association, at 3½ per-cent interest for Assurances, and at 4 per-cent for Annuities. |

| | Policies. | Bonuses.* | Premiums. | Premiums. | Sums Assured and Bonuses.* | Office Yearly Premiums. | Net Yearly Premiums. | Net Liability. |
|---|---|---|---|---|---|---|---|---|
| ASSURANCES. | | | | | | | | |
| I.—With Participation in Profits. | | £ | £ | £ | £ | £ | £ | £ |
| For Whole Term of Life | 14,973 | 6,226,008 | 195,721 | 153,207 | 3,026,679 | 2,711,576 | 2,122,153 | 904,527 |
| Limited number of Premium Payments | 498 | 183,610 | 7,414 | 5,763 | 71,609 | 73,371 | 56,994 | 14,615 |
| Premiums ceasing with Joint Lives | 1 | 500 | 18 | 14 | 258 | 184 | 145 | 113 |
| Increasing Assurances (Ultimate Sums Assured) | 83 | 76,850 | 1,871 | 1,460 | 27,813 | 28,748 | 22,483 | 5,330 |
| Joint Lives | 261 | 87,733 | 4,148 | 3,247 | 50,633 | 48,330 | 37,821 | 12,812 |
| Last Survivor | 5 | 2,332 | 31 | 24 | 677 | 628 | 486 | 190 |
| Endowment Assurances | 812 | 197,150 | 8,568 | 6,676 | 114,888 | 95,233 | 74,130 | 40,758 |
| Extra Premiums payable (temporary) | .. | .. | 2,145 | .. | 1,072 | .. | .. | 1,072 |
| Total Assurances with Profits | 16,633 | 6,774,183 | 219,916 | 170,391 | 3,293,629 | 2,958,070 | 2,314,212 | 979,417 |
| II.—Without Participation in Profits. | | | | | | | | |
| For Whole Term of Life | 560 | 551,907 | 17,997 | 16,231 | 284,785 | 220,822 | 199,152 | 85,633 |
| Limited number of Premium Payments | 2 | 900 | 42 | 36 | 379 | 42 | 36 | 343 |
| Increasing Premiums | 2 | 2,500 | 45 | 41 | 1,055 | 1,132 | 1,029 | 26 |
| Premiums ceasing with Joint Lives | 3 | 300 | 12 | 11 | 216 | 65 | 59 | 157 |
| Future Premiums commencing after Joint Lives fail | 2 | 12,070 | 308 | 280 | 4,272 | 3,408 | 3,098 | 1,174 |
| Increasing Assurances (Ultimate Sums Assured) | 76 | 105,900 | 2,303 | 1,960 | 39,108 | 37,866 | 32,226 | 6,882 |
| Joint Lives | 6 | 5,900 | 381 | 347 | 4,034 | 3,150 | 2,864 | 1,170 |
| Last Survivor | 6 | 10,200 | 212 | 192 | 4,294 | 3,445 | 3,132 | 1,162 |
| Endowment Assurances | 6 | 2,700 | 152 | 138 | 1,790 | 1,400 | 1,273 | 518 |
| Short Periods and Survivorships | 78 | 68,390 | 912 | (a) .. | 754 | .. | .. | 754 |
| Miscellaneous Contingent Assurances | 5 | 13,000 | .. | .. | 650 | .. | .. | 650 |
| Endowments | 16 | 3,500 | 54 | (b) .. | 1,970 | .. | .. | 1,969 |
| Extra Premiums payable (temporary) | .. | .. | 196 | .. | 98 | .. | .. | 98 |
| Total Assurances without Profits | 762 | 777,266 | 22,614 | 19,236 | 343,405 | 271,330 | 242,869 | 100,536 |
| TOTAL ASSURANCES | 17,395 | 7,551,450 | 242,530 | 189,627 | 3,637,034 | 3,229,400 | 2,557,081 | 1,079,953 |
| Deduct Reassurances | .. | 282,399 | 8,682 | 7,303 | 138,562 | 104,908 | 88,889 | 49,672 |
| NET AMOUNT OF ASSURANCES | .. | 7,269,051 | 233,848 | 182,324 | 3,498,472 | 3,124,492 | 2,468,192 | 1,030,281 |
| ANNUITIES. | | Per annum. | | | | | | |
| Immediate | 330 | 13,999 | .. | .. | .. | .. | .. | 105,101 |
| Survivorship and Deferred | 8 | 745 | 382 | 300 | .. | .. | .. | 1,508 |
| TOTAL of the RESULTS, being the whole present NET LIABILITY for LIFE ASSURANCES and ANNUITIES | | | | | | | | £1,136,890 |

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* The Bonus Additions included in this column amount to £34,535, and are Reversions payable at death, only if the lives attain to at least their expectation of life from the ages at entry. The great mass of the assurances participate in the surplus by cash bonuses, and not by bonus additions to the sum assured.

† These tables were furnished in 1839 to the Association by the late Mr. Griffith Davies, as having been deduced from the experience of several London Life Offices, and show a mortality somewhat less than that of the Equitable Experience Tables published by him in 1825.

(a) The net premiums for short-period and survivorship assurances were not used in the Valuation, a proportion of the current year's premiums being held as the liability.

(b) The end[illegible] premiums are mostly returnable in the event of death. The total amount of office premiums received for these, with accumulated interest thereon, is stated [illegible] liability; the [illegible] being used

### *Valuation Balance Sheet as at 5th April* 1871.

| Dr. | £ | *s.* | *d.* |
|---|---|---|---|
| To Net Liability under Assurance and Annuity Transactions, as per preceding Summary Statement | 1,136,889 | 12 | 8 |
| To Surplus . . . . . . . . . . | 131,530 | 9 | 11 |
| | £1,268,420 | 2 | 7 |

| Cr. | £ | *s.* | *d.* |
|---|---|---|---|
| By Life Assurance and Annuity Funds as per Balance Sheet under Schedule 2, deposited with the Board of Trade in July last . . . . | 1,307,409 | 1 | 7 |
| Add further Values of Government Annuities and other Stocks, and Value of Contingent Reversion and Bonuses on Policies held from other Offices, since ascertained . . . . . . . . . . | 10,683 | 2 | 2 |
| | £1,318,092 | 3 | 9 |
| Deduct Values of Reassurances (included among the Assets in Schedule 2 as "Policies with other Offices"), now deducted from the Net Liability in this Balance Sheet, as required by the Act . . . . | 49,672 | 1 | 2 |
| | £1,268,420 | 2 | 7 |

In Class A a participating policy must be in force for six years in order to entitle it to share in the profits. In Class B bonus additions are made to every participating policy in force, whatever its standing.

(1) The amount of surplus existing at 5th April 1871, as per the above Balance Sheet, is £131,530. 9*s.* 11*d.*

(2) The amount of surplus divided among the policyholders is £66,833. 5*s.* 10*d.* in cash and in present value for bonus additions. The policies which participate are 12,928 for £5,189,887 of sums assured. £12,500 of the surplus is added to the paid-up capital or guarantee fund, and the remainder is left unallocated.

(3) In Class A the cash bonuses are 37½ per-cent of the next annual premiums on policies opened on or before 5th April 1855; 30 per-cent of the next annual premiums on policies opened from 6th April 1855 to 5th April 1856; and 25 per-cent of the next annual premiums on all subsequent participating policies opened on or before 5th April 1866.

In Class B the contingent reversionary bonus additions to the sums assured, as before explained, are at the same rate per annum per £100 at all ages. The following are specimens for policies of £100:—

| Duration of Policy | One Year's Premium paid. | Two Years' Premiums paid. | Three Years' Premiums paid. | Four Years' Premiums paid. | Five Years' Premiums paid. |
|---|---|---|---|---|---|
| Bonus Addition . . . | £4 | £8 | £12 | £16 | £20 |

The average rate of interest yielded by the funds actually invested (exclusive of all uninvested moneys and floating balances) at 5th April of the following years was:—

| | £ | s. | d. | |
|---|---|---|---|---|
| 1868 . . . . . . | 4 | 8 | 7 | per-cent. |
| 1869 . . . . . . | 4 | 9 | 7 | „ |
| 1870 . . . . . . | 4 | 9 | 11 | „ |
| 1871 . . . . . . | 4 | 9 | 6 | „ |

The values allowed at present for the surrender of assurance policies and for endowments and endowment assurances, are the results of a correct valuation of each policy by the mortality table in use by the Association, under deduction of a varying percentage. The following are specimens of such values for whole of life policies and endowment assurances:—

VALUES OF POLICIES OF £100, EXCLUSIVE OF BONUSES.

| Age at Entry. | Assurances for Whole of Life. | | | | | | | | | | | | Endowment Assurances Payable at Death or Age 60. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Duration. | | | | | | | | | | | | Duration. | | | | | | | | |
| | 5 Years. | | | 10 Years. | | | 20 Years. | | | 30 Years. | | | 5 Years. | | | 10 Years. | | | 20 Years. | | |
| | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. | £ | s. | d. |
| 20 | 2 | 4 | 5 | 6 | 8 | 7 | 13 | 18 | 1 | 23 | 9 | 9 | 3 | 9 | 10 | 10 | 6 | 6 | 24 | 0 | 5 |
| 30 | 2 | 16 | 4 | 8 | 3 | 5 | 18 | 13 | 1 | 29 | 18 | 6 | 5 | 6 | 3 | 15 | 17 | 7 | 39 | 1 | 9 |
| 40 | 4 | 1 | 6 | 11 | 15 | 3 | 24 | 8 | 2 | 38 | 10 | 6 | 9 | 14 | 7 | 29 | 8 | 10 | | ... | |
| 50 | 5 | 10 | 3 | 15 | 0 | 0 | 31 | 14 | 10 | 47 | 17 | 3 | | ... | | | ... | | | ... | |
| 60 | 7 | 8 | 6 | 20 | 18 | 6 | 41 | 1 | 7 | 55 | 6 | 5 | | ... | | | ... | | | ... | |

The Association do not transact business at other than their European rates of premium. The temporary extra premiums in the accompanying tabular statements of the policies include any charges for residence abroad; and these are arbitrary, not being based on specific mortality tables for the foreign climates.

Policies on unhealthy or under average lives, assured at higher premiums than the ordinary rates for their true ages, are dealt with in the valuations as of the higher ages corresponding to the premiums charged.

# CORRESPONDENCE.

## TABLES DEDUCED FROM Mr. SAMUEL BROWN'S COMBINED INDIAN EXPERIENCE.

*To the Editor of the Journal of the Institute of Actuaries.*

Sir,—Having lately taken occasion to compute complete tables for annuities and annual premiums of assurance based on Mr. Samuel Brown's valuable tables of mortality deduced from the combined experience of the Bengal, Madras, and Bombay Military Funds, and the Bengal and Madras Civil Service Funds, respectively (Tables A(1) and A(4), pp. 208–210, vol. xvi of the *Journal of the Institute*), I enclose copies thereof, which you may, perhaps, think worthy of a place in the *Journal.*

The calculation has brought to light four typographical errors in the original tables. In Table A(1), $\log p_{75}$ should be $\bar{1}\cdot96056$, $\log p_{84}$ should be $\bar{1}\cdot92467$; and in Table A(4), $\log p_{16}$ should be $\bar{1}\cdot99258$, and $\log p_{27}$ $\bar{1}\cdot99268$.

I am, Sir,
Your most obedient servant,
H. AMBROSE SMITH.

2 *King William Street, London,*
14 *May* 1874.

*Annuities and Annual Premiums, at* 3 *per Cent, per Brown's Indian Table* A(1), *Military Lives,* Institute Journal, *vol. xvi, pp.* 208 *and* 209.

| Age. | Annuity. | Annual Premium. | Age. | Annuity. | Annual Premium. |
|---|---|---|---|---|---|
| 15 | 16·9909 | ·026457 | 55 | 11·9168 | ·048293 |
| 16 | 16·8880 | ·026777 | 56 | 11·6485 | ·049935 |
| 17 | 16·7865 | ·027097 | 57 | 11·3670 | ·051734 |
| 18 | 16·6859 | ·027416 | 58 | 11·0719 | ·053711 |
| 19 | 16·5867 | ·027735 | 59 | 10·7619 | ·055894 |
| 20 | 16·4892 | ·028052 | 60 | 10·4361 | ·058316 |
| 21 | 16·3930 | ·028368 | 61 | 10·0932 | ·061020 |
| 22 | 16·2984 | ·028683 | 62 | 9·7526 | ·063875 |
| 23 | 16·2053 | ·028996 | 63 | 9·4145 | ·066894 |
| 24 | 16·1145 | ·029304 | 64 | 9·0791 | ·070089 |
| 25 | 16·0253 | ·029610 | 65 | 8·7466 | ·073474 |
| 26 | 15·9384 | ·029911 | 66 | 8·4173 | ·077062 |
| 27 | 15·8539 | ·030207 | 67 | 8·0914 | ·080869 |
| 28 | 15·7720 | ·030497 | 68 | 7·7688 | ·084915 |
| 29 | 15·6930 | ·030779 | 69 | 7·4497 | ·089222 |
| 30 | 15·6166 | ·031055 | 70 | 7·1342 | ·093812 |
| 31 | 15·5434 | ·031321 | 71 | 6·8223 | ·098714 |
| 32 | 15·4666 | ·031603 | 72 | 6·5163 | ·103917 |
| 33 | 15·3860 | ·031902 | 73 | 6·2169 | ·109438 |
| 34 | 15·3013 | ·032219 | 74 | 5·9241 | ·115298 |
| 35 | 15·2124 | ·032555 | 75 | 5·6382 | ·121518 |
| 36 | 15·1185 | ·032914 | 76 | 5·3594 | ·128122 |
| 37 | 15·0198 | ·033297 | 77 | 5·0881 | ·135129 |
| 38 | 14·9155 | ·033706 | 78 | 4·8246 | ·142560 |
| 39 | 14·8057 | ·034142 | 79 | 4·5691 | ·150435 |
| 40 | 14·6901 | ·034608 | 80 | 4·3222 | ·158766 |
| 41 | 14·5683 | ·035107 | 81 | 4·0841 | ·167568 |
| 42 | 14·4393 | ·035644 | 82 | 3·8479 | ·177150 |
| 43 | 14·3029 | ·036221 | 83 | 3·6124 | ·187681 |
| 44 | 14·1583 | ·036844 | 84 | 3·3959 | ·199401 |
| 45 | 14·0054 | ·037517 | 85 | 3·1357 | ·212670 |
| 46 | 13·8434 | ·038244 | 86 | 2·8885 | ·228043 |
| 47 | 13·6721 | ·039031 | 87 | 2·6292 | ·246416 |
| 48 | 13·4904 | ·039885 | 88 | 2·3503 | ·269352 |
| 49 | 13·2980 | ·040814 | 89 | 2·0410 | ·299712 |
| 50 | 13·0943 | ·041825 | 90 | 1·6847 | ·343357 |
| 51 | 12·8784 | ·042928 | 91 | 1·2574 | ·414161 |
| 52 | 12·6533 | ·044116 | 92 | ·8732 | ·504056 |
| 53 | 12·4184 | ·045398 | 93 | ·5372 | ·621425 |
| 54 | 12·1733 | ·046785 | 94 | ·2468 | ·772931 |

*Annuities and Annual Premiums, at* 3 *per Cent, per Brown's Indian Table* A(4), *Civil Lives,* Institute Journal, *vol. xvi, pp.* 209 *and* 210.

| Age. | Annuity. | Annual Premium. | Age. | Annuity. | Annual Premium. |
|---|---|---|---|---|---|
| 15 | 19·3222 | ·020085 | 56 | 12·0243 | ·047653 |
| 16 | 19·2453 | ·020268 | 57 | 11·7207 | ·049486 |
| 17 | 19·1642 | ·020467 | 58 | 11·4116 | ·051443 |
| 18 | 19·0789 | ·020678 | 59 | 11·0972 | ·053538 |
| 19 | 18·9889 | ·020902 | 60 | 10·7766 | ·055788 |
| 20 | 18·8943 | ·021140 | 61 | 10·4496 | ·058213 |
| 21 | 18·7926 | ·021393 | 62 | 10·1198 | ·060804 |
| 22 | 18·6880 | ·021666 | 63 | 9·7872 | ·063576 |
| 23 | 18·5781 | ·021951 | 64 | 9·4514 | ·066554 |
| 24 | 18·4626 | ·022255 | 65 | 9·1118 | ·069766 |
| 25 | 18·3410 | ·022577 | 66 | 8·7680 | ·073248 |
| 26 | 18·2133 | ·022921 | 67 | 8·4196 | ·077041 |
| 27 | 18·0790 | ·023288 | 68 | 8·0644 | ·081199 |
| 28 | 17·9379 | ·023678 | 69 | 7·7020 | ·085789 |
| 29 | 17·7896 | ·024095 | 70 | 7·3315 | ·090897 |
| 30 | 17·6343 | ·024538 | 71 | 6·9512 | ·096634 |
| 31 | 17·4712 | ·025012 | 72 | 6·5832 | ·102744 |
| 32 | 17·3072 | ·025497 | 73 | 6·2276 | ·109226 |
| 33 | 17·1415 | ·025996 | 74 | 5·8875 | ·116067 |
| 34 | 16·9749 | ·026507 | 75 | 5·5638 | ·123234 |
| 35 | 16·8072 | ·027031 | 76 | 5·2583 | ·130659 |
| 36 | 16·6381 | ·027569 | 77 | 4·9758 | ·138224 |
| 37 | 16·4679 | ·028122 | 78 | 4·7184 | ·145749 |
| 38 | 16·2960 | ·028691 | 79 | 4·4925 | ·152950 |
| 39 | 16·1226 | ·029276 | 80 | 4·3042 | ·159399 |
| 40 | 15·9477 | ·029879 | 81 | 4·0585 | ·168543 |
| 41 | 15·7716 | ·030498 | 82 | 3·8924 | ·175259 |
| 42 | 15·5873 | ·031161 | 83 | 3·7148 | ·182983 |
| 43 | 15·3940 | ·031871 | 84 | 3·5201 | ·192073 |
| 44 | 15·1920 | ·032633 | 85 | 3·4067 | ·197788 |
| 45 | 14·9806 | ·033450 | 86 | 3·1894 | ·209592 |
| 46 | 14·7589 | ·034330 | 87 | 2·9427 | ·224534 |
| 47 | 14·5268 | ·035279 | 88 | 2·6575 | ·244288 |
| 48 | 14·2837 | ·036303 | 89 | 2·3221 | ·271861 |
| 49 | 14·0286 | ·037414 | 90 | 1·9209 | ·313243 |
| 50 | 13·7611 | ·038619 | 91 | 1·4306 | ·382300 |
| 51 | 13·4804 | ·039933 | 92 | 0·9922 | ·472808 |
| 52 | 13·1965 | ·041314 | 93 | 0·6083 | ·592620 |
| 53 | 12·9092 | ·042768 | 94 | 0·2773 | ·752390 |
| 54 | 12·6184 | ·044304 | 95 | ... | ·970874 |
| 55 | 12·3237 | ·045929 | | | |

## ON THE FORMULA USED BY MR. JELLICOE IN THE GRADUATION OF THE EAGLE INSURANCE COMPANY'S EXPERIENCE.

*To the Editor of the Journal of the Institute of Actuaries.*

SIR,—Having recently had occasion to examine the method of graduation described by Mr. Jellicoe in his valuable paper "On the Rate of Mortality in the Eagle Insurance Company", in vol. iv of the *Journal,* I found that no demonstration was given of the working formula, as set forth on p. 206. In a footnote on that page, Mr. Jellicoe says, "The mode of obtaining this expression I defer till a future occasion, when I propose to examine all the methods that have been devised for the purpose in question." As the proposed explanation has not been published, a short demonstration of the formula will probably be considered of sufficient interest, even at this interval of time, to justify its insertion in the pages of the *Journal.* At any rate, future students will be saved some time and trouble in obtaining the formula for themselves.

The method of graduation is as follows:—If $\Sigma q'_x$ denote the sum of the unadjusted rates of mortality for $n$ successive ages beginning with age $x$, and $\Sigma q'_{x+n}$ denote a similar quantity beginning with age $x+n$, then the adjusted rate of mortality for ages $x$ to $x+2n-1$ inclusive, is made to increase in geometric progression, the common ratio $r$ being found from the equation,

$$r^n = \frac{\Sigma q'_{x+n}}{\Sigma q'_x} = p$$

$$\text{or} \quad r = p^{\frac{1}{n}}.$$

Taking, now, $a$ as the starting age, and denoting the adjusted rate of mortality at that age by $q_a$, we have

$$\begin{aligned}\Sigma q'_a = m &= q_a + q_{a+1} + q_{a+2} + \ldots\ldots + q_{a+n-1}\\ &= q_a + p^{\frac{1}{n}} q_a + p^{\frac{2}{n}} q_a + \ldots\ldots + p^{\frac{n-1}{n}} q_a\end{aligned}$$

whence $\quad q_a = \frac{m}{1-p}\left(1 - p^{\frac{1}{n}}\right)$

$$q_{a+1} = \frac{m}{1-p}\left(p^{\frac{1}{n}} - p^{\frac{2}{n}}\right)$$

$$\ldots\ldots\ldots$$

$$\begin{aligned}q_x &= \frac{m}{1-p}\left(p^{\frac{x-a}{n}} - p^{\frac{x-a+1}{n}}\right)\\ &= \frac{m}{p-1}\left(p^{\frac{x-a+1}{n}} - p^{\frac{x-a}{n}}\right)\\ &= 10^{\log\frac{m}{p-1} + (x-a+1)\frac{\log p}{n}} - 10^{\log\frac{m}{p-1} + (x-a)\frac{\log p}{n}}\\ &= \Delta 10^{\log\frac{m}{p-1} + (x-a)\frac{\log p}{n}}\end{aligned}$$

which is the formula as given on p. 206, with the slight difference of $p-1$ in lieu of $1-p$. This error in Mr. Jellicoe's formula did not, however, affect the results, in consequence of $\log\frac{m}{1-p}$ having been taken as equivalent to $\log\frac{m}{p-1}$.

I am, Sir,
Your obedient servant,
WM. SUTTON.

21 *Fleet Street*,
16 *September* 1874.

## Mr. HARDY'S VALUATION TABLES.

*To the Editor of the Journal of the Institute of Actuaries.*

Sir,—It may be useful to point out that at page 9 of Mr. Hardy's *Valuation Tables*, $\log a_{55}$ is printed as ·028563 instead of ·082563, and $\log a_{83}$ as ·849818 instead of ·489818.

I am, Sir,
Your most obedient Servant,
GEO. M. LOW.

*English and Scottish Law Life Assurance Office*,
120 *Princes Street, Edinburgh*,
14 *September* 1874.

Mr. E. Justican has communicated the following correction in the same work:—

Page 22, $x+n=50$, $P_{29.\overline{21}|}$ should be ·03900 and not ·03991.

# INSTITUTE OF ACTUARIES.

## FIRST YEAR'S EXAMINATION, 1873.

### I.

1. Given $x=·06253$, $y=·12785$, $z=·000912$.
Find the value to 5 decimal places of $\frac{x^2+y-3z}{4x-y^2-7z}$.

2. Express $\frac{1}{3}+\frac{1}{3^3}+\frac{1}{5\times3^5}+\frac{1}{7\times3^7}+\frac{1}{9\times3^9}$ correctly to 5 places of decimals.

3. Solve the equation

$$\frac{25-\frac{1}{3}x}{x+1}+\frac{16x+4\frac{1}{5}}{3x+2}=5+\frac{23}{x+1}.$$

4. Solve the equations

$$\frac{x^2+1}{4x^2-1}=\frac{x}{1+2x}-\frac{1}{4}.$$

$$\left.\begin{aligned} x^2+xy=a^2 \\ y^2+xy=b^2 \end{aligned}\right\}$$

5. The sum of the first 9 terms of an arithmetical series is 126, and the 10th term is 29. Find the sum of the first 15 terms of the series.

6. Assuming the formula for the sum of $n$ terms of a geometrical series, whose first term and common ratio are given: if $S_n$ represent this sum, find the value of $S_1+S_2+S_3+\ldots\ldots+S_n$.

7. The difference between the 4th and 5th terms of a geometrical series is twice the difference between the 5th and 6th terms, and the sum of the series *ad infinitum* is equal to 8: find the first term and common ratio.

8. The coefficient of the 5th term of $(1+x)^n$ is 6 times that of the preceding term: find the coefficient of the last term but two.

9. Show that the coefficient of the $(n+1)$th term of $(1+x)^{2n}$ is twice the coefficient of the $(n+1)$th term of $(1+x)^{2n-1}$.

10. Expand $(ax-x^2)^{\frac{2}{5}}$ to 5 terms, and write down the general term of $(a^{\frac{1}{3}}-x^{-\frac{1}{3}})^{-4}$.

11. Find the number of combinations of 12 things taken $r$ together; $r$ being such that the number of combinations formed may be the greatest possible.

12. How many different permutations can be made out of the letters in the word "*Examination*" taken all together; and in how many of such permutations will *e* be the first letter?

13. Out of 20 white and 8 black balls, how many collections can be made, each composed of 3 white and 2 black balls?

14. Given $\log 2=\cdot3010300$, and $\log 3=\cdot4771213$.

Find the logarithms of 675, ·00216, and $\frac{1}{\sqrt{6}}$

15. If $\left(1+\frac{1}{n}\right)^n$ be expanded by the binomial theorem, and $n$ be increased without limit (so that quantities of the form $\frac{1}{n}$, $\frac{2}{n}$, &c. may

be neglected), to what well known function in the theory of logarithms will such expansion become equal?

16. From the series $\log_e(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\ldots.$ deduce a converging series, from which the logarithm of either of two consecutive numbers can be found when we know that of the other.

17. Define what is meant by the modulus of a system of logarithms.

18. What is the probability that if a shilling be tossed four times it will fall head once at least?

19. There are $n$ different urns containing black and white balls; the whole number of balls in each urn being respectively $m_1$, $m_2$, $m_3$, .... $m_n$, and the number of white balls respectively $p_1, p_2, p_3, \ldots. p_n$. A ball is taken *at random* from one of the urns; what is the probability of its being a white ball?

20. From a collection of 36 balls, comprising 10 green, 12 white, and 14 red balls, 3 are to be drawn. What is the probability that two out of the three drawn will be green and the other white?

21. Find $\Delta^n a^x$ when $x$ is variable, the increment of $x$ being unity.

22. Given log 235=2·3710679.
„ 236=2·3729120.
„ 237=2·3747483.
„ 238=2·3765770.
Find log 23563.

23. State the respective merits of book-keeping by Single and Double Entry.

24. What are the principal books required in book-keeping by Single Entry and Double Entry respectively?

25. What do the Trial Balance Sheet and the Final Balance Sheet contain, and what is the difference between them?

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## SECOND YEAR'S EXAMINATION, 1873.

### I.

1. Two lives $x$ and $y$ being proposed, find the probability that $x$ will die before $y$.

2. Two offices have each £1,000,000 assured. Office A by 1000 policies of £1000 each, and office B by 2000 policies of £500 each. Assuming all the ages equal and the rate of mortality to be 1 per cent per annum, find an expression for the probability in each case that the claims will amount to £20,000 in the course of a year.

3. Deduce a formula for finding the value of a deferred annuity certain.

4. What is meant by "force of interest"? Show how th "force" is obtained from the "rate" of interest.

5. The Guardians of a Poor Law Union are desirous of borrowing

£5000; the loan to be discharged and the interest paid by an equal half-yearly charge upon the rates extending over thirty years. Assuming that the rate of interest at which the loan is obtained is 5 per cent per annum, find what the half-yearly charge should be. Given $v^{60}$ at $2\frac{1}{2}$ per cent to be ·2272836.

Draw up a schedule showing what portions of each of the first four payments are applicable for the payment of interest and the discharge of the capital account.

6. Describe the ordinary form of a mortality table; and mention the tables which are generally constructed from it in order to make it available for monetary calculations.

7. Given a table of the values of annuities on two joint lives for every combination of quinquennial ages, explain how to approximate to the value of an annuity for two intermediate ages.

8. Give a proof of the expression

$$A_x = v(1+a_x) - a_x.$$

9. State the approximate increase to be made to the value of a life annuity payable yearly when the same is to be paid half-yearly, quarterly or monthly.

10. Given a table of annual premiums for assurances for the whole of life, how would you construct a table of half-yearly premiums?

11. Find a formula for the value of a reversionary annuity so as to return one rate of interest while the annuity is in reversion, and another rate when it is in possession.

12. What addition should be made to the value of a curtate annuity payable yearly $(a_x)$, to obtain the value of a complete annuity payable half-yearly $(\overset{c}{a}{}_x^{(2)})$?

13. A sum of money $(s)$ is to be applied in the purchase of an annuity on three lives, $x$, $y$, $z$, such that the annual payment while all three are alive shall be A, when one has died $\frac{3}{4}$ A, and when only one survives $\frac{2}{3}$ A. Find the value of A.

## II.

14. Explain the principles upon which Mr. Woolhouse has constructed his "Improved Theory of Annuities and Assurances", and state briefly the advantage of the proposed system.

15. Investigate the formula for finding the annual premium for a contingent assurance. If asked to quote a premium, how would you proceed?

16. How would you proceed to form D and N columns; and what checks would you use to secure accuracy?

17. Show how to construct a table of premiums for the assurance of £1 for the term of one year, by a continuous process.

18. Show that $D_x - M_x = (1-v)N_{x-1}$.

19. Required the value of a policy for £100, effected at age 40, which has been in force for 5 years and 7 months.

$$\begin{aligned} \text{Given } P_{40} &= 2{\cdot}589 \\ a_{40} &= 17{\cdot}176 \\ a_{45} &= 15{\cdot}594 \\ a_{46} &= 15{\cdot}260 \end{aligned}$$

20. Give a form of endorsement to be placed upon a policy in order to substitute a half-yearly for an annual premium.

21. Draw up a form of Policy Register, omitting the consideration of bonus.

22. What does the following formula represent

$$\frac{D_x}{D_{x+n}}(P_x - P^1_{x\overline{n}|}).$$

Prove your answer.

23. A at the age of $x$ effected an ordinary whole life assurance at a premium $P_x$; $n$ years having elapsed he wishes to convert it into a contingent assurance, the counter life being aged $y$; what premium should he pay on the new policy?

24. Investigate an expression for the $n$th presentation to a living.

25. Find a formula to express the amount to which a life annuity-due will on the average accumulate at the end of the year of death, supposing each payment to be invested and accumulated at compound interest.

# JOURNAL

OF THE

# INSTITUTE OF ACTUARIES

AND

# ASSURANCE MAGAZINE.

## *On the Rate of Mortality among Adult Government Emigrants on the Voyage to Australia, during the Years* 1847–1861 *inclusive, as determined from the Reports of the Emigration Commissioners. By* J. J. McLAUCHLAN, A.F.A., *of the Scottish Equitable Life Assurance Society.*

THE object of the following paper is to describe an investigation recently made, to determine, from observed facts, the risk incurred by persons making voyages to the Australian Colonies. This investigation was carried out under the superintendence of Mr. Sprague, at whose suggestion the following account of it was prepared.

For more than thirty years past, an Annual Report has been presented to Parliament by the Emigration Commissioners; and a set of those Reports, extending from the 1st to the 25th inclusive (with the exception of the 22nd), supplied the required material. Among the other duties of the Commissioners, is that of chartering ships for the conveyance of emigrants, either wholly or partly at the public expense, to certain of the colonies; and in the Appendixes to their 8th and subsequent Reports, they have given returns of the number of ships and emigrants despatched by them annually to each of the colonies. Among other information, the returns contain the number of adult emigrants of each sex (ages not stated) embarking in each vessel, the duration of the voyage, and the number of adult males and females that died before reaching their destination. Those returns for the years 1847–1861 inclusive, as summarized in the 24th Report, pp. 80, 81, were selected to form the basis of the present enquiry.

As but a small proportion of the total number of emigrants proceeded to Western Australia, Queensland, and New Zealand, the investigation was confined to the mortality among those making the voyages to New South Wales, Victoria, and South Australia. From the summary referred to above, the following Table (A) is extracted. It shows the number of ships despatched to New South Wales, Victoria, and South Australia respectively, the total numbers of adult males and females embarked for each voyage, and the total numbers of each sex that died before reaching their destination, during the period over which the observations extend.

TABLE A.

| Voyage. | No. of Ships. | Males embarked. | Males died. | Females embarked. | Females died. |
|---|---|---|---|---|---|
| New South Wales......... | 281 | 33,152 | 160 | 36,051 | 241 |
| Victoria...................... | 313½* | 27,814 | 190 | 50,232 | 399 |
| South Australia ......... | 225½* | 22,217 | 115 | 25,315 | 161 |
| Total ................. | 820 | 83,183 | 465 | 111,598 | 801 |

The deaths entered in the summary are exclusive of those arising from the wreck of vessels. From the 20th Report, p. 17, and the Appendixes to the succeeding Reports, we learn that the only loss of life by wreck during the period 1847–1861, was in the case of the "Guiding Star", which sailed for Melbourne in 1855, and never reached her destination. By a comparison of the original return for 1855 with the summary, we see that this vessel is included in the number of those despatched; the 328 adult emigrants—118 males and 210 females—who sailed on board of her, being included in the number embarked, but not in the number dying upon the voyage. They are not included in the numbers embarked as given in Table A.

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* One ship, in 1853, conveyed emigrants both to Victoria and South Australia.

Table B is similar to Table A, except that it contains the deaths arising from the loss of the "Guiding Star", distributed over the three Australian voyages in proportion to the number of emigrants embarked upon each. For purposes of comparison, Table B may be considered fairly to represent the mortality experienced.

TABLE B.

| Voyage. | No. of Ships. | Males embarked. | Males died. | Females embarked. | Females died. |
|---|---|---|---|---|---|
| New South Wales......... | 281⅓ | 33,199 | 207 | 36,119 | 309 |
| Victoria...................... | 313⅚ | 27,853 | 229 | 50,326 | 493 |
| South Australia ......... | 225⅚ | 22,249 | 147 | 25,363 | 209 |
| Total .................. | 821 | 83,301 | 583 | 111,808 | 1011 |

In order to know whether the rate of mortality shown by Tables A and B is excessive or otherwise, it is necessary to compare it with that which prevails among a similarly constituted body of persons in this country; and with this view it was resolved to calculate the expected deaths according to Dr. Farr's English Life Table, No. III. As a preliminary to doing so, it is necessary to make some assumption as to the ages of the persons embarking, these not being given in the annual returns, or elsewhere in the Reports; and also to determine the average duration of each of the three voyages during the period 1847–1861.

There are contained in the Appendix to the 25th Report (pp. 72–78), tables giving an analysis of the mortality in ships despatched by the Commissioners to each of the Australian Colonies, for a period of years ending 31st December 1864,—the number of years being 10 for New South Wales, Victoria, and South Australia, 9 for Western Australia, and 5 for Queensland. From those tables may be obtained, classified according to age, the total number of adults, without distinction of sex, that died on each of the voyages during the above periods. The numbers dying in each of the following intervals of age are grouped together, 10–19, 20–29, 30–39, 40–49, 50–59, 60 and upwards. This information is contained in the following Table C, which shows the ages of the adult emigrants who died on the voyage to each of the Australian Colonies during certain periods of years ending 31st December 1864.

TABLE C.

| Voyage. | Deaths, 13-19. | Deaths, 20-29. | Deaths, 30-39. | Deaths, 40-49. | Deaths, 50-59. | Deaths, 60 and upwrds. | Deaths at all Adult Ages. |
|---|---|---|---|---|---|---|---|
| New South Wales, 1855–64 | 29 | 83 | 27 | 21 | 7 | 6 | 173 |
| Victoria, 1855–64 ........... | 17 | 60 | 13 | 8 | 4 | 1 | 103 |
| South Australia, 1855–64 .. | 13 | 36 | 12 | 14 | 3 | 0 | 78 |
| Western Australia, 1856–64 | 1 | 2 | 1 | 0 | 0 | 0 | 4 |
| Queensland, 1860–64 ...... | 4 | 16 | 3 | 1 | 2 | 2 | 28 |
| Total ................. | 64 | 197 | 56 | 44 | 16 | 9 | 386 |

It will be noticed that the first interval of age in Table C is 13–19, not 10–19, as given in the original analysis of mortality. Previous to the year 1856, all emigrants of 14 years of age and upwards are returned as adults, and appear in the Summary under that head. A change in regard to this was, however, introduced by the Passengers' Act of 1855, which fixed the adult age at 12, and from the succeeding year the numbers of adults given in the returns and summary include all persons of 12 years old and upwards. The change having been made about the middle of the period under observation, the adult age was, for the purpose of this inquiry, taken at 13. In order to determine the number dying in the interval of age 13–19, required for Table C, from the number dying in the interval 10–19, we observe that, according to the English Table III, about 72 percent of the annual deaths among persons of the ages 10–19 are those of persons above 13 years of age; and the numbers given in Col. 2, Table C, are obtained by taking 72 percent of the numbers given in the Report as dying between ages 10 and 20.

It was also assumed that all emigrants described in the analyses of mortality as over 60 years of age might without serious error be taken to be under 70. It does not seem probable that assisted passages would be granted to persons of greater age than this, even if they were supposed willing to emigrate.

The totals of the columns of Table C thus represent the numbers of adults, in each of seven intervals of age, who died on the voyage to Australia, during a certain period of years. It was considered not unreasonable to assume that the deaths appearing in Tables A and B might be distributed between the different ages in the proportions indicated by Table C; that is, that the number of deaths at each age might be supposed to bear the same proportion to the number at all ages, for each of the voyages during the period 1847–1861, as for the whole Australian voyages from 1855–1864 inclusive. This is not, however, quite satisfactory, as the analyses of mortality, from which Table C is prepared, give only the total number dying at each age without distinction of sex, and we are therefore obliged to assume that the male and female deaths take place at the same ages, which may or may not be true.

Having thus obtained the ages of the dying, the ages of the emigrants embarking may be obtained by assuming that the normal rate of mortality among them is increased or diminished by their special circumstances, in the same proportion at all ages. It was therefore assumed that the ratio of the number of lives entering upon any year to the number dying in that year, according to the rate of mortality prevailing on the Australian voyages, was equal to that of the English Life Table III, multiplied by a constant quantity.

To facilitate the determination of the numbers exposed to risk, the following Table D was constructed. It contains the required values of the function $\frac{L_x - L_{x+n}}{l_x - l_{x+n}}$ (see *English Life Table*, pp. 26, 27, 30, 31), the numerator of which represents $l_x + l_{x+1} + l_{x+2} + \ldots\ldots + l_{x+n-1}$, being the whole number of persons entering upon risk at ages $x$ to $x+n-1$ inclusive; and the denominator, $d_x + d_{x+1} + d_{x+2} + \ldots\ldots + d_{x+n-1}$, being the annual number dying between the ages $x$ and $x+n$. The fraction thus represents the ratio which the number of persons of all ages from $x$ to $x+n-1$ entering upon risk bears to the annual number of deaths between ages $x$ and $x+n$; assuming the lives to be distributed over the different ages in the tabular proportions.

TABLE D.

| $x$ | $n$ | $\frac{L_x - L_{x+n}}{l_x - l_{x+n}}$ Males. | $\frac{L_x - L_{x+n}}{l_x - l_{x+n}}$ Females. |
|---|---|---|---|
| 13 | 7 | 171·14 | 161·39 |
| 20 | 10 | 110·33 | 105·48 |
| 30 | 10 | 89·43 | 87·11 |
| 40 | 10 | 65·70 | 70·76 |
| 50 | 10 | 42·56 | 48·84 |
| 60 | 10 | 22·55 | 25·13 |

Results not differing greatly from the above may be obtained by taking the averages for the required periods, of the numbers living at each age corresponding to one death per annum, as given by Dr. Farr (*English Life Table*, pp. 34, 35). The results that would be thus obtained were, however, considered unsuitable, as representing the number of persons *living through* the year, corresponding to one death per annum, instead of the number *entering upon* the year. It is not necessary to calculate separately the number of persons living at each age, as will be explained further on.

The average durations of the voyages to New South Wales, Victoria, and South Australia, might be exactly determined by adding together the whole number of days occupied on the voyages to each colony, made during the period under observation, and dividing by the number of such voyages. It was, however, considered that a sufficiently accurate result might be obtained by proceeding as follows. We observe that,

| | |
|---|---|
| 38 voyages to New South Wales, in 1848 and 1849, occupied | 4131 days; that |
| 38 voyages in 1858, 1859, 1860, and 1861, occupied . . | 3917 days; the |
| 76 voyages thus occupying . . . . . . . | 8048 days. |

The average duration of those 76 voyages is found to be 106 days, which was assumed fairly to represent the average duration of the voyage during the whole period from 1847 to 1861. It may be mentioned in passing that while the average duration of 38 voyages in 1848–1849 was 109 days, that of the same number in 1858–1861 was only 103 days. For the purpose of obtaining the average duration of the voyages to Victoria and South Australia, it is observed that in the year 1854,

| | |
|---|---|
| 33 voyages to New South Wales occupied . . . . | 3305 days, |
| 33 voyages to Victoria occupied . . . . . . | 3359 days, |
| 33 voyages to South Australia occupied . . . . | 3282 days. |

We may thus obtain the ratios of the duration of the Victorian and South Australian voyages to that of the New South Wales voyage in the year 1854; and the same ratios may fairly be assumed to hold during the whole period under observation, so that the average duration of the Victorian voyage may be taken at $\frac{3359}{3305} \times 106 = 108$ days; and that of the South Australian voyage at $\frac{3288}{3305} \times 106 = 105$ days.

The following method was devised and adopted by Mr. Sprague, for applying the above data to the calculation of the expected number of deaths:—

Let N = the whole number of males or females embarking upon any given one of the three voyages, as contained in Tables A and B.

Let $d_1, d_2, \ldots . d_6$ = the whole numbers dying on all the Australian voyages, in the 1st, 2nd, 3rd intervals of age, respectively, being the totals of the columns of Table C.

Let $\frac{1}{m}$ = the duration of the voyage, expressed as a fraction of a year. The number of deaths occurring among N persons during a voyage occupying $\frac{1}{m}$th of a year being $d_1+d_2+\ldots . +d_6$, the corresponding number of annual deaths is $m(d_1+d_2+\ldots . +d_6) = m\Sigma d$.

Let $n_1, n_2, \ldots . n_6$ = the numbers of persons entering upon risk, corresponding to one death per annum, in the 1st, 2nd, 3rd intervals of age respectively, according to the English Table III, as given in Table D.

If the rate of mortality prevailing were that expressed by English Table III, the number of persons embarked in each interval of age would be,

$$mn_1d_1,\ mn_2d_2,\ \ldots .\ mn_6d_6;$$

and the whole number embarked, $m\Sigma nd$.

In accordance with previous supposition, let the true rate of mortality be such that the actual numbers of persons embarked corresponding to each death in the several intervals of age are,

$$pn_1,\ pn_2,\ \ldots .\ pn_6.$$

The numbers embarking at the different ages are thus,

$$mpn_1d_1,\ mpn_2d_2,\ \ldots .\ mpn_6d_6,$$

the sum of which, $mp\Sigma nd$, should equal N, the whole number embarked. By equating them we obtain the value of $p$, thus,

$$\mathrm{N} = mp\Sigma nd,$$

$$p = \frac{\mathrm{N}}{m\Sigma nd}.$$

Substituting the value of $p$ in the expressions obtained for the numbers embarking in each interval of age, they become

$$N\frac{n_1d_1}{\Sigma nd},\ N\frac{n_2d_2}{\Sigma nd},\ \ldots\ldots N\frac{n_6d_6}{\Sigma nd},$$

the sum of which is equal to N.

$\frac{1}{n}$ being the annual rate of mortality according to the English Table III, and $\frac{1}{nm}$ the corresponding rate of mortality on the voyage, the expected deaths in each interval of age are as follows,

$$\frac{Nd_1}{m\Sigma nd},\ \frac{Nd_2}{m\Sigma nd},\ \ldots\ldots \frac{Nd_6}{m\Sigma nd},$$

and the total expected deaths on the voyage,

$$\frac{N\Sigma d}{m\Sigma nd},\ \text{the percentage of deaths being}\ \frac{100}{m}\cdot\frac{\Sigma d}{\Sigma nd}.$$

It will be observed that the value of the last obtained expression is dependent on those of $m$ and $n$, the former of which varies with the voyage, and the latter has different values for males and females. The following tables (E and F) contain the expected mortality on each of the voyages, calculated by means of the above formulas, the values of $\Sigma nd$ and $m$, made use of in the construction of the tables, being as follows,

| | $m$ | | $\Sigma nd$ |
|---|---|---|---|
| New South Wales . . | ·34433 | Males . . . | 41471 |
| Victoria . . . . . . | ·33796 | Females . . | 40110 |
| South Australia . . . | ·34762 | | |

TABLE E.—*Showing the Mortality (excluding that arising from Wrecks) prevailing amo Adult Government Emigrants, on the Voyage to Australia* 1847–1861, *compared wi that to be expected according to English Table III.*

| Voyage. | MALES. | | | | | | FEMALES. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Number embarked. | Expected deaths. | Actual deaths. | Expected deaths percent. | Actual deaths per-cent. | Excess of actual deaths percent. | Number embarked. | Expected deaths. | Actual deaths. | Expected deaths percent. | Actual deaths per-cent. | Exce of actu death perce |
| New South Wales | 33,152 | 89 | 160 | ·270 | ·483 | ·213 | 36,051 | 101 | 241 | ·279 | ·669 | ·390 |
| Victoria ......... | 27,814 | 76 | 190 | ·275 | ·683 | ·408 | 50,232 | 143 | 399 | ·285 | ·794 | ·509 |
| South Australia | 22,217 | 60 | 115 | ·268 | ·518 | ·250 | 25,315 | 70 | 161 | ·277 | ·636 | ·359 |
| Total ......... | 83,183 | 225 | 465 | ·270 | ·559 | ·289 | 111,598 | 314 | 801 | ·281 | ·718 | ·437 |

ABLE F.—*Showing the Mortality (including that arising from Wrecks) prevailing among Adult Government Emigrants on the Voyage to Australia,* 1847–1861, *compared with that to be expected according to English Table III.*

| Voyage. | MALES. | | | | | | FEMALES. | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Number embarked. | Expected deaths. | Actual deaths. | Expected deaths percent. | Actual deaths per-cent. | Excess of actual deaths percent. | Number embarked. | Expected deaths. | Actual deaths. | Expected deaths percent. | Actual deaths per-cent. | Excess of actual deaths percent. |
| New South Wales | 33,199 | 90 | 207 | ·270 | ·623 | ·353 | 36,119 | 101 | 309 | ·279 | ·853 | ·574 |
| Victoria ......... | 27,853 | 77 | 229 | ·275 | ·822 | ·547 | 50,326 | 143 | 493 | ·285 | ·981 | ·696 |
| South Australia | 22,249 | 60 | 147 | ·268 | ·661 | ·393 | 25,363 | 70 | 209 | ·277 | ·824 | ·547 |
| Total ......... | 83,301 | 227 | 583 | ·272 | ·700 | ·428 | 111,808 | 314 | 1011 | ·281 | ·904 | ·623 |

It will be seen from these tables that the extra risk incurred by Government emigrants on a voyage to Australia, is caused, not so much by the danger of wreck, as by the greater intensity with which the causes ordinarily producing mortality act on a sea voyage. It further appears that the extra risk attaching to the voyage is much greater for women than for men—a result that was quite to be anticipated. The mortality on the voyage to Victoria, both among males and females, is somewhat greater than that on the other voyages. It was not considered necessary to inquire into the special circumstances to which this is due. The general result is, that the extra risk attaching to the voyage, would be covered by an extra premium of about 8*s.* 6*d.* percent for males, and 12*s.* 6*d.* percent for females.

The Government emigrants among whom the above mortality prevailed were chiefly drawn from the class of artizans and agricultural labourers. The accommodation provided for them is considerably inferior to that which is generally obtained by persons belonging to the assuring classes. It may therefore be supposed that the rate of mortality found to prevail among the Government emigrants will considerably exceed that which insurance companies may expect to experience. It must be taken into account, however, that all the emigrants are select lives, being subjected to a strict medical examination to test their eligibility to receive an assisted passage; as the Colonial Governments endeavor to secure that the labor of the emigrants introduced to the colonies shall benefit them for some years. The strictest precautions are also taken by the Legislature and the Commissioners, to secure the health and safety of the emigrants on the voyage. In the case of persons proceeding to Australia in steamships, it is natural to suppose that the extra risk will be reduced nearly in the same proportion as the duration of the voyage.

It is to be regretted that the Reports do not contain any information as to the ages of the emigrants embarking annually. If an analysis, on a similar plan to the analyses of mortality given for each of the voyages, but distinguishing the sexes, were given, dealing with the number embarked instead of the number dying, it would greatly facilitate such an investigation as the present; and would further enable us to state what proportion of the increased mortality occurred at each age. Even if this were not done, it is much to be desired that in the analyses of mortality on each of the voyages, the sexes were distinguished, as is done in the returns.

The administration of the Passengers' Acts by the Emigration Commissioners, has enabled them to supply, in the appendixes to their Annual Reports, much interesting statistical information, which, however, from its want of completeness, it has not been found practicable to turn to account.

Since the year 1854, there has been given an annual return of the ships and emigrants despatched from the United Kingdom, showing the number of vessels wrecked, and of lives lost by wreck. Had the vessels been arranged according to destination, it would have been possible to estimate the extra risk arising from the chance of wreck, which it is natural to suppose must vary considerably with the length of the voyage, and the quarter of the world to which it is made. They are, however, arranged according to port of departure only.

Returns are also given of the mortality on board passenger ships proceeding to different parts of the world. As, however, we have no information as to the age or sex of either living or dying, it is impossible to state, with any approach to certainty, what the extra risk incurred on such voyages is.

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## *Report by* Mr. MALCOLM *and* Mr. HAMILTON, *Assistant Secretaries to the Board of Trade, upon the Accounts and Statements submitted to the Board of Trade, under the "Life Assurance Companies Act,* 1870". *(Presented to Parliament by Her Majesty's Command.) Ordered, by the House of Commons, to be Printed,* 10 *July* 1874.

*To the Right Honourable the President of the Board of Trade.*

SIR,—In compliance with your instructions, we have the honour to submit to you the following observations upon the Accounts and

Statements submitted to the Board of Trade in pursuance of the Life Assurance Companies Act, 1870.

A "company" is defined for the purposes of the Act to mean "any person or persons, corporate or unincorporate, not being registered under the Acts relating to Friendly Societies, who issue, or are liable under policies of assurance upon human life within the United Kingdom, or who grant annuities upon human life within the United Kingdom."

One hundred and twenty-five such companies, excluding the branch establishments of four American companies, appear to exist in the United Kingdom; and Revenue Accounts and Balance Sheets have been received from this number since the passing of the Act.

The Blue Book for the year 1873, from which the table* given in Appendix (A) has been compiled, contains the annual accounts of 120 of these companies. Of the remaining five companies who failed to deposit their returns in time for publication, the total annual income from premiums does not amount to £10,000. The table may therefore be taken as practically complete. The first column of it shows the income of each company arising from premiums for assurance and the consideration money for annuities. As the Act requires that such income should be stated, less the amount of reassurance premiums paid over to other assurance offices, the sum of these items represents the total premium income of the assurance business of British offices. This amounts to £10,824,093. The extent of the business done by the American companies with residents in the United Kingdom is not known at present.

The next column of Appendix (A), gives the percentage of the working expenses of each company to its premium income, excluding considerations† for annuities, as accurately as the returns permit. A reference to this column will show the very different proportions of their premium income which the different offices absorb in conducting their business. Thus, of each pound entrusted to them by the assured, some companies spend as little as 1*s.*, others spend 2*s.* 6*d.*, others 5*s.*, others 10*s.*, and others the whole; and a few not only do this, but get into debt, trusting to be set right by means of future payments from the assured. In some instances at least the proportion year by year absorbed is so great that the companies can in no sense be said to exist for the benefit of those who insure in them.

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* It has been thought better for various reasons to omit the figures contained in this table.—Ed. *J. I. A.*

† Had the consideration money for annuities been included in making this calculation, the percentages would have in a few cases been somewhat reduced. Neither the inclusion nor the exclusion of this item is strictly correct, but the method adopted admits of but a very small margin of error.

The third column of Appendix (A) shows the average accumulated life fund during the last official year of each company, and the fourth column shows the rate of interest made. The total of the life funds of all the companies in the United Kingdom amounts to no less a sum than £94,260,592, and the average gross* interest made by them is at the rate of 4·45 per-cent approximately.

This information is not only useful as affording a means of judging of the rate of interest which may be employed in assurance calculations, but is also interesting as an indication of the income obtainable from investments which may be classed as safe.

As nearly as can be at present determined, the total amount of all classes of assurances effected with the companies in the United Kingdom is £338,882,752, of which the sum of £317,682,774 is to be paid on the deaths of single lives, in consideration of premiums according to one or other of the various plans adopted by assurance companies. The total of the annuities of all kinds granted by them is £502,718, of which about £413,631 yearly is now in actual payment. See Appendix (B).

The 3rd section of the Life Assurance Companies Act, 1870, requires that any company established in the United Kingdom after the passing of the Act, or which commences after the passing of the Act to carry on business within the United Kingdom, shall be required to deposit £20,000 in the Court of Chancery, to be returned to the company so soon as its Life Assurance Fund accumulated out of the premiums has amounted to £40,000.

No new company has been established since the passing of the Act, and only one company established out of the United Kingdom has within this period begun to carry on the business of life assurance within the United Kingdom. This is an American Company, the Continental of New York. The deposit was made in this case; but as the accumulated life fund of the company was far more than £40,000, the deposit was immediately paid out again.

The Act (Sections 14 and 15) makes certain provisions with respect to the amalgamation of life assurance companies, or transfer of business from one to another, the principle of which is, that proposals for amalgamation are to be submitted to the Court of Chancery, and are to be subject to confirmation by the Court. No amalgamation or transfer in pursuance of these sections has been carried out.

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* Where the companies have returned the net interest received after deduction of income tax, a proportionate addition has been made to the amount for the purpose of this calculation. Allowance has also been made for reversions. The valuation of the fund is necessarily taken as given in the accounts.

The 7th section of the Act requires every company at certain intervals to cause an investigation to be made into its financial condition by an actuary, and to cause an abstract of the report of the actuary to be made in the form prescribed in the fifth schedule to the Act, and to be sent to the Board of Trade.

The Board of Trade have now received abstracts of such actuarial investigations from the greater number of the assurance companies, and a cursory examination would convince any person acquainted with the theories of valuation of the necessity of calling attention to the elementary principles involved in such processes.

A valuation is based on an estimate of the present value of the future debts of a company, and on an estimate of the present value of its future receipts. By placing the present value of the receipts, together with the company's accumulated funds, against the present value of the debts, its object is to show whether the company will be able or unable to pay such debts as they arise, and also what amount of surplus, if any, is available for division in the shape of profit or bonus.

This information is given in the replies to the questions prescribed in Schedule 5. Those replies contain, no doubt, a great many particulars which most people, especially those actively engaged in business, would scarcely spare the time to read carefully through. But the substance of the information as to the merits of the valuation may be obtained upon reference merely to the short answers given to Questions 3, 4, and 5, namely,

1. The rate of mortality employed.
2. The rate of interest employed.
3. The portion of the future premium income reserved in the valuation for future expenses and profits.

Upon these three points the estimate of the net liability of a company under its policies almost entirely depends. The valuation of the investments held by a company is also an important element in determining its financial position. This is a matter which can only be determined by a careful and intelligent consideration of particulars, and may involve a certain amount of difficulty, but the difficulty is common to other undertakings besides assurance companies whose funds are similarly invested. The Act does not require the companies to show the method by which their assets are valued; it merely requires the results to be stated.

## 1. *The Rate of Mortality.*

It must be remembered that a premium is the sum of two parts: the larger part dependent upon the table of mortality, and the rate of interest adopted, is called the *net, pure, true, mathematical,* or *risk* premium, and is that annual sum which, being invested year by year, is calculated to meet exactly all claims as they arise; the smaller part, known as *"loading"*, is the provision for expenses and profits.

The mortality tables employed in valuations arrange themselves in two classes, and are described respectively as *true* and *hypothetical.* True tables are those that were originally constructed to represent actual mortality and have been found by experience to answer their purpose more or less faithfully. When the valuer has recourse to this class of tables, provision for future expenses and profits is made by the simple process of withdrawing an assigned portion of the premiums to be received from the computation, that is, treating the premiums to be received as less than they really will be. Hypothetical tables are those in which the rates of mortality are either made or known to be fictitiously heavy,* the more rapid mortality being such as to carry with it a similar provision.

As regards the comparative *scientific* advantages of valuations based on the true and hypothetical mortalities respectively, actuaries are not at one, and both methods are in use by companies of the highest reputation, but there is no question that the hypothetical process does not give a simple and complete statement of the financial condition of the office, and that it cannot be so easily understood, and its results so accurately appreciated by others than actuaries, as the other method. The hypothetical system, however, may require a fund to be kept in hand larger than that required by a true table, even where the "net premium" system, as hereafter explained, is employed.

Of true tables in use for net premium valuations, we believe that the "Combined experience of 17 offices", and the new "Institute of Actuaries" (both, it will be noticed, records of the experience of assurance offices), are those requiring the largest funds to be kept in hand. The "Carlisle" has been and is still a great favourite, though it produces results differing appreciably from the above. Other tables, approaching fairly near to actual mortality, are occasionally used. It is, however, a serious matter for the consideration of the companies, how far it is judicious to employ tables bringing out smaller reserves than are required by tables skilfully constructed upon information furnished from records of correct observations upon lives placed under conditions similar to those that will surround the lives concerned in the contracts of which the values are to be estimated.

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* In hypothetical tables the mortality is usually such as results from supposing the gross premiums charged by the office to be net premiums, the object being apparently to represent the "loading" by a corresponding increase in the rate of mortality.

The following table exhibits a few selected specimens of the liability of a company under policies for £100, payable at death, using net premiums, and 3 per-cent interest, as determined from different tables of mortality, the premiums being just due and not paid.[*]

| Description of Policy. | TABLES OF MORTALITY. | | | | |
|---|---|---|---|---|---|
| | Institute of Actuaries ($H^M$). | 17 Offices. | Carlisle. | Equitable (Davies). | English (Males) (No. 3). |
| | £ | £ | £ | £ | £ |
| Issued at age 25, and 5 years in force | 5·311 | 4·981 | 5·119 | 5·146 | 5·106 |
| „ 30, and 10 „ | 12·898 | 12·677 | 11·739 | 11·691 | 12·691 |
| „ 35, and 15 „ | 23·949 | 24·082 | 21·253 | 21·864 | 23·318 |
| „ 40, and 20 „ | 38·183 | 38·266 | 36·664 | 33·607 | 36·950 |
| „ 45, and 25 „ | 53·859 | 53·537 | 51·828 | 47·973 | 52·468 |

It will be observed that nothing approaching a fixed relationship exists between the results of one table and those of another, and to tell the exact effect of any change in the table of mortality, there must be two valuations of all the policies of a company, one for each basis. To give even a rough estimate of the substitution of one table for another is extremely difficult, inasmuch as companies differ greatly as to the ages of the assured, and the duration of the policies. It may, however, be asserted that the substitution of one mortality table for another, may cause a variation in the resulting liability of 10 per-cent.

### 2. *The Rate of Interest.*

In forming an opinion as to the suitability of the rate of interest adopted in a valuation, reference should be made to the

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[*] This table should be read thus: For every policy issued five years ago on the lives of persons then aged 25 the Institute of Actuaries ($H^M$) table requires that £5·311 should be under investment at not less than 3 per-cent, and so on, for the other policies, and the other tables of mortality.

Consolidated Revenue Account, or to the Annual Revenue Accounts given in Schedules 1 or 3, in order to determine the rate of interest that the company has realized during the past few years. This may be done by comparing the total clear interest shown in the life revenue account to have been made, with the mean or average amount of the capital and funds producing it. In this way the effect of any unproductive portion of capital or fund is duly taken into account.

It would be unreasonable to adopt for valuation purposes a rate of interest higher than that so determined, and it would be dangerous to adopt a rate as high. The primary object of an assurance company should be, to be safe, to secure the certainty that its engagements will be promptly met, despite the bad debts that may arise on its loans, and the losses that may accompany the realization of its investments at reduced prices. In fixing the interest for valuation purposes, there should obviously be deducted from the rate of interest made in the past a margin amply providing for all the possibilities of unfortunate advances and fallen prices. Although prices may rise and unexpected gains be discovered, a company is not thereby relieved from the responsibility of taking all possible care to avoid a deficiency. Apart from the fact that a continuous rise in prices would indicate a continuous fall in the productiveness of future investments, it must ever be borne in mind that the safety of assurance is supremely important, while the maintenance of any excess in the fund is comparatively a trivial matter. In the early days of assurance, if the trusted Northampton table had involved errors of charge which led to deficiencies instead of surpluses, the business of life assurance would have been seriously retarded; besides any excess of fund is invested, and in the end the additional interest on it will nearly or quite repay whatever sacrifice may be made in the present.

On an average, the rate of interest made by the offices does not reach 4½ per-cent. Many of the cases in which a higher rate appears are shown by the accounts to be attended by exceptional circumstances. It follows that, in order to determine the liability under the contract of an office (that is looking forward to existence in the future as well as the present), a 4 per-cent valuation should be adopted with great hesitation, if at all.

The following table exhibits specimens of the values of the same policies using net premiums and the Carlisle table, as determined for different rates of interest:—

| Description of Policy. | Rates of Interest. | | |
|---|---|---|---|
| | 3 per-cent. | 3½ per-cent. | 4 per-cent. |
| | £ | £ | £ |
| Issued at age 25, and 5 years in force | 5·119 | 4·661 | 4·252 |
| ,, 30, and 10 ,, | 11·739 | 10·813 | 9·963 |
| ,, 35, and 15 ,, | 21·253 | 19·893 | 18·615 |
| ,, 40, and 20 ,, | 36·664 | 35·121 | 33·659 |
| ,, 45, and 25 ,, | 51·828 | 50·374 | 48·961 |

It will be observed that this table again affords no simple law of the exact effect of a change in the rate of interest. It may, however, be stated that a change from a lower to a higher rate of interest of only 1 per-cent, may make an alteration of 8 per-cent on the liability.

### 3. *The portion of the premium income to be reserved for future expenses and profits.*

In the matter of reserve for future expenses and profits, in cases where true mortality and interest are employed, the directors of companies would do well to require a valuation based on net premiums as provided for in the schedule. If a smaller reserve than that arising on a net premium valuation as explained below is deducted from the future premiums, more or less of the profit portion of them is anticipated and absorbed.

But whatever may be thought of the merits of a net premium valuation, the greatest care should be taken by directors and by the public in insisting that the proportion of reserve for expenses never falls below the proportion of expense to premium income, as shown in the Consolidated or Annual Revenue Accounts. The sum of commission, expenses of management (so called), and any other item falling into the same class, should be placed against the total premiums incoming in the same period, and the proportion of such sum to such total then calculated.

The test is one that some companies very much object to, and for obvious reasons; they argue that the expenditure in starting a policy includes the extra charges of medical fees, policy stamps, &c., which do not recur on account of existing business; to which the reply is, that when safety is the chief object, it is out of place to fix the minimum reserve with such extreme nicety as this would imply in any established business. Some companies urge that they will reform their management, and reduce the rate of expenditure in future. Here, again, safety being the chief object, the wise course is to make the larger provision, and at the same time to insist upon the reformation being effected. It has been urged also that a portion of management expenses, generally that portion described as extension expenses, is fairly to be regarded as an investment made by the shareholders in the purchase of business, and is therefore chargeable to capital. If these words were used in the sense that the outlay in this behalf is really to be borne by the shareholders, nothing could be said against it, except in the interest of the shareholders themselves; but this is not the case, such a charge may be made against capital this year, but on the first opportunity it will be deducted from the life fund in order to restore the capital. The whole procedure is simply a book-keeping expedient, by which the charge against the life fund is deferred, and the burden borne by it in the future instead of in the present.

To make clear what a valuation is, the following analysis of a premium into its component parts is given.

At the age of 30 the annual premium for the assurance of £100, to be paid at death (the assured to share in the profits of the company), may for the sake of example be taken at £2. 10*s*. Assuming the rate of mortality exhibited in the Institute of Actuaries Life Tables $H^{M(5)}$, and assuming that a company may reckon upon a clear 3½ per-cent interest upon its accumulated funds (whether productive or unproductive), in the case of an ordinary office the premium may be approximately subdivided into four parts, thus:—

| | £ | *s.* | *d.* |
|---|---|---|---|
| Net premium, that is, the portion of the total premium required to provide for current and future claims | 1 | 17 | 0 |
| Commission to agent (say) | 0 | 2 | 6 |
| Working expenses (say) | 0 | 3 | 6 |
| Margin or profit | 0 | 7 | 0 |
| | £2 | 10 | 0 |

It will be readily understood that mathematicians can easily combine the arithmetical effects of compound interest with the simple chances of a person living or dying year by year throughout the remaining period of possible existence, and that the *cash* or *present* value of an annuity on that life, as well as the cash or present value of the reversion of £100 at death, may be accurately determined. That is of course "accurately", on the supposition that the life in question is one of a large number assured, for though there are few things so uncertain as the duration of the life of any one individual, observation has proved that the duration of the collective life of a large number is practically certain.

In order to represent the circumstances attending valuation, it may be supposed that 10 years have elapsed, and that the company wishes to ascertain the profit or loss resulting upon the 10 years' trading, and that the portion of the monies accumulated in respect of this policy, after paying the share of the current claims and expenses, has reached £18. 5*s.* 10*d.* The person who effected the insurance for £100 at the age of 30 has been assured 10 years, and is consequently now 40 years of age. It may be further supposed that experience has agreed sufficiently well with both the assumed rate of mortality and the assumed rate of interest to justify the use of both in the valuation. The contract between the office and the assured may be looked at in its two chief divisions, the money to be paid by the office and the money to be paid by the assured, and the latter may be further analyzed as indicated below.

The office has then to provide a reversion of £100 at death, the discounted or present value of which is, at the age of 40 . . . . . . . . . . £42. 16*s.* 11*d.*

The assured, having just paid the current premium, has still to pay £2. 10*s.* annually, which may be analyzed as follows:—

| | £ | *s.* | *d.* |
|---|---|---|---|
| 1. An annuity of £1. 17*s.* for net premium, the present value of which at age 40 is . . . . . . . | 29 | 8 | 4 |
| 2. An annuity of 2*s.* 6*d.* for commission „ „ | 1 | 19 | 9 |
| 3. An annuity of 3*s.* 6*d.* for office expenses „ „ | 2 | 15 | 8 |
| An annuity of 7*s.* for profit „ „ | 5 | 11 | 4 |
| In all, an annuity of £2. 10*s.*, the present value of which is | £39 | 15 | 1 |

Now the office in selecting the items for and against itself, so as to arrive at its net liability under the policy, certainly can take no credit for the £1. 19*s.* 9*d.* commission, for this represents the present value of monies that the agent will deduct, and which therefore the company will not ever receive. Nor can any credit properly be taken for the £2. 15*s.* 8*d.* expenses; for, by hypothesis, this represents the provision made to pay charges which arise in the collection and investment of premiums concurrently with the incoming of the premiums themselves, charges which the company cannot possibly avoid. With regard to the £5. 11*s.* 4*d.* for prospective profit, a considerable difference of opinion exists, and a considerable variety of practice has prevailed. And while it must be admitted that ordinary companies do have a hold upon such future profits, either directly by the continuance of the policy, or indirectly by its forfeiture or surrender, yet if they take credit in their valuation for any portion of this amount, they necessarily confer undue benefits upon present members to the injury of new members, and the process resembles that of a spendthrift anticipating for present use the income to be derived in future years from a settled estate.

Omitting from consideration the annuities for future commission, future expenses, and future profits, and taking into account the accumulated reserve, or fund of the company, £18. 5*s*. 10*d*., in the case mentioned above, the estimated surplus attained would be calculated in this way:—

| | £ | *s*. | *d*. |
|---|---|---|---|
| To the credit of the company will stand— | | | |
| The present value of the net premium, £1. 17*s*. yearly | 29 | 8 | 4 |
| Amount of accumulated fund in hand . . . | 18 | 5 | 10 |
| | 47 | 14 | 2 |
| From which deduct present value of the reversion of £100 . . . . . . . . . . . | 42 | 16 | 11 |
| Surplus (already actually realized) . . . . | £4 | 17 | 3 |

It will hardly be believed that the Board of Trade could have had submitted to them for acceptance under the Life Assurance Companies Act, of 1870, valuations in which the future profit, future expenses, and even future commission, have been turned into present value, and the whole represented as profit. These accounts have been rejected as being manifestly incorrect and misleading, and amended returns requested. Other companies have treated the provision for future profits, and more or less of the provision for future expenses as profit. These have been accepted with considerable hesitation, on the ground that an extravagant management in past years *may* become somewhat more economical in the future. A third class of accounts show that the future profits have been trenched upon, and these the Board of Trade have apparently no power whatever to interfere with. It is probably desirable that the department should not have such power, but it is most undoubtedly in the interest of the public that there should grow up a sufficient perception of the main facts of valuation to enable the public themselves to recognize when a safe and judicious method has not been followed.

No technical knowledge is necessary for the appreciation of these points. Technical knowledge is required to make and adjust the calculations, not to fix these important data. This is a matter for which the directors are responsible, and to comprehend what the data are is all-important for the assured. For, as will appear from what has been said, a company by using one table of mortality instead of another, may keep £900,000 in hand instead of £1,000,000: by taking a rate of interest higher only by 1 per-cent than may be prudent, the £900,000 would be reduced to £830,000, and taking a business of the ordinary type, it may be asserted with little chance of error, that a small reserve of premium income in place of a larger one may have a final effect so great as to bring the £830,000 down to £600,000. It thus follows that of two companies under exactly the same particulars of business, the one will be cautious, and keep in hand a million to meet its engagements; the other, perhaps anxious to distribute large bonuses, will deem it sufficient to keep in hand a fund of £600,000, and will recklessly publish and divide the remaining £400,000 as profit.

The wide range in the results of valuing life business as indicated above, by no means reaches the extreme limits of hypothetical mortality, speculative interest, and impracticable reserve sometimes submitted to the Board of Trade. The preceding results have been deduced from mortality tables placed in the category technically known as "true", from the ordinary limits of interest, and from reserves neither exceeding the full loading as a maximum, nor below a tenth of the gross premiums (a proportion greater than that for which several good offices are managed) as a minimum.

It will be readily understood, then, how easy it is for the responsible officers of an assurance company to "value up", and to produce a flattering actuarial balance sheet, and it need hardly be pointed out that the public are readily misled by large present bonuses, and this practice is apparently sometimes adopted by young offices with the view of attracting business.

But taking insurance business as it exists in this country, where adequate premiums are charged, and lives selected with care, the public cannot be misled if when seeking an office in which to effect an insurance they select one which transacts its business at a small percentage of working cost, and does not anticipate its profits.

We have, &c.,
(Signed) W. R. MALCOLM.
R. G. C. HAMILTON.

*Board of Trade*, 2 *July* 1874.

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## APPENDIX (A).

### *Compiled from the Annual Returns for* 1873.

*Notes.*—(1.) In making the calculations of the rates of interest all profits or losses made on reversions, the sale of stock, &c., and losses by bad debts, have been omitted. The portion of the funds invested in the purchase of reversions, in the few cases in which the accounts show the amounts so invested, and that they belong wholly to the Life Department, and are sufficiently large to appreciably affect the percentage of interest, has been deducted before calculating such percentage.

(2.) It has not been possible to adopt a uniform course with regard to Income Tax on dividends, inasmuch as the accounts do not generally tell whether or not the tax has been deducted from the interest received. It is very desirable that companies should show the Income Tax on the credit side of the revenue account as an independent item, so that the full reproductive power of money may be readily obtained.

(3.) As far as possible the rate of expenses has been calculated on the gross premiums, but the accounts do not always show the reduction by bonus as a separate item.

| Date of Establishment. | Company. | Amount of Premiums and Consideration for Annuities (less Reassurances). | Proportion of Expenses of Management and Commission to Premium Income (excluding consideration for Annuities). | Mean Amount of Life and Annuity Fund during the Year. | Rate of Interest. | Remarks. |
|---|---|---|---|---|---|---|
| | | | | | | |

## APPENDIX (B).

TOTALS *of* SUMS ASSURED *and* ANNUITIES GRANTED *by English, Scotch, and Irish Offices, compiled from Returns made up to various dates received at the Board of Trade before the* 10*th June* 1874.

| | ASSURANCES. | | | ANNUITIES. | | |
|---|---|---|---|---|---|---|
| | Whole Life. | Other Classes. | Total. | Immediate. | Other. | Total. |
| | £ | £ | £ | £ | £ | £ |
| The Accounts received from 105 offices show | 307,291,135 | 20,419,907 | 327,711,042 | 411,479 | 87,984 | 499,463 |
| Twenty Companies have not yet deposited their Statements, but the sums assured and Annuities granted are estimated, namely . | 10,391,639 | 780,071 | 11,171,710 | 2,152 | 1,103 | 3,255 |
| Total . . £ | 317,682,774 | 21,199,978 | 338,882,752 | 413,631 | 89,087 | 502,718 |

The above amounts are exclusive of the business done by four American Companies trading in the United Kingdom, but the extent of whose English business is not at present known.

# *On the Usefulness of Mathematical Studies to the Actuary.— An Address to the Actuarial Society of Edinburgh. By the Honorary President,* T. B. SPRAGUE, M.A., *Vice-President of the Institute of Actuaries.*

[Read 13 November 1874.]

IT may perhaps be thought by some persons, that the subject upon which I am to address you tonight, is one that requires no urging on your attention. It is admitted on all hands, I may be told, that a good actuary must necessarily go through a course of mathematical training, and nothing further need be said upon the point. But if an apology for the choice of my subject is required, I would remind you that a mathematician takes nothing for granted. Whereas many people will say that it is self-evident that "two and two make four", and that this neither requires nor admits of proof, the mathematician knows that it is a proposition, which is not in the true sense of the word self-evident, but can be strictly proved. While we deal with simple matters of this kind, little harm is likely to result from assuming as self-evident, propositions which admit of proof. But the case is very different when we come to more complicated and obscure questions; and not the least advantage to be derived from a mathematical training, is the gradual forming of the habit of mind which requires strict proof to be furnished of many important propositions which are usually taken for granted. I purpose, then, instead of taking for granted the use of mathematical studies, to examine exactly why and how they are of service to the actuary; and I hope that the conclusions I shall arrive at, may be of some assistance to those among you who wish to extend their mathematical reading beyond the point that is by common consent considered indispensable to the actuary.

With this object, I will consider in succession some of the branches of mathematics most commonly read.

To commence with elementary geometry, or Euclid. Not only is the study of Euclid a necessary preliminary to other geometrical studies, but it is also of great use to a student by accustoming him to a course of strict reasoning. Any person who thoroughly masters the first book of Euclid, acquires a practical knowledge of what correct reasoning consists in, and is less likely to be misled by the examples of fallacious reasoning that we so constantly meet with in all kinds of subjects. Another advantage of the study of Euclid consists in the precision of the language employed. Every kind of geometrical figure about which we have to reason, receives a distinctive name, which cannot be misunderstood; so that the mere mention of the name is sufficient to inform us precisely, and without any ambiguity, of the nature of the figure denoted by it. The habit of using language in this strict and precise manner is of inestimable value, not only to the actuary, but in every department of science, nay, in every walk of life. Without precision of language, clearness of thought is unattainable; and when we see words used in a loose and inexact manner, we shall generally not be wrong in concluding that it indicates confusion of thought. It must not however be supposed that this advantage is in any way peculiar to the study of Euclid or of mathematics; for there are many other subjects with an equally exact nomenclature, which, if systematically studied, will in a similar way foster habits of precision of language.

The advantages to be derived from a study of algebra are of quite a different character. The first and one of the principal uses of algebraical notation, is the ease with which it enables us to take in at a glance, a long series of complicated operations, which, if described in words, would require very much more time and attention to follow, and would then not be so clearly apprehended. We feel this advantage even in the case of the simplest propositions. When we compare the very simple proposition,—"If the sum of two numbers is added to their difference, the total is equal to twice the greater number", with its algebraical equivalent, $(a+b)+(a-b)=2a$; or the proposition,—"If the sum of two numbers is multiplied by their difference, the product is equal to the difference of their squares", with the equation $(a+b)(a-b)=a^2-b^2$, we at once recognize the superior simplicity and clearness of the formulas. It is true that in these cases the use of the terms "sum" and "difference", gives a compactness to the verbal statements of the propositions, which renders them only a little less easy to follow than their symbolical expression; but this is not often the case. If we take a somewhat more complicated formula, such as $(a+b)^2=a^2+2ab+b^2$, which would be thus stated in words: "The square of the sum of two numbers is equal to the sum of their squares increased by twice the product of the numbers", the advantage is more decidedly in favour of the formula as compared with the verbal statement. Still more is this the case when we proceed to more lengthy expressions, as, for instance, if we attempt to give a verbal description of the proposition, $(a+b)^3=a^3+3a^2b+3ab^2+b^3$; or still more, $(a+b-c)^2=a^2+b^2+c^2-2bc-2ca+2ab$. Precisely similar to this is the advantage gained in the study of life contingencies by the substitution of formulas instead of verbal descriptions. Although it was formerly the practice for writers on life contingencies, to give verbal descriptions (or "Rules") for finding any required benefit, this must simply be regarded as a concession to the imperfect mathematical acquirements of their readers, and we may hope it is no longer necessary. The justice of the above remarks will, I am sure, be admitted by every algebraist; and I therefore think I am justified in laying down the principle, that every actuarial student will find it well worth his while to go through a long course of algebra, for the purpose of rendering himself sufficiently familiar with algebraical symbols, to realize for himself the superior advantages afforded by the use of formulas in comparison with verbal descriptions.

It may be useful to consider these advantages a little more closely, and I will take an illustration from the well known D and N tables. To the present company I may submit without explanation the equations,

$$a_x = \frac{N_x}{D_x}, \quad \mathbf{a}_x = \frac{N_{x-1}}{D_x}.$$

If we wish to describe, without the use of algebraical symbols, the truths which these equations set forth, we should have to say for the first equation, "In order to find the value of an ordinary annuity for the life of a nominee of any assigned age, we must divide the number which stands opposite to that age in the column N, by the number which stands opposite to it in the column D"; and for the second, "In order to find the value of an annuity payable in advance for the life of a nominee of any assigned age, divide the number in the column N, standing against the age one year less than the assigned age, by the number in the column D against the age". Let us now examine as to the precise points in which the superiority of the symbolical notation consists. In the first place, instead of the phrase "the assigned age", we substitute the letter $x$. Secondly, "the number standing in the column D opposite the age $x$" is denoted by $D_x$; and similarly with $N_x$. "The value of an annuity on the life of a nominee of the age $x$" is denoted by the symbol $a_x$; and that of an annuity payable in advance, by $\mathbf{a}_x$. Lastly, we have the symbol, $=$, which denotes that the quantities between which it is placed are equal to each other.

The advantage arising from the use of algebraical symbols is of the same nature as that arising from the use of the ordinary Arabic numerals, over the Roman numerals, or the verbal statement of a number. Thus, when we write 1874, we take in the meaning much more quickly than when it is written in the old-fashioned Roman numerals, MDCCCLXXIV, or in the words "*one thousand eight hundred and seventy-four*".

In connection with this subject, I may fitly make a few general remarks on life contingency notation; and the rather so, as there appears to me to exist in some quarters a tendency to undervalue its importance. From what I have said it will have appeared that the actuary's notation is a sort of symbolical language. As such it will react upon the ordinary language employed by him. Thus, when we adopt the recognized symbols, $\mathring{a}_x$ and $\bar{a}_x$, we feel more necessity for some simple name by which to describe the annuities they represent; and accordingly the invention of the names "complete annuity" and "continuous annuity", instead of the descriptive phrases "annuity payable up to the day of death", or "annuity secured by land", for the first, and "annuity payable by momently instalments" for the second, is a real gain. I cannot leave this subject without uttering a word of warning against ill-considered changes in the meanings of words and symbols. When a technical word like "annuity", which has a well settled and universally understood meaning, is used with a slightly different meaning, a serious inconvenience, not to say injury, is inflicted upon the student of life contingencies. This, as you are probably aware, has been done by one well known writer on life contingencies, who employs the word to denote what is commonly denoted by "an annuity-due", or "an annuity payable in advance". The same writer has also altered the meaning of the well known symbol $N_x$, so that, although he retains the equation "an annuity on a life of the age $x = \frac{N_x}{D_x}$", neither side of the equation has the meaning ordinarily attributed to it. I find it difficult to moderate my language in speaking of the mischievousness of changes like this, and will only say further, that they must have the effect of very seriously diminishing the scientific value, and interfering with the general use of writings, in which they are adopted.

In comparing together the various works that have appeared in this country on the subject of life contingencies, nothing is more remarkable than the gradual adoption and improvement of a system of notation, to express the various benefits considered. In the earliest works on the subject, such as William Dale's *Introduc-*

*tion to the Study of Annuities,* we have no notation of any kind. In Morgan's works we have a little introduced, but it is widely different from anything that is at present in use. Baily, Milne, Griffith Davies, and David Jones, have all made in succession a greater use of notation; and it is not a rash prediction to say that the next work on life contingencies, whoever may be its author, will show still further progress in the same direction.

Passing on now to consider the subject of coordinate geometry, I need only remind you how often it is found desirable, if not necessary, to lay down graphically the curve that represents various values of a function for different values of the variable; as for example when the abscissa is taken as the age, and the ordinate as (1) the rate of mortality, or (2) the number out of which one will die in a year. Suppose that we have drawn a diagram of this nature, and that we wish to describe the characteristics of the curve, a knowledge of coordinate geometry enables us to do this in the simplest and clearest manner. We are able to point out that at one age a *maximum* occurs, at another a *minimum,* and at a third a *point of inflection;* also that in certain cases one or other of the coordinate axes will be an *asymptote* to the curve; and those who are familiar with the properties of algebraical curves, will require no explanation of the terms we have used. But we cannot study the properties of curves fully without the aid of the Differential Calculus, and thus we are led at once to the conclusion that a knowledge of the Differential Calculus is useful to the actuary.

I come next to the Theory of Probabilities, upon which the whole of the theory of life contingencies ultimately rests. Although little or no use is made of the theory of probabilities in deducing the various formulas for single and annual premiums, yet that theory explains to us the reason for the use we always unhesitatingly make of the ordinary mortality tables, and shows us how far we may safely rely upon the conclusions drawn from them. If this theory were better understood by actuaries, we should not so often meet with the mistaken views that are frequently embodied in the reports and the prospectuses of insurance companies. We should not, for instance, be told that "out of 1,000 persons of a given age alive at the beginning of a year, it is certain that so many will die within the year"; nor that "because the claims have exceeded the expectation in one year, it is to be anticipated that this will be counterbalanced by their falling short of the expectation in a subsequent year". Nor would the reports of companies state that "the claims have fallen on large amounts, although the number of deaths has not been above the expectation". It would take too long to explain the inaccuracies of these various forms of speech. I must therefore content myself with saying that they all indicate fundamentally erroneous notions on the subject of probabilities. To take only one simple instance:—Suppose that a coin has been tossed twenty times, and that each time "head" has come uppermost. Suppose further that the coin is accurately shaped, so that there is no reason why "head" should fall uppermost rather than "tail." Then, if the coin is tossed again twenty times, is the fact that head has fallen uppermost twenty times, any reason for believing that tail is more likely to turn up than head in the second series of throws? This is an idea that habitual gamesters at the continental gaming tables find it almost impossible to divest themselves of, and arguments of some plausibility may be adduced in favour of it. Thus it may be said, Since there is no reason why head should come uppermost rather than tail, there will, when the coin is tossed a large number of times, be as many heads as there are tails. If, then, there has been a run of heads, it is necessary in order that this equality may hold, that there should be subsequently a run of tails. This reasoning, however, is altogether unsound. In the first place, we may remark that if head has come uppermost twenty times running, this affords a very strong reason for believing that there is something in the constitution of the coin that renders head more probable than tail. If, however, we suppose that the coin is an evenly balanced one, the correct conclusion is that the occurrence of head for any number of times in succession, however large, is utterly without effect upon the subsequent throws. How then is this to be reconciled with the admitted equality in the number of heads and tails that will prevail in the long run? Simply by observing that the theory informs us that this equality will only prevail when the number of trials is increased indefinitely; and when this is done, any finite number of heads that have been observed in succession—twenty, fifty, a hundred, or any larger number—will vanish in comparison with the larger number of trials.

The Theory of Probabilities is an extremely difficult study in its highest branches, but it is one well deserving the attention of the actuary. It teaches us how to calculate, not only the number of claims that we may expect out of a known number of lives under observation, but also the probability of the number of deaths falling short of, or exceeding the expectation. When we find the claims exceeding the expectation, it will enable us to judge whether this is to be attributed to accidental fluctuations, or whether it indicates that the lives have been badly selected. It also teaches us what are the real advantages to be gained in life insurance companies by an increase of the number of lives at risk, and shows the actuary how he may best make use of the practice of re-insurance, to spread his risks over a wider area.

The subject of Finite Differences is of especial importance to the actuary, as teaching him the rules of Interpolation. It is unfortunate that interpolation is usually treated in a brief and unsatisfactory manner, both in works on algebra and in treatises on life contingencies; but its applications are so numerous and important, that any amount of time and labor spent in gaining a knowledge of it, by reading the scattered papers on the subject, will be well bestowed. In the hands of a skilful actuary, an interpolation judiciously performed will often be the means of saving a vast amount of labour, either by rendering direct calculation unnecessary, or by furnishing a check of the accuracy of results otherwise obtained. Although it is possible to demonstrate all the formulas for interpolation, by means of ordinary algebraic processes, yet a knowledge of the Differential Calculus and of the method of Separation of Symbols of Operation and Quantity, (or Calculus of Operations) will present these formulas in a wholly different light. That knowledge will also give the actuary so much more command over the formulas, and enable him so much more readily to effect any required transformation of them, that I do not hesitate upon this ground alone, to recommend every actuarial student to make himself acquainted with those branches of mathematics.

Let us now consider somewhat in detail what are the actuary's ordinary duties. He has to calculate premiums for special risks, to calculate the surrender values of policies of all kinds, to advise as to the values of life interests and reversions offered as security for loans. He may be also called upon for his opinion as to the amount of outlay that may be prudently and advantageously incurred for the purpose of extending the business; but beyond all question his most important and responsible duty is to make the periodical valuations or investigations into the position of his company. As regards all these matters, it may be alleged with much truth that definite rules are either laid down in the text-books for the ordinary cases that present themselves, or are readily learnt in the course of an actuary's training. If the actuary cultivates private practice, he will be consulted not only with reference to subjects such as those already mentioned, but also as to the solvency of friendly societies, where the principal benefit involved, namely, sick allowance, is of a wholly different kind from those with which he is familiar in his office practice. He will be consulted also as to the values of next presentations and advowsons, as to the division of property between the life-tenant and the reversioner, and with regard to a variety of other questions which call for special individual treatment. Whenever the question proposed to an actuary differs, even to a slight extent, from those with which he is previously familiar, some modification of the formula to be used will probably be necessary. In these cases it is that an extended mathematical knowledge becomes useful. It is a comparatively simple thing to follow the demonstration of a formula in a text-book. But it requires much greater familiarity with mathematics, to be able to deduce a new formula when the circumstances of the case are slightly altered; and an actuary requires to have considerable familiarity with mathematical processes, before he can feel much confidence in the accuracy of a new formula deduced by himself. This is perhaps the principal reason for which an extended mathematical course of reading may be urged upon the actuarial student; and when this is the object, it matters comparatively little what branch of mathematics he studies. The important thing is for him to become thoroughly accustomed to algebraical processes, to be able readily to transform a formula into a new shape, and readily to recognize a familiar formula when he finds it transformed into an unfamiliar shape in the writings of others. It is quite true that in dealing with practical questions mathematical skill will not supersede the necessity for that tact which only experience can give. For instance, when an experienced actuary is asked to quote a premium for some very remote risk, as, for instance, a premium for an insurance payable if three or more young and healthy lives all die during the lifetime of a much older one, he will know that it would be labor lost to spend his time upon calculating the net premium for such a contingency; as the loading upon such insurances is in practice very often actually greater than the net premium, and the whole question resolves itself into considering how small a premium can be charged, considering the commission and other expenses; and how large a premium can be obtained, bearing in mind the rates of competing offices. So again, if asked to compute a table of ascending-scale premiums, he will be well aware that, taking into account the very great probability of policies of this kind being discontinued, by far the most important point for him to consider is the proper adjustment of the premium payable during the first term of, say, five years; and that it is essential that the premium payable for that term should somewhat exceed the premium for a term insurance of five years, in order to compensate for the option that is given to the assured under an ascending-scale policy, of continuing or dropping it at his pleasure at the end of the first five years.

One remark may be made before quitting this subject. It is that an actuary has to deal even more with figures than with formulas. He must submit every new formula to the test of actual numerical calculation. Very much in consequence of the absence of an established theoretical law of mortality, a numerical calculation is throughout as important as the demonstration of formulas, if not of greater importance. For this reason it will be found useful to the student to study those branches of mathematics in which numerical calculation occupies the most prominent position. Thus, the Theory of Logarithms, together with the practical methods of calculating tables of logarithms, will be found of special value. The subject of Analytical Trigonometry, also, is of great importance, as the formulas for the solution of triangles and for similar problems, admit of being so easily applied numerically. Perfect familiarity with trigonometrical formulas and processes will not only be useful to the student, as enabling him to study to advantage the differential and integral calculus, but also as accustoming him to associate inseparably the idea of a mathematical formula with its numerical application. In many branches of applied mathematics, it has been too much the custom in treatises designed for the English student—especially, perhaps, for the student who is reading for mathematical honors at the universities—to disregard the numerical applications of the formulas proved. This is particularly the case in such subjects as plane astronomy, which, as presented in the Cambridge text-books, is simply a collection of propositions, of the true bearing of which a student, in the absence of numerical applications, can never become fully cognizant.

In addition to the subjects already mentioned, an actuary is at all times liable to be called on to advise his directors on important questions of policy, such as the adoption of new rates of premium, new conditions of assurance, or new methods of valuation. In life insurance, as in other departments, whether of science or business, changes are constantly taking place. As regards what may be called the scientific basis of insurance, most important changes have, within no very long period, taken place in the mortality table employed in valuations. The first companies which calculated their premiums upon a scientific basis, used the Northampton three per cent. table. Subsequently a class of offices arose which used the Northampton four per cent. table. The latter, I believe, have all seen reason to alter their basis; but a small number of the former still adhere to the use of their original table. After the publication of the Carlisle table by Mr. Milne, the Northampton table very properly began to be held in less esteem; and for a long series of years the Carlisle table was the decided favorite among actuaries. More modern tables, such as the Experience of the Seventeen Offices, Davies's Equitable Experience, and Dr. Farr's English tables, have been competitors for public favor and have been adopted by many offices, but have failed to displace altogether the Carlisle table. The still more recent Institute table promises to be a formidable competitor to all others; partly on account of its intrinsic merits, but very much also for the reason that contributed so much to the general use of the Carlisle table, namely, the great number of the auxiliary tables based upon it which have been published. When an actuary is called upon to advise as to the abandonment of one standard of valuation and the adoption of another, a variety of points have to be considered, as to the decision of which the ordinary text-books will give him little or no assistance. He will have to consider how far the proposed data of mortality and interest will correspond with those likely to be experienced by his company, and what will be the effect of adopting elements which are admittedly different from those likely to be experienced.

Changes are constantly occurring also as regards the practical conditions and conduct of the business. Formerly it was the rule that lives assured must not proceed beyond the limits of Europe; and in making the voyage to and from the continent, certain ports were specified as the extreme ones to which the life assured might proceed,—the navigation of the Bay of Biscay, on the one side, and of the mouth of the Elbe, on the other, being considered too dangerous to be encountered without a substantial extra premium. Some of the most conservative companies still retain such conditions, and we see traces of them in the practice of other companies not quite so conservative, which notify the assured that they will be permitted to proceed to North America and certain other places beyond the limits of Europe, without extra charge, provided they apply for permission beforehand.

Some alterations, when once adopted by a few companies, are immediately forced upon the whole body, such for instance as the abolition of entry money, and the payment of stamps and medical fees by the company and not by the proposer. Other changes, such as the omission of the clause declaring the forfeiture of the policy in case of death by duelling, by the hands of justice, or by suicide, may be adopted by a considerable number of companies without competition compelling the others to follow their example. Quite recently we have seen still more fundamental changes introduced as regards the limits of foreign residence. One of the oldest of the companies now charges one fixed rate of premium for all parts of the world beyond the usual limits, while one of the youngest has abolished extra premiums altogether in its practice, and accepts all lives, wherever resident and whatever their occupation, at one uniform rate of premium. The conditions of other companies provide that no extra premium shall in any case be payable after the policy has been five years in force, while the conditions of another class provide that this privilege shall only be enjoyed if the life assured has not proceeded beyond the limits during the five years. The actuary, whether he considers these changes as improvements or the contrary, should always be on the watch to observe their effect; and he will not be able to form a sound opinion as to their probable tendency, unless he has been accustomed to investigate a variety of problems for himself.

It is true that the questions that thus come forward from time to time are frequently discussed by writers in the insurance periodicals, and the student is therefore rarely left to his own resources in forming an opinion upon them. But in order that he may be able even to follow and appreciate the reasonings of others, it is essential that he should have a considerable familiarity with algebraical methods and processes. As regards, however, the practical questions which from time to time force themselves upon the notice of the actuary, it cannot be considered satisfactory to form an opinion at second hand. The fully equipped actuary will not be content unless he is able to grapple with each of these questions as it arises, and give an answer to it himself. It will then be of the utmost service to him to compare his answer with those that are given by other writers, who perhaps approach the subject from a different side; and whether the answer which the actuary has in the first instance arrived at, shall ultimately prove to be right or wrong, the process of investigating it will be equally improving to him. It may possibly be the more improving when he finds that his first conclusion is vitiated by some error of reasoning, that he has been enabled to detect by comparing his solution with those of other writers. For this reason, I cannot help considering that the habit of taking a part in the controversies that are started from time to time in the insurance periodicals, is an extremely improving one for the actuarial student. Among other advantages of this course, it is no slight moral discipline to argue one side, whether it be the right or the wrong one, of a disputed question, bringing forward and stating forcibly all the proper arguments and illustrations, and exposing in forcible language the supposed errors of an antagonist, while avoiding carefully all undue warmth of language, and resisting every temptation to indulge in offensive personalities.

It is scarcely necessary for me to add that an actuary's advice is most likely to be followed, when it is not put forward simply as his opinion, but is well defended by cogent arguments founded on undeniable facts. When a change of policy is proposed, as for instance when the question of a reduction of the rate of premiums is proposed, or the adoption of more liberal conditions of insurance as regards foreign residence, the questions that are put to the actuary by his Directors, or that force themselves spontaneously upon his own mind, may often be best answered by an examination of the records and experience of his own company. It may, for example, be considered desirable to deduce a mortality table and calculate premiums of insurance, from the facts furnished by one particular class of policies issued by the company. This is a matter requiring great care, first in the extraction of the facts, and then in their subsequent treatment. It can scarcely be said that the extraction of the facts is a process for which much mathematical knowledge is required; but when the facts have been extracted, the process of adjusting them so as to deduce a satisfactory mortality table, that shall proceed without any abrupt changes and shall yet not deviate too far from the original facts, is a problem requiring for its complete solution the very highest mathematical attainments. Not only is an intimate knowledge of the Differential and Integral Calculus useful for this purpose, but a knowledge of the Method of Least Squares may also be extremely serviceable. Until we have come to this point, we have been able only to speak of a knowledge of the differential calculus as serviceable in a general way to the actuarial student; but when we come to the present part of our subject, and examine the methods that have been employed with most success to the graduation of mortality tables, we are compelled to say that no actuary can be considered to be fully qualified for the discharge of all the duties he may be called upon to undertake, who has not a competent knowledge of the differential and integral calculus. It is not to be forgotten, moreover, that extensive use has of late years been made of the Calculus, to obtain more exact values of annuities payable by half-yearly, quarterly, or other instalments; also of complete annuities; and in a variety of other questions, especially in the Theory of Continuous Annuities. It is not yet essential for an actuary to be familiar with these investigations, but it is impossible to say how soon it may become so; and, in the meantime, a study of them cannot fail to give the actuarial student a greater command over the simpler parts of his subject. I have hitherto confined my remarks entirely to the use of mathematics for the actuary, strictly so called; but some of you no doubt aspire to being hereafter selected to fill the more influential and responsible position of Manager, and a few words may therefore with advantage be said upon the relative positions of the Actuary and the Manager. A pure actuary may be regarded as a man of science, whose business it is to work out the problems submitted to him. The manager, on the other hand, is the representative of the company, whose business it is to communicate with the outer world. He informs himself, by correspondence, and by mixing freely with the supporters of his company, with the public at large, and with the representatives of other companies, what are the wishes of the public as regards rates of premium and conditions of insurance,—what are the expectations of the agents as to commissions and allowances, and of the borrowers and their solicitors when loan proposals are under consideration. He must constantly require calculations to be made by the actuary for the purpose of testing either some existing practice of the company or the safety of some proposed change. Comparing the functions of the actuary and the manager, as thus stated, it is perhaps difficult to say which of them is the more important; but a thoroughly competent manager should be able, if circumstances ever render it necessary, to work out the answers to those questions for himself. If he does not actually do so, it will be because he thinks his time may be more advantageously occupied. In the same way as he may give instructions to one clerk for making a lengthy return required for business purposes, to another as to the answers to be made to letters requiring attention, so he will delegate to the actuary the actual calculations required to answer the various questions of policy which present themselves to him. In other words, the manager should himself be a skilled actuary, in order to discharge his duties in a thoroughly efficient manner; and thus what I have said as to the usefulness of mathematics for the actuary, may be applied advantageously to those who aim at being ultimately managers.

May the Actuarial Society have before it a long career of usefulness, and train up a succession of accomplished and skilful actuaries, who may in their turn become the successful managers of prosperous English and Scottish Companies!

## *On Mortality Fluctuations. By* JAMES R. MACFADYEN, *of the Legal and General Life Assurance Society.*

[Extracted from the *Insurance Times* of New York.]

THERE is nothing immutable in this world, and certainly life companies are no exception to the general rule; for in them, from year to year, we find fluctuations of every kind—in amount of new assurances—in expenses of management—in value of assets—in realized interest—in the death claims—in fact, in every branch and item of their business. Of all these variations there is none so important as that concerning the mortality experienced. The fluctuations in the other elements of a life office's affairs are trifling as compared with this, and can without difficulty be prepared for. But in the perturbations of the death claims from year to year, we are dealing with a matter that cannot be calculated on in advance—an element which, if violent, may endanger the safety of the company itself; and, even when this is not the case, will often prove a serious inconvenience to it. I have called violent mortality fluctuations *inconvenient* to a life office. To men acquainted with the practical as well as the theoretical side of assurance, the word will not seem exaggerated. They, at least, know how important it is that the bonus to policyholders and the dividend to shareholders, do not retrograde. The moment the bonus tide begins to ebb, there is an outcry; and variations in the mortality may thus create disgust in those already connected with an office, and at the same time add another difficulty to the already sufficiently difficult task of its agents and canvassers.

The object, then, of the following paper is, in the first place, to examine the causes of the fluctuation in question; and, in the next, to see whether it is not possible to suggest a remedy, or at least alleviation, of an undoubted evil, constantly experienced in life assurance business.

First, as to the causes. The larger the number of lives in a mortality table, the less effect will the influence peculiar to the individual have on the death curve.* In order, then, that actuarial forecasts may be made with reasonable hopes of fulfilment, large numbers are indispensable. If we have this essential, unforeseen fluctuations will be rare indeed. For, though we know little, and can predict less, of the term of the acceptances by which the individual man pays nature's debt, yet the action of our creditor is so uniform, that the actuary can, in the mental clearing-house, set the long-dated bills against those of shorter term, and thus foretell the number falling due from year to year. In thus elaborately setting forth all this, I may seem to be wasting time over a matter that requires no elaboration whatever. We might think so, did not insurance officials seem to believe that, with what is called a true mortality table as basis, their death claims are necessarily provided for; and whether, as a matter of theory, this is really believed or no, there is not a doubt of the fact that there are life offices carrying on business with a number of policies so trifling, that prediction as to their claims is impossible. In companies with a very small number of risks, the insurance of life is not founded on mathematical principles at all. Such offices, instead of ruling the laws of chance, are their sport. And notwithstanding the zero in their favour of selection, or even the double zero of selection and loading, it would be far from surprising if in the roulette game such offices are playing, Death were occasionally to break the bank.

In a multitude, then, there is safety, and in that only. A sufficiently large number of risks is the first condition of a life company's freedom from death perturbations. It is not, however, the only one. Death's harvest may, in two consecutive years, show the same number of sheaves, but not the same quality of grain. The claims may have fallen on large rather than on small policies. This, however, is a form of fluctuation that has been foreseen and provided against by most offices. Almost all have a maximum risk sum, beyond which they will not go on an individual life. It is perfectly true that in many instances it would be rather a puzzling problem to ascertain the principle on which the particular figure had been arrived at; and it occasionally seems to bear not overmuch relation to the actual circumstances of the company. However, notwithstanding its empiricism, I believe that in practice the method has on the whole worked well, and has proved a tolerably efficient guard against severe mortality fluctuations from this source. I therefore need not enlarge on this point; and I am the more inclined to pass it by, as it has already been most ably discussed by Mr. Sprague in the thirteenth volume of the *Journal of the Institute of Actuaries.*

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* It does not follow from this that the mortality table founded on the largest number of lives is necessarily the best for adoption. The table ought to be on all fours with the mass of the lives assured, and so the nature of the business to be transacted must be considered in choosing a table. Thus a company, the bulk of whose risks lay in the more Southerly American States, would hardly be safe in adopting a table of lives drawn from the Union generally. And on the same principle, a table founded on the experience of a nation at large is not the best for use by a life office, since assurance companies create a new subdivision by medical selection.

I have hitherto been dealing with sources of perturbation *outside* our basis mortality table,—which are not allowed for in it, and which therefore, unless otherwise guarded against, may endanger the stability of a life company. This is the only class that is dangerous, and I do not propose to consider at length any other. It may be well, however, to remind the reader that there are causes, which may be called *inside* the basis table, tending to increase the claims from year to year. Thus the strain on a life office augments with the average age of those assured in it. The young *may* die, but the old *must.* And so there will be a tendency to growth in the claims, either absolutely or proportionately. Against this tendency, business energy and medical selection fight with temporary success. Fresh lives of high vitality are constantly thrown into the mass. If, in ratio to the total risk number, there are many thus added, the evil day when the waste shall exceed the nutriment, may be staved off for the time; but sooner or later it must come. The most progressive of companies has in it the germs of decay, though not necessarily of death. It need not become extinct. The waste and the growth may each alternately have the mastery, and thus the office may, moon-like, wax and wane and wax again. Of a kindred nature is another cause productive of increase in the annual mortality claims. In companies that declare their profits in the form of a reversionary addition, this bonus accretion will tend to augment the sums payable from year to year.

As I have already said, both of these causes of claim-increase are within the basis-mortality-table—are provided for by it—and are therefore not dangerous to life offices. I have, however, thought it expedient to point them out, since they press most heavily on the older societies. If then statistics show that, despite being thus handicapped, the longer-established companies have less variation from year to year in their claims than their younger compeers, we are led to the conclusion that it is to their ordinarily greater number of risks that the older offices owe their freedom from the fluctuations that so violently shake their less broadly-rooted brethren.

I will now proceed to show from actual figures what I have already analyzed theoretically—namely, that the essential cause of fluctuation is insufficient numbers: and that, in proportion as the risks are multiplied, the deviations from the basis-mortality-table diminish in violence. The following statistics are compiled from the returns made to the Board of Trade under the Life Assurance Companies Act. The number of policies is one of the items given in the fifth schedule. The claim amount, which deducts reassurances, but includes reversionary bonus, is taken from the first schedule (the revenue account).

The fifth, which is the valuation schedule, not being an annual return under the Act, which was only passed in 1870, all the offices have not yet given it, so the following figures had to be based on the returns already made. All companies furnishing three revenue accounts, and at least one valuation schedule, have been included in the table:—

| | | | Percentage of preceding year's claims formed by difference between that year's amount and that of its successor. | | |
|---|---|---|---|---|---|
| Number of Risks in all. | No. of Offices. | Policies in them. | *Second* year as compared with *First.* | *Third* year as compared with *Second.* | Average annual fluctuation. |
| Offices each with less than 2000 ... ... | 12 | 10,527 | 120·5% | 24·3% | 72·4% |
| Offices each with between 2000 and 2999 | 5 | 12,366 | 31·4 | 18·8 | 25·1 |
| Offices each with between 3000 and 3999 | 7 | About 23,868 | 19·6 | 24·7 | 22·2 |
| Offices each with between 4000 and 4999 | 8 | 36,212 | 16·0 | 18·4 | 17·2 |
| Offices each with between 5000 and 9999 | 18 | About 126,734 | 12·7 | 21·2 | 17·0 |
| Offices each with over 10,000 ... ... ... | 14 | 230,900 | 13·7 | 14·4 | 14·1 |
| Offices each with *less* than 5000 ... ... | 32 | 82,973 | 58·4% | 22·0% | 40·2% |
| Offices each with *more* than 5000 ... ... | 32 | 357,634 | 13·2 | 18·2 | 15·7 |

Comment on the above results is unnecessary. They show, as plainly as figures can show, the intimate connection between numbers and fluctuations. Even if, instead of taking the wider and therefore more correct view—namely the average of two years' variations—we contented ourselves with examining the individual annual fluctuation, we still find our data pointing unmistakeably in the same direction. As the numbers increase, the oscillations decrease. In the second year, as compared with the first, it is so, save in a single instance. In the third year, as compared with the second, it is so also, with two exceptions;—all exceptions, however, are brought back to rule in the average of the two results. The figures cannot be explained away on the ground that the fluctuation may in any great part arise from death striking at larger policies one year than another. As I have already said, offices have at present in action a precaution against this. Besides, extreme fluctuations from such a cause would not be felt merely in the smaller companies. And it thus becomes impossible to account on any such hypothesis for the story told by the preceding figures. It is still less important that the policy-numbers given, correspond to only one of the three years. The variation is trifling, and would have no practical effect on the results. On the other hand, I believe that, had I the means of taking individual lives, and not policies, as the measure, the result would have been still more striking. Be that as it may, the figures, even as they stand, show forcibly enough the part numbers play in the mortality of a life office.

Various curious results might be deduced from the extended table of which the preceding is a summary. For instance, by throwing the returns of various offices together, and comparing the mortalities of the new companies thus created, both with each other and with those of single societies, we might, were our basis broad enough, get some hints as to the limits of death-oscillation in British offices with large risk numbers;—or, by comparing the Registrar-General's report from year to year with the corresponding policy-claims, we might be able to determine whether an increase in the amount paid for any year by assurance offices generally, was due to a death-wave sweeping over the whole country. As it is, with so narrow a basis as the preceding figures give, it would be worse than useless to go into such enquiries. The figures are however quite broad enough to furnish all that we have aimed at, —namely, to show that fluctuations are mainly dependent on risk numbers.

It now remains, having diagnosed the disease, to study the remedies. The obvious cure for mortality fluctuations is, of course, an increased business. To point this out, however, as the specific for the fluctuation malady, would be a mockery. Few companies require to be told that new business is a desirable thing; and if they do not set about getting it from more palpable considerations, they are scarcely likely to be influenced by this rather recondite reason. Nevertheless, however desirable and desired a large business may be, there will always exist many offices that do not possess it. With the best and most energetic management, there will still be many offices that for a long period cannot have sufficient risk-numbers to avoid dangerous fluctuations. I will therefore examine what measures can be devised to give to small companies the risk numbers which they have not.

In the doctrine of averages, time and number are convertible terms. Infinitesimal numbers and infinite time must equal infinite numbers and infinitesimal time. And thus a life company with few risks will, by sufficiently lengthening its valuation period, (assuming, of course, it can pay its way in the meantime), be enabled to set its years of plenty against its years of famine, and so declare a non-fluctuating profit, despite the deviations from year to year. The valuation date should, then, be fixed with some reference to the risk numbers.

As a corollary to this, it follows that it is in the highest degree imprudent for a small office to divide annually *all* the profit realized. Fluctuations damaging to the welfare, perhaps even dangerous to the stability of the company, *must* ensue. On the other hand, for various reasons, it is equally undesirable that the valuation date should be very long delayed. And thus there are many offices with so few risks that they can neither divide profits frequently, with safety, nor protract their bonus period to such a length as the number of their policies demands. What follows, then? Must it be held that small offices should be incontinently extinguished? To this I cannot assent. Young companies must necessarily be small companies—and young societies certainly have their uses, not only to insurance officials and to the press, but as really, if more indirectly, to the provident public also. Their competition has prevented the older offices from becoming fossils, which would otherwise have been useful mainly to the student of life insurance geology as petrifications of the primary period. And from this competition the public has, with various drawbacks, serious enough, derived the benefit, in reduced rates and more liberal conditions. What, then, can be done by small offices to avoid mortality fluctuations? There is, it seems to me, only one remedy, namely, an extension of the reassurance system. I will therefore devote the remainder of this paper to an analysis of a method by which this subdivision of risks can be carried out. I may say in advance that I do not consider that in the adoption of such a measure there are no drawbacks of any kind. The method is at best an attempt to furnish by artificial means a substitute for the natural food, and, like all such efforts to supply nature's place, it must of necessity be imperfect. Infants brought up by hand can scarcely have the vigour of those that draw their nourishment from the breast. But hand nurture is better than none at all, and it is in this spirit that I proceed to indicate a scheme for the subdivision of risks.

In a scheme in which so much depends upon details, which can only be settled by the special circumstances of the case, it is not possible to lay down all points absolutely. And thus in the following analysis, unless the details suggested are stated to be universally applicable and therefore of the very essence of the matter, they are not to be considered necessary or even specially desirable. As already said, the scheme here indicated is simply an extension of the existing reassurance system. Let two or more life companies doing the same class of business, combine to offer each other a portion of their new risks; and let this be done although the parent office might even of itself be able to retain the whole amount of the proposed assurance. Assuming then for the moment, the new risks in each office to be of the same amount and value, and that all are shared to the same extent with the other companies, each society will have exactly the same amount of new business, with exactly the same profit-producing power, as if it had never joined the combination; but will hold that business on twice, thrice, &c., as many lives as in that case. This is the principle of the plan here indicated for the diminution of mortality fluctuations, and it now remains to suggest details for its practical working.

As is now done under the ordinary mode of reassuring surplus risks, let the society to which the original proposal is made, lay before the company or companies working with it, all the information it has itself on the case, stating at the same time, how much of the total amount is retained by itself, and how much it offers to the society before which the case is laid; and let this latter office then accept or decline the share of the assurance thus offered. It may be pointed out that it is not here intended that the acceptance of the original risk by the parent company should be dependent on the completion of the reassurances. Except in under-average cases (to which my observations are not for the present to be understood to apply), the parent office accepts or declines, exactly as if it retained the whole amount itself. Though it is thus not necessary to delay the acceptance of the original proposal till the reassurances are completed, still it is essential that such reassurances as the parent office proposes to make, should be carried out so soon after its own acceptance of the risk as may be practicable. It is also vital to the scheme, to remember that though societies enter into such a combination, no one of them has any obligation either to give off any risks at all or to accept any offered. Each office gives such number of assurances, of such nature, and of such amount, as it pleases; and the other societies accept or reject them, just as they may choose.

The details of a scheme like this can be best indicated in discussing the objections to it, so I will now proceed to do this.

*First.*—It may be urged that the company reassuring has no security that for the amount given off it will receive a like amount in return. It is proposed to meet this objection in two ways—the first being to take care that the amount reassured contains the minimum of profit-producing power; and the second, to have between the combining offices, say, monthly balances of the reassurance accounts, and possibly still oftener than monthly towards the close of the financial year; and at such balancings, or, indeed, at any time, the company that finds itself much the creditor can, if it pleases, stay its hand till the debt is extinguished. At the end of each year, the balance should be struck solely with regard to the amounts then remaining at risk. It may be well to point out that the greater the difference in the amount of the new business of the combining offices, the oftener the balancing should be effected.

*Second.*—But it may be urged that assurances can be equal in amount and yet be of different profit-producing power. Thus, a company in return for profit cases might get non-profit—for joint lives, term risks—for endowment assurances, half-premium policies. Leaving aside, for the moment, the question of varying age, I will proceed to examine this objection. In order to avoid bonus complications, it is absolutely essential to the working of the scheme, that, whatever the original policy may be, the reassurance be without profits;—and still further, the non-profit premiums under which the transactions are carried out, cannot be the ordinary "without participation" scale of rates, but must be a special table of premiums, representing, as nearly as may be, the cost price (including expenses) of the risk undertaken. This point is vital; first, because the temptation to give less reassurances than are received (objection 1) must be reduced to a minimum; and, secondly, because the office which has to give its own bonus to its policy-holder, must retain within itself the whole profit-producing power. The scale of premium for these interchanged risks should therefore be, as nearly as possible, a cost-price scale—care being taken in fixing it to see its bearing on the premium used for valuation purposes.

The facts that reassurances are yearly balanced according to the amounts *remaining at risk*, that they contain the minimum of profit-producing power, and above all, that each company retains the right of declining cases it does not consider an equivalent for those given by itself, are in themselves a sufficient safety-guard against this second objection. However, as an additional precaution, reassurance accounts between societies might be kept in three divisions—one containing the ordinary whole-life equal premium class, another those in which the premium exceeded this, and a third where it fell short of it. This is a complication, however, upon which, I think, there is no need to insist. The only other point to be examined under this head is that of the ages. Even if the class and amount of assurance were the same, the premiums taken and given off might, through diverse entry ages, be different. Since however the reassurances are, as nearly as may be, taken at cost price, this is a matter of little moment; and this the more especially when we remember that risks are yearly balanced by the amounts remaining in force;—and, above all, that an office entertaining a suspicion that its fellow in the combination is unfairly selecting its offered cases, has the power of declining those risks which it considers less valuable than its own.

Unless selection is exercised, the entry ages in reassurance proposals will be practically the same in each company, since they are assumed to do the same class of business. It may be well, under this section, to consider the question of selection. This may arise when in exchange for better risks there are offered either inferior lives or policies of less profit-producing power. With regard to selection of the former kind, it may be pointed out that combining at all is only suggested in reference to offices doing much the same class of business; that also the reassurance risk is accepted at the same time, and (unless there is a distinct fraud,—a thing equally practicable under reassurances at present) on the same data as the parent office; and that, also, the society reassured with knows the amount retained by that parent company. The precautions against selection of the second kind have been already discussed. It will be noticed that if this scheme is carried out to any appreciable extent, selection in either form can hardly occur, unless some one of the combining offices has a new business so much larger than its fellows as to be able, while keeping pace with them, risk for risk, in the *amounts* exchanged, to select the least eligible of its larger number for reassurance. For this reason it may be considered expedient that the societies combining have new business not only of much the same nature, but of much the same extent. This, however, is not absolutely necessary. The principal security against selection is the power of the office selected against to reject ineligible lives.

*Third.*—It may be pressed as a reason against the working of such a scheme as is here analyzed, that the various societies may have very different rules in regard to the conditions and terms of their policies, extra premium for climate and occupation, surrender values, and the like. This objection can only be met by a comparison of the practice of the individual offices proposing to work together. Each company will require to see whether it can, without appreciable loss to itself, take reassurances from the others on their conditions. (It is obvious that the rules by which the parent office is bound to its policyholder, must be binding on all guarantees to that office.) Practically there are but few regulations so vital that a difference of practice in regard to them will render the exchange of guarantees in even large numbers impossible. On these few the practicability of working the scheme will depend. The same difficulty is got over every day in respect of ordinary reassurances.

*Fourth.*—It may be argued that since a large number of risks will be exchanged between offices that are parties to such a combination, a period will at length arrive when a dangerously great proportion of the assets of each society, will be dependent on the management of all the other companies which are partners in the scheme. This is undoubtedly a very serious consideration. The objection, however, may be met by making it a necessary preliminary to every such arrangement between offices, that, in addition to the personal security of the company reassured with (which is, of course, pledged by the handing over of their guarantee), each society have the real security of a deed conferring a lien on the value of the risks underwritten with it by each of the other offices, for the due payment of those it has underwritten with that office. When it is remembered that the exchanges between the companies will be of practically the same value, it will be seen that the danger is reduced by this safetyguard to a very small fraction. In fact it would be a vanishing fraction, and disappear altogether, were it not for the possibility of a rush by the lives assured in a sinking office, to secure the surrender value of their policies; and thus there would be a chance of the strong office having its mortgage security diminished, without a corresponding diminution of the assurances locked up in the sinking company. This point will however be provided for—in part, at least—by the condition that the surrender value paid by a society reassured with, shall not exceed that paid for a like proportion of the risk by the parent office to its policyholder. When we remember the facilities of ascertaining a company's condition given by the enforced publication of its accounts—the exceeding unlikelihood of such a rush as we are discussing—the fact that, even if it does occur, it *must* be among only the recent and therefore least valuable policies (a part of the business that will not affect at all the strong office, since the interchange of risks will probably have long ceased); when we remember all this, it seems that, with the aforesaid mortgage, practically no loss can ensue to a solvent company from its partner's insolvency. And we are led the more powerfully to this conclusion when we remember the difference between the surrender values that would be paid by a sinking company and the amounts representing its liability. Altogether, with the precautions pointed out, the objection I have been urging can, I believe, be successfully encountered. It may be considered expedient, however, for offices proposing to trade together in this way, to see that there is no great discrepancy in their actuarial basis, and that their assets are of a like good quality.

*Fifth.*—I have reserved for the last the argument that, in consequence of the operation of such a plan as is here suggested being to increase the number of the risks, without augmenting the total amount of the new business, it will swell the working expenses. This is, to a certain extent, undoubtedly true; and it is therefore inexpedient for a company whose average policy is very small, to enter into such a combination. There is a point in the subdivision of assurances where the good is outweighed by the evil; but to most societies that point is far below one-half or one-third of their average policy. And it is precisely the companies which can best bear subdivision of their risks, that have the fewest of them, and have thus the greatest need of such a system. The increased expense of moderately-sized subdivisions is not much. In reassurances on the method here proposed, there will be no medical fees, no commissions, no advertising, not one of the expenses incidental to the *procuring* of new business, nothing, perhaps, except the policy stamp and a little additional labour to the office staff:—an extra expense, no doubt; but one, in most instances, not to be compared with the benefit resulting from the reduction of the death fluctuations. To under-average assurances, which however would have to be treated as a class apart, the plan seems specially applicable. It would diminish the risk of such assurances, since; if the best thing in these cases is to have a correct mortality table to go by, the second best, in the necessary absence of the first, is to have a minimum sum at risk. If such arrangements for under-average lives were common, we should be more frequently spared the spectacle of an under-average proposer practically putting his proposal up to auction, to be knocked down, not to the highest but to the lowest bidder—that is, to the company most willing to undersell its neighbours.

If such a plan as is here suggested were applied to under-average risks, it would be expedient that the assurance (almost necessarily non-profit) be completed, and the rate fixed, not by the parent office alone, but by the whole combination, which would receive their share of *this* premium. It may be remarked that a combination on a similar principle for the assuring of under-average life, has been in existence in Scotland for many years, and has, I believe, worked well.

The association of offices for the purpose of diminishing fluctuations, is applicable to investments as well as to mortality. Whether it is expedient to adopt such a plan, in order to make investing safer, is a question upon which I will not now enter. It is exactly the principle on which the associations for the purchase of foreign stocks and other financial operations are founded.

I have now made what anatomists would call a "preparation" of a system for diminishing mortality fluctuation by creating larger risk numbers. As has been already stated, it is here advocated not as a thing desirable in itself, but as a *pis aller*. The multitude of precautions I have pointed out, may possibly suggest the idea that the plan would prove too cumbrous for working. I do not consider, however, that in actual practice it would be nearly so difficult to carry out as it may seem on paper. I have, in my objections, assumed a friction greatly exceeding what may be looked for as likely to occur. In calculating the strain on a bridge, the engineer allows for a tension and compression far beyond the utmost weight that can with any reason be expected to come upon it. So in fencing the preceding plan of reassurances with so many precautions, I have assumed that each office entering such a combination does so with the deliberate intention of overreaching its partners, so far as the law will permit. If any business practice were treated in this fashion, a plan to ensure its safe working would look equally cumbrous.

In fact, as has been already said, the scheme here elaborated is simply an extension of the present reassurance system, and every difficulty I have examined, now exists in that system; though, in ordinary practice, they are either overlooked, or, from the paucity of reassurance cases, ignored. I have simply projected these difficulties on a larger surface, and they consequently stand out more clearly. Both the disease of mortality fluctuations and its remedies have now been discussed. If freedom from severe mortality fluctuations is sought, large risk-numbers are indispensable, let them be procured how they may. It is only offices which have such numbers that can enjoy this freedom, while companies that have not such numbers may at any time be deprived of the stability they have hitherto seemed to possess.

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## *Economical Rates of Reserve and Premium.*

[From the *Spectator*, of New York: Chicago.]

WHATEVER difference of opinion may exist as to the rate of interest upon which a life company should be required to estimate its reserve, there is none, we believe, in relation to the desirableness of uniformity in the rate in the several states. We have recently had so many illustrations of the absurd effect of discordance among the states, that we are disposed, more than ever, to press the advantages of a uniform standard of reserve upon the attention of the public. The spectacle presented of a company declared solvent by one insurance department and insolvent by another, though the schedule of assets is the same in each examination, is calculated to impress the popular mind with suspicion as to the existence of common sense in the administration of the laws governing the life insurance business. It is a fact that a company may be legally sound on this side of an imaginary line, called, we will say, the boundary line between New York and Connecticut, and unsound on the other, and it all comes of the fact that on this side the company's reserve is valued at four-and-a-half per cent, and on the other at four per cent.

Whether public opinion can be sufficiently interested in the correction of this anomalous condition of things, to force the legislatures of states to agree to some standard which shall be alike in all, may be questioned. Probably the uniformity we desire cannot be had until there is a national bureau of insurance established by congress. The most direct road to a complete systematizing of governmental supervision is that which leads to national supervision, for it would seem to be a tiresome task to induce the states severally to fall into line with uniform laws.

In reference to what should be the rate of interest recognized as safe and equitable, if a uniform standard of valuation should be fixed, the position of the *Spectator* is well known. We reiterate an oft expressed opinion when we say that the standard prevailing in most of the states is unnecessarily and even injuriously high. Massachusetts makes it four per cent; New York, four-and-a-half per cent; Pennsylvania alone makes a concession—and that in favor of stock companies only—of six per cent. The four or the four-and-a-half per cent standard is largely more than is requisite for safety, because there is not the slightest prospect that in the life time of any of the existing American companies the value of money will be nearly as low as that standard. The tendency is notoriously the other way. It is true that there is an immense supply of capital in this country, and that it is continually augmented by the flow hither of European capital seeking better investment than it can obtain in Europe; but the demand for money to develop our mighty mineral resources, our manufactures, and our means of transportation, is insatiate, and must continue until our broad area is compactly settled, and we are a *finished* nation, in the same sense that nations on the continent of Europe are finished—having reached their limit. When we consider the extent of our territory yet untouched by the axe and plough, we will perceive that the day when the Union will be as densely populated as European countries will not be reached for at least one or two centuries to come.

An intelligent opinion of the value of money, in the United States, for many years to come, was written by Judge McKinnon, of Chicago, a practical financier, and published in the *Spectator* a few years ago. We quote a striking extract:

"Were the Mississippi river the western boundary of our continent, the case would be different. The fact is, that all the states lying on its eastern border are yet in their infancy, coupled with the further fact that to the west of the mighty river lies an empire fertile in all the products of the earth, and in whose bosom lies hidden millions of treasure; to finish the unfinished portion of this great highway, and to utilize and develop the vast and varied extent of country beyond, must long create no ordinary demand for money. The west is not only unfinished but it is a live country, filled with live, enterprising people from every land and every clime, whose mission is the development of this empire. To do this they must and will have money, for they are not lenders, they are borrowers. What will be the rates of interest twenty, thirty and fifty years hence, in the above cases, can only be conjectured. I will venture to say, however, that in twenty years the average rate in this section, on the kinds of property I have mentioned, will be very little, if anything, below 8 per cent. In thirty years it may be 7 per cent, and in fifty years it may be 6 per cent."

Should a uniform standard of valuation be recognized, we believe that 5 per cent could be trusted as entirely safe. The lowering of the standard to that point (accompanied, of course, by a reduction of premiums) would necessarily effect one very desirable change in the administration of the business of our life insurance companies. It would enable buyers of insurance to retain in their own hands a considerable part of the funds which are now *superfluously* heaped up in the coffers of the companies. The difference between the official valuation at four per cent, and interest earnings at six and seven per cent, is millions of dollars. This money is not needed, as we have shown, to ensure solvency. It is taken from the people, however, upon that presumption, and becomes a fund to tax the extra oversight of company officials. Admitting that these gentlemen are consciously influenced by the best motives only in the management of this fund, the fact still remains that as it is superfluous, it continually tempts to extravagant and unnecessary expenditure, if to nothing worse. Admitting, too, that surplus is eventually returned to policyholders in the shape of dividends, still it has to bear the cost of management. Policyholders have the right to claim that they can manage this surplus as advantageously to themselves as others can, and therefore that they should be furnished with insurance at the lowest price, consistent with the safety of the companies.

# HOME AND FOREIGN INTELLIGENCE.

## SCOTTISH WIDOWS' FUND LIFE ASSURANCE SOCIETY.

*Established* 1815.

### REPORT BY THE MANAGER.

### I.—Introductory Narrative.

The period of seven years ending 31st December 1873 has not, as regards the progress of the Society, been marked by any changes, except such as have arisen from the continued extension and prosperity of its business. In life assurance affairs generally, the disastrous failure of the Albert and European companies, and the subsequent passing of the Act 33 and 34 Vict. Cap. 61 (August 1870), requiring the public registration of the accounts and valuations of every company carrying on business, have produced very important and beneficial effects. The Society's long-continued practice of publishing balance-sheets and valuations had showed the practicability and value of such publication, and the 'Summary and Valuation of Policies', required by the Act, is substantially the same as that in the Society's last septennial report.

(1.) *Increase of Business.*

The amount of New Assurances transacted during each year of the last septennial period was as shown in the following—

Vidimus of New Business Transacted.

| Year of Issue. | Policies issued. | Amount Assured. | | | First Premiums thereon. | | |
|---|---|---|---|---|---|---|---|
| | | £ | s. | d. | £ | s. | d. |
| During Year 1867 | 1,356 | 811,410 | 13 | 4 | 27,595 | 9 | 5 |
| Do. Year 1868 | 1,311 | 711,608 | 0 | 3 | 24,222 | 12 | 9 |
| Do. Year 1869 | 1,298 | 732,377 | 12 | 8 | 25,375 | 19 | 8 |
| Do. Year 1870 | 1,700 | 965,627 | 15 | 1 | 31,667 | 10 | 9 |
| Do. Year 1871 | 1,897 | 1,091,205 | 0 | 0 | 33,976 | 6 | 2 |
| Do. Year 1872 | 2,156 | 1,213,800 | 8 | 0 | 39,039 | 15 | 11 |
| Do. Year 1873 | 2,340 | 1,480,765 | 9 | 3 | 47,428 | 16 | 3 |
| Total . . | **12,058** | **£7,006,794** | **18** | **7** | **£229,306** | **10** | **11** |
| The New Business of the previous Period ending 1866 was . . . . | 8,112 | 5,461,884 | 7 | 9 | 185,396 | 10 | 8 |
| Excess of Present Period | **3,946** | **£1,544,910** | **10** | **10** | **£43,910** | **0** | **3** |

1. The Business thus transacted does not include any Business reassured with other Offices.

2. The Agencies of the Society being confined to the three Kingdoms, the risks undertaken are those only which are connected with a Home Business.

3. There has been a strict adherence to the Society's Rule of paying Commission to its regularly appointed Agents only, Solicitors introducing business being allowed no more than a single fee.

### (2.) *Extension of Branches and Agencies.*

During the last seven years the Directors have still further carried out the system of establishing Branch Offices under the charge of Resident Secretaries, by opening two of such offices in Bristol and Newcastle, as well as by further strengthening those previously opened. The plan of the Directors in regard to these local centres of superintendence is now completed, no further appointments of that kind being at present considered desirable. The great convenience of these Local Offices, and the value of the superintendence which they enable the Head Office to exercise over the remotest Agencies, no doubt much more than repay the expense of maintaining them in a state of efficiency.

The purchase and erection of suitable buildings for the Branch Offices have been effected on terms which yield a return on the amount expended equal to the average derived from the Society's other investments, and the properties acquired are all in first-class and improving situations. At the same time, it has been considered proper by the Ordinary Board of Directors to raise at once a Depreciation Account for these buildings, and to carry to the credit of it at the present time a sum of £12,500. This amply provides for any diminution of market value which can possibly occur within the next seven years.

### (3.) *Interest Realized.*

The average rate of Interest realized during the seven years has been:—

| | £ | *s.* | *d.* | |
|---|---|---|---|---|
| In 1867 . . . . . | 4 | 7 | 6 | per-cent. |
| „ 1868 . . . . . | 4 | 5 | 0 | |
| „ 1869 . . . . . | 4 | 5 | 4 | |
| „ 1870 . . . . . | 4 | 7 | 1 | |
| „ 1871 . . . . . | 4 | 7 | 0 | |
| „ 1872 . . . . . | 4 | 4 | 10½ | |
| „ 1873 . . . . . | 4 | 6 | 0 | |

It seems now to be generally understood and admitted that, especially in large consolidating Loans, the money of a Lending Corporation like the Society possesses an element of value which cannot belong to the funds of private persons, and on this ground better rates are willingly paid on large Loans for extended terms.

### (4.) *Expenses of Management.*

The whole expenses of management, including commission and every other charge at the Head Office, Branches, and Agencies, have during the seven years been at the rate of £7. 7*s.* per-cent on the total revenue. This, though an increase on the preceding period, is more moderate (I believe) than the rate of any similar Office doing a business like that of the Society, through numerous Agencies, and largely increasing its new business from year to year. The rate of expense necessarily rises in the first instance in proportion to the extent of new business. The development of the District Offices and Agencies under them has, during this last septennium, led to new charges which, with a proper regard to the permanent interests of the Society, are unavoidable.

### (5.) *Increase of Resources.*

The net funds and revenue have increased during the seven years as shown below:—

| | Funds. | | | Annual Revenue. | | |
|---|---|---|---|---|---|---|
| | £ | *s.* | *d.* | £ | *s.* | *d.* |
| 1867. Jan. 1, commencement of Septennium | 4,371,995 | 0 | 1 | 589,440 | 9 | 8 |
| 1867. December 31, end of 1st year . . | 4,541,871 | 13 | 1 | 588,065 | 9 | 2 |
| 1868. „ end of 2d year . . | 4,670,913 | 3 | 2 | 596,276 | 8 | 4 |
| 1869. „ end of 3d year . . | 4,857,145 | 12 | 8 | 613,387 | 6 | 4 |
| 1870. „ end of 4th year . . | 4,947,089 | 8 | 11 | 641,086 | 7 | 11 |
| 1871. „ end of 5th year . . | 5,168,261 | 2 | 11 | 663,702 | 6 | 4 |
| 1872. „ end of 6th year . . | 5,439,201 | 13 | 5 | 698,236 | 6 | 1 |
| **1873. Close of Septennium . . .** | **5,626,854** | **8** | **11** | **733,354** | **1** | **10** |

Thus the increase which has taken place in the Society's resources in the course of the septennium is as under:—

**Funds . £1,254,859 8 10 Annual Revenue . £143,913 12 2**

### (6.) *Mortality Experience.*

The following table exhibits the actual experience of the Society in number of deaths and amount of claims compared with the expectations under the Carlisle table, and also with the expectations of three other well known tables:—

COMPARATIVE RESULTS OF SEVEN YEARS, 1867-1873.

| | Number of Deaths. | Amount of Claims. | Difference in favour of the Society. | |
|---|---|---|---|---|
| | | | Deaths. | Claims. |
| | | £ | | £ |
| Actual Experience of Society . . . | 1928 | 2,765,754 | ... | ... |
| Expected— | | | | |
| By Carlisle Table . . . . . . | 2171 | 2,810,981 | 243 | 45,227 |
| „ New Experience of Assured Lives, 1863 | 2139 | 2,857,944 | 211 | 92,190 |
| „ Experience of seventeen Offices, 1834 . | 2208 | 2,948,338 | 280 | 182,584 |
| „ English Life Table, No. II., Males . | 2386 | 3,114,440 | 458 | 348,686 |

The remarkably favourable character of the Society's Experience is thus clearly proved by every test which can be applied to it.

In the amount of claims actually paid (£2,765,754), there was included £770,780 of bonus additions. Excluding the non-profit policies amounting to £67,531, the additions were thus, *on an average*, no less than 40 per-cent of the sums originally assured by all the policies becoming claims during the last seven years.

## II.—Investigation of the Society's Affairs at 31st December 1873.

### (1.) *Ascertainment of Surplus.*

The Carlisle Table of Mortality and 3 per-cent interest, which together form the basis of the Society's system of calculation, have been used throughout the valuation (including the annuities which at last investigation were taken at 3½ per-cent), and the 'loading' or addition to the net premiums has been wholly excluded from the estimate, the net premiums alone having been taken credit for as an asset in expectation. All possible anticipation of future profit has thus been effectually guarded against. The Guarantee Fund being fixed on the principle of a percentage on liability, the separate valuation of each policy has been unavoidable. To insure perfect accuracy these values were deduced in duplicate by separate computers. The sum of the values was also compared and reconciled with the results of another valuation by the method of lists classified according to age. After applying these several checks, it is impossible to doubt the absolute correctness of the results obtained.

* * * * *

It will be observed—

| | | | | | | |
|---|---|---|---|---|---|---|
| The total surplus at the end of the seven years is . . | | | | £1,275,321 | 4 | 11 |
| *Whereof—* | | | | | | |
| (1.) There has been already paid as contingent prospective bonuses on policies becoming claims during the seven years . . . . . | £73,697 | 5 | 10 | | | |
| (2.) There has been carried to the credit of 'Depreciation of House Property Account' . . . . | 12,500 | 0 | 0 | | | |
| | | | | 86,197 | 5 | 10 |
| Leaving . . . . . . | | | | £1,189,123 | 19 | 1 |

***Now to be distributed among the Members in terms of the Laws.***

## III.—Division of Surplus among the Policyholders.

The whole of the undivided surplus shown above will now, subject to the approval and order of the Extraordinary Court, be apportioned as follows:—

| | | | |
|---|---|---|---|
| (1.) In providing a Guarantee Fund at the rate of £5 per-cent on the value of the total liabilities. This requires a sum of . . . . . . . . . | £267,251 | 15 | 10 |
| (2.) In providing a reversionary bonus on all participating policies at the rate of £1. 13*s*. per-cent per annum on sums assured and previous bonuses. This requires a sum of. . . . . . . . | 907,732 | 4 | 1 |
| (3.) Leaving unappropriated the sum of . . . . | 14,139 | 19 | 2 |
| | £1,189,123 | 19 | 1 |

The members are aware that by the Laws the share of the Guarantee Fund attached to each policy is a cash addition on its value at the same rate as that at which the whole fund is constructed (£5 per-cent, *see above*), payable at death if that event happens before another investigation, with 3 per-cent interest from 31st December last.

The Bonus of £1. 13*s.* per-cent will be calculated, as on previous occasions, in the compound form upon original sums and all former bonus additions in existence at 31st December 1873.

In order to show the increasing scale of Additions under the present division, some examples may be given:—

EXAMPLES OF ADDITIONS TO POLICIES OF £1000 FOR THE SEVEN YEARS ENDING 31ST DECEMBER 1873.

| Year of Entry. | Original Sum Assured and Bonuses to 31st December, 1866. | | | New Bonus at 33*s.* per-cent per Annum. | | | | | | In addition to the New Bonus, the Shares of Guarantee Fund to be attached to the Policies are approximately as under. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | Amount of New Bonus. | | | Annual Percentage on Original Sum Assured. | | | | | |
| | £ | *s.* | *d.* | £ | *s.* | *d.* | £ | *s.* | *d.* | £ | *s.* | *d.* |
| 1817 | 2444 | 3 | 4 | 282 | 6 | 0 | 4 | 0 | 8 | 118 | 2 | 10 |
| 1820 | 2071 | 10 | 6 | 239 | 5 | 3 | 3 | 8 | 5 | 99 | 1 | 11 |
| 1825 | 1939 | 17 | 3 | 224 | 1 | 1 | 3 | 4 | 0 | 89 | 3 | 5 |
| 1830 | 1808 | 3 | 8 | 208 | 16 | 11 | 2 | 19 | 8 | 78 | 3 | 6 |
| 1835 | 1663 | 2 | 7 | 192 | 1 | 10 | 2 | 14 | 11 | 66 | 19 | 3 |
| 1840 | 1512 | 17 | 5 | 174 | 14 | 9 | 2 | 10 | 0 | 53 | 19 | 9 |
| 1845 | 1377 | 16 | 11 | 159 | 2 | 10 | 2 | 5 | 6 | 42 | 9 | 9 |
| 1850 | 1266 | 11 | 7 | 146 | 5 | 10 | 2 | 1 | 10 | 33 | 1 | 1 |
| 1855 | 1168 | 5 | 10 | 134 | 18 | 9 | 1 | 18 | 7 | 23 | 11 | 4 |
| 1860 | 1080 | 10 | 0 | 124 | 16 | 0 | 1 | 15 | 8 | 15 | 6 | 2 |
| 1865 | 1023 | 0 | 0 | 118 | 3 | 2 | 1 | 13 | 9 | 9 | 9 | 3 |
| 1867 | 1000 | 0 | 0 | 115 | 10 | 0 | 1 | 13 | 0 | 7 | 8 | 6 |
| 1870 | 1000 | 0 | 0 | 66 | 0 | 0 | 1 | 13 | 0 | 4 | 2 | 9 |
| 1873 | 1000 | 0 | 0 | 16 | 10 | 0 | 1 | 13 | 0 | 0 | 19 | 6 |

It is right to inform the Extraordinary Court, that besides the valuation of the Society's liabilities by the Carlisle table of mortality and 3 per-cent interest, I have at the same time made valuations by other *data* of calculation. In particular, I have deduced the results by the Actuaries' New Experience table 1863, assuming interest at 3 per-cent. It is not necessary that I should here give any statement in figures of the differences which appear in this comparison. It is enough to say that the check or comparative estimate of liability alluded to shows results which fully sustain and justify the declaration of bonus recommended in this report.

## IV.—CONCLUSION.

In submitting this report, it gives me peculiar satisfaction to refer to the evidence which the results now declared afford, that the explanations in my last septennial report, 1866, were strictly correct. I there stated that the current profit of that period was such as fully to sustain the previous rate of additions, and that the unexpected diminution of bonus then arising was due to the fact that the old Northampton system of valuation had not left in 1859, as supposed, an undivided balance of nearly £200,000, and that *in consequence* the new Guarantee Fund had to be constituted almost wholly out of current profit, thus reducing the bonus, for that division, to £1. 3*s.* per-cent, instead of £1. 13*s.* per-cent, the rate which the profit of the period, if all given in bonuses, would have yielded. The amount of the profit now ascertained as accruing from the last seven years gives a bonus of £1. 13*s.* per-cent per annum, and besides, adds to the Guarantee Fund, maintaining it at the required amount of £5 per-cent on the total liability. This result will be regarded by any one who fairly compares the two periods as an ample confirmation of the correctness of the explanation which I gave in my last report. The increased *ratio* of profit during last seven years, while it is decidedly in advance of the preceding period, is seen to be so to no greater extent than might be expected from the circumstances and probabilities of the case. In the period ending in 1866, the profit or surplus was £834,183. In that now ending it is £1,085,764.

### *The following additional particulars are taken from the returns under the Fifth and Sixth Schedules of the Act.*

The principles adopted in the valuation and distribution of profits among the policyholders are determined by the Deed of Constitution and Bye-laws of the Society.

(1.) A valuation is made every seven years.

(2.) The valuation is made on the principle of estimating the present value of the Society's obligations under life policies and annuities and the present value of the future *net* premiums receivable by the Society. The difference between these two values is the net liability.

Each policy was separately valued (in duplicate). The sum of the values of the ordinary class was also compared and reconciled with the results of another valuation by the method of lists according to age.

In all cases where a higher premium than the ordinary tabular rate had been charged the policies were valued at the "constructed" age, and not at the "real" age. Where extra premiums were charged for foreign residence, a proportion was reserved to meet the risk for the unexpired time.

(3.) The surplus is divided into two funds, named respectively the "*Bonus Fund*" and "*Guarantee Fund*", the latter fund being fixed at not less than £5 per-cent of the value of the Society's total liabilities at the time of valuation, including the value of the new bonus. The "Bonus Fund" is divided in the form of an annual percentage of additional sum assured payable at death for each annual premium paid during the septennium, calculated on the original sum assured, and also on all previous bonuses in existence at the time. The Guarantee Fund is divided in the form of a cash addition of five per-cent on the value of each policy. Interest is allowed on that addition at the rate of £3 per-cent per annum from the time of division till payment of the claim after death.

The table of mortality used throughout the valuation is the Carlisle table.

The rate of interest assumed in all the calculations is three per-cent.

The entire "loading" or addition to the net or pure premium is excluded in every instance from the calculation, and is accordingly reserved as a provision for future expenses and profits.

### CONSOLIDATED REVENUE ACCOUNT *for the Seven Years, commencing 1st January* 1867, *and ending 31st December* 1873.

| | | | | £ | *s.* | *d.* |
|---|---|---|---|---|---|---|
| Amount of funds on 1st January 1867, the beginning of the Septennium | | | | 4,371,995 | 0 | 1 |
| Premiums (after deduction of Re-assurance Premiums) | 3,034,120 | 18 | 4 | | | |
| Consideration for Annuities granted | 36,587 | 5 | 7 | | | |
| Interest | 1,463,381 | 14 | 7 | | | |
| Profit on Investments | 623 | 5 | 9 | | | |
| | | | | 4,534,713 | 4 | 3 |
| | | | | £8,906,708 | 4 | 4 |

| | £ | *s.* | *d.* |
|---|---|---|---|
| Claims under Policies (after deduction of Sums Reassured) | 2,770,153 | 15 | 6 |
| Surrenders | 121,688 | 1 | 2 |
| Annuities | 54,703 | 17 | 6 |
| Commission | 70,282 | 8 | 9 |
| Expenses of Management | 263,025 | 12 | 6 |
| | 3,279,853 | 15 | 5 |
| Amount of funds on 31st December 1873, the end of the period | 5,626,854 | 8 | 11 |
| | £8,906,708 | 4 | 4 |

### ABSTRACT OF THE VALUATION OF THE POLICIES (see pp. 438, 439).

The profits are distributed among all the participating policies, existing at the date of valuation, subject only to this special proviso, that the shares allocated to policies of less than five years' standing only vest and become payable after the lapse of five full years.

The Bonus Fund is divided every seven years in the form of a reversionary addition to the sum assured, but policyholders have the option of allowing the declared bonuses either to remain as an addition to the sums assured payable at death, or of surrendering them for their present value in cash. That value may also be commuted or converted into an equivalent deduction from the future annual premium payable under the policy.

## ABSTRACT OF THE VALUATION OF TH

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | | | | |
|---|---|---|---|---|---|---|---|
| | Number of Policies. | Sums Assured. | Premiums. | | | Loading. | |
| | | Original Sums Assured and Bonus Additions at 31st December 1873. | Gross Annual Premiums payable after giving effect to all Reductions and Redemptions. | Net Annual Premiums contained in Col. 2. | Proportion of Net Premiums contained in Col. 3 to next Renewal. | Annual Loading on Net Premiums in Col. 3. | Loading on Single Premiums and parts of Premiums redeemed. |
| I. ASSURANCES. | | Col. 1. | Col. 2. | Col. 3. | Col. 4. | Col. 5. | Col. 6. |
| 1. With Participation. | | £ | £ | £ | £ | £ | £ |
| For Whole Term of Life | 22,010 | 16,010,396 | 442,773 | 344,138 | 228,427 | 98,635 | 1,134 |
| Do. definite number of Premiums | 242 | 168,285 | 8,042 | 6,221 | * .. | 1,821 | 714 |
| Do. do. decreasing Premiums | 1 | 1,046 | 30 | 24 | * .. | 6 | 9 |
| Do. Premiums decreasing septennially | 1 | 506 | 21 | 17 | 7 | 4 | .. |
| Do. Premiums increasing quinquennially | 40 | 33,647 | 1,004 | 734 | 509 | 270 | .. |
| Do. Premiums increasing septennially | 2 | 2,279 | 66 | 50 | 51 | 16 | .. |
| Endowment Assurances, "One Life" | 611 | 183,356 | 8,941 | 6,821 | * .. | 2,120 | 17 |
| Do. do. "Joint Lives" | 3 | 603 | 39 | 29 | * .. | 10 | .. |
| Joint Lives | 115 | 62,568 | 3,123 | 2,382 | 1,489 | 741 | .. |
| Last Survivor | 18 | 30,831 | 413 | 326 | 275 | 87 | .. |
| Extra Premiums payable | .. | .. | 4,273 | .. | .. | .. | .. |
| 1. Assurances with Profits | 23,043 | 16,488,517 | 468,725 | 360,742 | 230,758 | 103,710 | 1,874 |
| 2. Without Participation. | | | | | | | |
| For Whole Term of Life | 645 | 598,467 | 19,903 | 18,417 | 9,123 | 1,486 | 6 |
| Do. definite number of Premiums | 4 | 9,600 | 305 | 285 | * .. | 20 | 8 |
| Do. Premiums increasing quinquennially | 282 | 204,875 | 3,327 | 2,314 | * .. | 1,013 | .. |
| Do. Premiums increasing septennially | 3 | 3,399 | 91 | 75 | 8 | 16 | .. |
| Contingent | 24 | 45,350 | 719 | 624 | 274 | 95 | 13 |
| Endowment Assurances | 102 | 38,000 | 1,915 | 1,803 | * .. | 112 | .. |
| Joint Lives | 7 | 12,700 | 679 | 585 | 208 | 94 | .. |
| Last Survivor | 16 | 16,300 | 346 | 297 | 214 | 49 | .. |
| Children's Endowments | 19 | 4,000 | 213 | 198 | * .. | 15 | 2 |
| Short Term Assurances | 35 | 56,950 | 914 | 671 | * .. | 243 | .. |
| Extra Premiums payable | .. | .. | 221 | .. | .. | .. | .. |
| 2. Assurances without Profits | 1,087 | 989,641 | 28,633 | 25,269 | 9,827 | 3,143 | 29 |
| Total Assurances | 24,130 | 17,478,158 | 497,358 | 386,011 | 240,585 | 106,853 | 1,903 |
| *Deduct* Reassurances | .. | 57,389 | 1,713 | 1,480 | 1,191 | 234 | .. |
| Net Amount of Assurances | 24,130 | 17,420,769 | 495,645 | 384,531 | 239,394 | 106,619 | 1,903 |
| II. ANNUITIES. | | | | | | | |
| Present and Contingent | 197 | 10,042 | 659 | 559 | 313 | 101 | 23 |
| Total of the Results | 24,327 | . . . | 496,304 | 385,090 | 239,707 | 106,720 | 1,926 |

Notation.—$S$=Sums Assured; $A$=Value of £1 payable at Death; $a$=Value of £1

Note.—The above formulas apply only to Whole Life Assurances maintained by equal Annual Premiums, which represent expressive of the respective contingencies involved. All Policies under which

* The Net Premiums payable under these Policies were valued by Annuities deferred

## LICIES, *as at* 31*st December* 1873.

| VALUATION OF THE POLICIES. | | | | | DISTRIBUTION OF THE SURPLUS.<br>At credit of the Profit and Loss Account . . £1,201,624<br>Less to Depreciation Account, £12,500, and Balance unappropriated, £14,140 . . . 26,640<br>£1,174,984<br>New Bonus, Col. 13 . . . . . . . £907,732<br>Guarantee Fund, Col. 15 . . . . . 267,252<br>1,174,984 | | | |
|---|---|---|---|---|---|---|---|---|
| ALUE OF LIABILITIES UNDER POLICIES | | | VALUE OF ASSETS UNDER POLICIES. | NET VALUE OF LIABILITIES, being excess of Col. 9 over Col. 10. | | | | |
| Value of ms Assured nd Bonuses in Col. 1. $(S \times A_x')$. | Value of paid-up Loading in Col. 6. $L \times (a_x'+1)$ – "*Proportion*" to next Renewal. | Total Value of Liabilities in Cols. 7 and 8. | Value of Net Premiums in Col. 3. $P_x \times (a_x'+1)$ – "*Proportion*" thereof in Col. 4. | | New Bonus at £1. 13s. per-cent per annum, on Col. 1. | Value of New Bonus in Col. 12. | Total Value of Policy and Bonuses to 31st December 1873, being Cols. 11 and 13. | Guarantee Fund, being £5 per-cent on Col. 14. |
| Col. 7. | Col. 8. | Col. 9. | Col. 10. | Col. 11. | Col. 12. | Col. 13. | Col. 14. | Col. 15. |
| £ | £ | £ | £ | £ | £ | £ | £ | £ |
| 9,017,658 | 12,198 | 9,029,856 | 4,848,388 | 4,181,468 | 1,527,314 | 890,474 | 5,071,942 | 253,597 |
| 76,582 | 4,003 | 80,585 | 50,162 | 30,423 | 11,534 | 5,523 | 35,946 | 1,797 |
| 721 | 89 | 810 | 54 | 756 | 121 | 83 | 839 | 42 |
| 313 | .. | 313 | 160 | 153 | 59 | 36 | 189 | 9 |
| 16,741 | .. | 16,741 | 14,484 | 2,257 | 2,909 | 1,516 | 3,773 | 189 |
| 1,012 | .. | 1,012 | 919 | 93 | 263 | 117 | 210 | 11 |
| 106,304 | 50 | 106,354 | 85,291 | 21,063 | 10,151 | 6,031 | 27,094 | 1,355 |
| 423 | .. | 423 | 263 | 160 | 62 | 44 | 204 | 10 |
| 38,236 | .. | 38,236 | 29,501 | 8,735 | 5,104 | 3,144 | 11,879 | 594 |
| 18,743 | .. | 18,743 | 4,018 | 14,725 | 3,251 | 2,077 | 16,802 | 840 |
| 1,946 | .. | 1,946 | .. | 1,946 | .. | .. | 1,946 | 97 |
| 9,278,679 | 16,340 | 9,295,019 | 5,033,240 | 4,261,779 | 1,560,768 | 909,045 | 5,170,824 | 258,541 |
| 331,414 | 65 | 331,479 | 249,110 | 82,369 | | | | |
| 3,760 | 11 | 3,771 | 3,504 | 267 | | | | |
| 92,975 | .. | 92,975 | 91,539 | 1,436 | | | | |
| 1,675 | .. | 1,675 | 1,667 | 8 | | | | |
| 6,269 | 116 | 6,385 | 5,136 | 1,249 | | | | |
| 29,713 | .. | 29,713 | 11,853 | 17,860 | | | | |
| 7,752 | .. | 7,752 | 7,262 | 490 | | | | |
| 8,487 | .. | 8,487 | 4,441 | 4,046 | | | | |
| 2,572 | 21 | 2,593 | 1,654 | 939 | | | | |
| 2,976 | .. | 2,976 | 2,509 | 467 | | | | |
| 83 | .. | 83 | .. | 83 | | | | |
| 487,676 | 213 | 487,889 | 378,675 | 109,214 | .. | .. | 109,214 | 5,461 |
| 9,766,355 | 16,553 | 9,782,908 | 5,411,915 | 4,370,993 | 1,560,768 | 909,045 | 5,280,038 | 264,002 |
| 31,187 | .. | 31,187 | 20,427 | 10,760 | 2,491 | 1,313 | 12,073 | |
| 9,735,168 | 16,553 | 9,751,721 | 5,391,488 | 4,360,233 | 1,558,277 | 907,732 | 5,267,965 | |
| 70,724 | 218 | 70,942 | 5,945 | 64,997 | .. | .. | 64,998 | 3,250 |
| **9,805,892** | **16,771** | **9,822,663** | **5,397,433** | **4,425,230** | **1,558,277** | **907,732** | **5,332,963** | **267,252** |

annually; $x$=Age at Entry; $x'$=Age at Date of Valuation; $L$=Loading; $P$=Net Premium.
over 95 per-cent of the entire business of the Society; the other classes of Assurances having been valued by formulas
extra premiums are payable have been valued at correspondingly advanced ages.
the exact proportions of a year to the dates of next Renewal respectively.

*The following are Specimens of the Bonuses allotted for the seven years ending 31st December 1873 to Participating Policies of £100, with their Cash Values, and Equivalent Deductions from Premiums:—*

| Age at Entry. | Duration of Policy. | Reversionary Addition. | Cash Value. | Equivalent Deduction from Premiums. | | Age at Entry. | Duration of Policy. | Reversionary Addition. | Cash Value. | Equivalent Deduction from Premiums. | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | For Life. | For 7 Years. | | | | | For Life. | For 7 Years. |
| | | £ s. d. | £ s. d. | £ s. d. | £ s. d. | | | £ s. d. | £ s. d. | £ s. d. | £ s. d. |
| 20 | 5 Years. | 8 5 0 | 2 11 9 | 0 2 7 | 0 8 6 | 20 | 25 Years. | 14 17 5 | 6 8 8 | 0 8 3 | 1 1 8 |
| 30 | | 8 5 0 | 3 0 10 | 0 3 5 | 0 10 2 | 30 | | 14 17 5 | 7 14 0 | 0 12 6 | 1 6 5 |
| 40 | | 8 5 0 | 3 11 5 | 0 4 7 | 0 12 0 | 40 | | 14 17 5 | 8 19 9 | 0 19 9 | 1 12 10 |
| 50 | | 8 5 0 | 4 5 5 | 0 6 11 | 0 14 8 | 50 | | 14 17 5 | 10 4 9 | 1 14 10 | 2 4 3 |
| 20 | 10 Years. | 11 19 0 | 4 1 6 | 0 4 4 | 0 13 6 | 20 | 30 Years. | 16 4 6 | 7 12 10 | 0 10 10 | 1 5 9 |
| 30 | | 11 19 0 | 4 15 10 | 0 5 9 | 0 16 1 | 30 | | 16 4 6 | 9 3 5 | 0 17 4 | 1 12 10 |
| 40 | | 11 19 0 | 5 12 6 | 0 8 0 | 0 18 11 | 40 | | 16 4 6 | 10 10 5 | 1 8 2 | 2 0 10 |
| 50 | | 11 19 0 | 6 15 1 | 0 12 9 | 1 4 2 | 50 | | 16 4 6 | 11 12 10 | 2 7 3 | 2 14 11 |
| 20 | 15 Years. | 12 13 8 | 4 13 7 | 0 5 3 | 0 15 7 | 20 | 35 Years. | 17 15 9 | 9 4 2 | 0 14 11 | 1 11 7 |
| 30 | | 12 13 8 | 5 9 9 | 0 7 1 | 0 18 6 | 30 | | 17 15 9 | 10 15 0 | 1 4 1 | 1 19 4 |
| 40 | | 12 13 8 | 6 11 4 | 0 10 8 | 1 2 5 | 40 | | 17 15 9 | 12 4 11 | 2 0 11 | 2 13 0 |
| 50 | | 12 13 8 | 7 13 4 | 0 17 3 | 1 8 0 | 50 | | 17 15 9 | 13 5 2 | 3 8 3 | 3 13 5 |
| 20 | 20 Years. | 13 14 0 | 5 9 11 | 0 6 7 | 0 18 6 | 20 | 40 Years. | 19 11 3 | 11 1 2 | 1 0 11 | 1 19 7 |
| 30 | | 13 14 0 | 6 9 0 | 0 9 2 | 1 1 9 | 30 | | 19 11 3 | 12 13 9 | 1 14 0 | 2 9 3 |
| 40 | | 13 14 0 | 7 14 10 | 0 14 8 | 1 7 9 | 40 | | 19 11 3 | 14 0 8 | 2 16 11 | 3 6 3 |
| 50 | | 13 14 0 | 8 17 8 | 1 3 10 | 1 14 6 | | | | | | |

*The following are Examples of the Portion of Guarantee Fund, exclusive of Interest thereon, attaching to Policies of £100, assuming the age at entry to be 35:—*

| Duration of Policy. | Portion of Guarantee Fund. | Duration of Policy. | Portion of Guarantee Fund. |
|---|---|---|---|
| | £ s. d. | | £ s. d. |
| 5 years . . | 0 10 7 | 30 years . . | 4 9 3 |
| 10 „ . . | 1 1 1 | 35 „ . . | 5 13 0 |
| 15 „ . . | 1 13 9 | 40 „ . . | 6 18 5 |
| 20 „ . . | 2 10 8 | 45 „ . . | 8 0 9 |
| 25 „ . . | 3 9 10 | 50 „ . . | 9 2 11 |

10. The value allowed for the surrender of policies, for the whole term of life, and for endowment assurances, is 75 per-cent of the net values deduced from the table employed in estimating the Society's liabilities. Participating policies receive in addition the cash value of the vested bonuses. This value is 85 per-cent of the net value ascertained by the same table. The following are specimens of the values allowed by the office:—

(1.) TABLE *containing examples of the Surrender Values allowed for Policies of* £1000, *for the whole term of Life, exclusive of Bonus Additions.*

| Age at Entry. | DURATION OF POLICY. | | | | |
|---|---|---|---|---|---|
| | Five Years. | Ten Years. | Fifteen Years. | Twenty Years. | Thirty Years. |
| | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* | £ *s.* *d.* |
| **20** | 34 0 2 | 70 13 3 | 107 15 9 | 150 9 3 | 244 6 3 |
| **25** | 38 8 0 | 77 5 8 | 121 19 8 | 166 5 6 | 285 16 11 |
| **30** | 40 19 8 | 88 1 10 | 134 15 6 | 191 14 3 | 330 15 0 |
| **35** | 49 16 8 | 99 4 3 | 159 8 10 | 232 10 5 | 367 5 2 |
| **40** | 52 17 10 | 117 8 3 | 195 13 10 | 274 19 0 | 414 3 10 |
| **45** | 69 8 3 | 153 12 8 | 238 18 1 | 308 18 3 | 460 7 6 |
| **50** | 92 16 2 | 186 15 8 | 263 18 6 | 351 17 3 | 487 1 3 |

(2.) TABLE *containing examples of the Cash Values allowed for the Surrender of* £100, *Vested Bonus Additions.*

| Present Age. | Cash Value of £100 of Bonus. | Present Age. | Cash Value of £100 of Bonus. | Present Age. | Cash Value of £100 of Bonus. |
|---|---|---|---|---|---|
| | £ *s.* *d.* | | £ *s.* *d.* | | £ *s.* *d.* |
| **20** | 28 16 8 | **40** | 40 1 8 | **60** | 56 10 10 |
| **25** | 31 7 6 | **45** | 43 5 0 | **65** | 60 9 2 |
| **30** | 34 1 8 | **50** | 47 2 6 | **70** | 64 18 4 |
| **35** | 36 16 8 | **55** | 51 5 10 | | |

No application for the surrender value of an endowment policy has ever been made to the office. Such value, however, would be determined according to the circumstances of each case.

The Society does not transact any other than home business, the Laws precluding the establishment of Agencies beyond the United Kingdom.

Extra premiums are charged in individual cases for foreign residence, according to the increased risk, as estimated by the Directors, when license for such residence is granted. The Society, however, transacts no business at other than European rates, and the extra premiums charged for foreign residence are discontinued on the return of the assured within the ordinary limits.

Policies are not granted to applicants regarded as being clearly under the average standard of healthy lives. The Society's tabular rates are in certain cases increased to cover supposed additional risks indicated by family history, or connected with hernia, gout, or the occupation of the assured; such lives are dealt with in all the calculations as if the advanced ages at which they have been accepted were the true ages.

## ROYAL EXCHANGE ASSURANCE CORPORATION.

*Established* 1720.

### QUINQUENNIAL VALUATION AS AT 31 DECEMBER 1870.

*This Company issues no Bonus Report, but the following particulars are extracted from the returns under the Fifth and Sixth Schedules of the Life Assurance Companies Act*, 1870.

The date up to which the valuation is made is 31st December 1870.

On the 31st December 1841, the "Old Series" of Non-participating Assurances was closed to new entrants, and a "New Series" was opened from the 1st January 1842, on the participating system, allotting two thirds of the Profits to Policyholders of the class entitled to share. The valuation of the Company's Liabilities on both series is made by ascertaining the present value of the sums at risk, and the present value of the future premiums, and by taking the difference as the amount of Cash Liabilities.

The distribution of Profits among the participating class is determined on the principle—"That the present value of the Bonus declared on each Policy (whether applied in increase of the sum assured, or in reduction of the future premiums) shall bear the same proportion to the gross amount of Profit realized by the Company in respect of that Policy, as the entire two thirds of the net surplus which is divisible among the assured shall bear to the aggregate amount of Profit on all the participating Policies." These principles were established by resolution of the Court of Directors.

On the "Old Series" the Northampton Table of Mortality was used with 3 per-cent interest. On the "New Series," the Equitable Table of Mortality with four per-cent, but the value is obtained by a calculation of the gross premiums on both sides of the account, which gives a higher reserve on the "New Series" than Equitable three per-cent net on both sides. On the Annuity Branch, the Equitable Table of Mortality, three per-cent interest.

No proportion of the future Profit is anticipated, and the whole of the "loading" is reserved.

Policies on the participating system on which three annual premiums have been paid, previously to the division period, are entitled to share in the profits.

The total amount of profit made by the company was:—

| | | | |
|---|---|---|---|
| Life Assurances, "New Series"—five years to 31st December 1870 . . . . . | £163,934 | 19 | 5 |
| Life Assurances, "Old Series"—eight months to 31st December 1870 . . . . | 1,980 | 4 | 7 |
| Annuities—eight months to 31st December 1870 | 1,429 | 17 | 7 |
| The amount of profit divided among Policyholders on the "New Series" (the only participating class) was . . . . . . . . | 109,289 | 19 | 8 |
| The number of Policies which participated was 4,219, assuring . . . . . . . | 3,304,019 | 0 | 0 |

The following are specimens of Bonus allotted to Policies of £100, effected at the undermentioned ages, on the last division of Profits at the 31st December 1870:—

| Age at Entry. | Policies in Force for | | | | |
|---|---|---|---|---|---|
| | 5 Years. | 10 Years. | 15 Years. | 20 Years. | 25 Years. |
| | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. |
| 20 | 4 0 0 | 4 0 0 | 4 0 0 | 4 0 0 | 4 0 0 |
| 30 | 4 0 0 | 4 0 0 | 4 0 0 | 4 0 0 | 5 0 0 |
| 40 | 4 0 0 | 4 0 0 | 5 0 0 | 5 0 0 | 6 0 0 |
| 50 | 5 0 0 | 5 0 0 | 6 0 0 | 6 0 0 | 7 0 0 |

| Age at Entry. | Bonus in Reduction of the Annual Premium allotted to Polices of £100. | | | | |
|---|---|---|---|---|---|
| | 5 Years. | 10 Years. | 15 Years. | 20 Years. | 25 Years. |
| | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. |
| 20 | 0 1 9 | 0 1 11 | 0 2 1 | 0 2 6 | 0 3 0 |
| 30 | 0 2 1 | 0 2 6 | 0 3 0 | 0 3 8 | 0 4 8 |
| 40 | 0 3 0 | 0 3 8 | 0 4 8 | 0 6 0 | 0 8 2 |
| 50 | 0 4 8 | 0 6 0 | 0 8 2 | 0 11 6 | 0 17 5 |

Note.—During the first fourteen years of the participating system the Bonus was declared septennially. It is now quinquennial. The "Old Series" have hitherto been valued *annually*, and the Annuities *triennially*, and the previous valuation was up to the 30th April 1870. With a view to the completeness of the returns required by Parliament, the Court of Directors resolved to have these Accounts made up for the eight months to the 31st December 1870, and henceforward to have them valued quinquennially at the same time as the "New Series."

## CONSOLIDATED REVENUE ACCOUNT—Life Assurance, "New Series"—for Five Years, commencing 1st January 1866, and ending 31st December 1870.

| | £ | s. | d. |
|---|---|---|---|
| Amount of Life Assurance Fund, "New Series," on the 1st January 1866, the beginning of the five years | 1,285,177 | 18 | 7 |
| Premiums, after deduction of reassurance Premiums | 670,211 | 7 | 0 |
| Interest and Dividends, less Income-tax | 300,724 | 8 | 11 |
| Increase in valuation of Stocks | 1,435 | 1 | 0 |
| | £2,257,548 | 15 | 6 |

| | £ | s. | d. |
|---|---|---|---|
| Claims under Life Policies (after deduction of Sums reassured) | 647,246 | 1 | 8 |
| Surrenders | 27,489 | 6 | 1 |
| Reversionary Annuities | 3,749 | 13 | 5 |
| Commission | 29,281 | 18 | 2 |
| Expenses of Management | 36,216 | 17 | 0 |
| Bad Debts | 67 | 6 | 11 |
| Profit of previous five years—Company's share carried off | 66,593 | 12 | 10 |
| Amount of Life Assurance Fund, "New Series," on the 31st December 1870, the end of the period | 1,446,903 | 19 | 5 |
| | £2,257,548 | 15 | 6 |

## SUMMARY AND VALUATION OF THE POLICIES, AS AT 31ST DECEMBER 1870.

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | | VALUATION. | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | "New Series," valued by Equitable Table, Interest 4 per-Cent, and "Old Series," by Northampton Table, Interest 3 per-Cent. | | | |
| | Number of Policies. | Sum Assured and Bonuses. | Office Yearly Premium. | Net Yearly Premiums if ascertained. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Yearly Premiums if computed. | Net Liability. |
| ASSURANCES—"NEW SERIES," | | | | See Valuation Columns | The Sums Assured and Bonuses are valued at the rate which would have been charged for new Assurances of like amount at the present ages of the lives; consequently the office premiums enter into the valuation both of sums assured and premiums, and the net yearly premiums are not computed. | | | |
| (*Less reassurances as stated below*). | | | | | | | | |
| I.—WITH PARTICIPATION IN PROFITS. | | £ | £ | | | | | £ |
| For whole term of life | 4,524 | 3,702,322 | 108,321 | | | | | 1,164,850 |
| Other classes—Joint Lives | 8 | 4,326 | 131 | .. | | | | 1,448 |
| Last Survivors | 30 | 25,641 | 432 | .. | | | | 9,803 |
| Contingent Assurances | 2 | 730 | 12 | .. | | | | 242 |
| Limited payment of Premiums | 18 | 19,764 | 1,133 | .. | | | | 10,838 |
| Extra Premiums payable | .. | .. | 880 | .. | | | | 440 |
| Cash Bonus not paid | .. | .. | .. | .. | | | | 193 |
| Total Assurances with Profits, "New Series" | 4,582 | 3,752,783 | 110,909 | .. | .. | .. | .. | 1,187,814 |
| II.—WITHOUT PARTICIPATION IN PROFITS. | | | | | | | | |
| For whole term of life | 399 | 328,370 | 11,556 | .. | .. | .. | .. | 56,676 |
| Other classes—Joint Lives | 4 | 10,550 | 499 | .. | .. | .. | .. | 723 |
| Last Survivors | 40 | 29,320 | 644 | .. | .. | .. | .. | 4,626 |
| Contingent Assurances | 14 | 21,100 | 778 | .. | .. | .. | .. | 2,990 |
| Endowment Assurances | 153 | 54,500 | 2,570 | .. | .. | .. | .. | 15,477 |
| Short Period Assurances | 40 | 29,281 | 462 | .. | .. | .. | .. | 230 |
| Reversionary Annuities (payable) | 5 | .. | .. | .. | (Annuities £980 per annum payable.) | | | 12,564 |
| " " (pending) | 15 | .. | 334 | .. | (Annuities £1,210 pending.) | | | 1,516 |
| Limited Payment of Premiums | 4 | 3,100 | 120 | .. | .. | .. | .. | 295 |
| Extra Premiums payable | .. | .. | 117 | .. | .. | .. | .. | 58 |
| Total Assurances without Profits, "New Series" | 674 | 476,221 | 17,080 | .. | .. | .. | .. | 95,155 |
| Total Assurances, "New Series" | 5,256 | 4,229,004 | 127,989 | .. | .. | .. | .. | 1,282,969 |
| The reassurances amount only to | .. | 41,700 | 1,269 | .. | .. | .. | .. | .. |
| ASSURANCES—"OLD SERIES," | | | | | | | | |
| (WITHOUT PARTICIPATION IN PROFITS). | | | | | | | | |
| *No reassurances in this Branch.* | | | | | | | | |
| For whole term of life | 455 | 326,136 | 9,762 | .. | .. | .. | .. | 201,327 |
| Other classes—Joint Lives | 2 | 400 | 14 | .. | .. | .. | .. | 162 |
| Last Survivors | 8 | 7,010 | 93 | .. | .. | .. | .. | 3,625 |
| Extra Premiums payable | .. | .. | 35 | .. | .. | .. | .. | 17 |
| Total Assurances, "Old Series" | 465 | 333,546 | 9,904 | .. | .. | .. | .. | 205,131 |
| Net amount of Assurances | 5,721 | 4,562,550 | 137,893 | .. | .. | .. | .. | 1,488,100 |
| Adjustments, if any | .. | .. | .. | .. | .. | .. | .. | .. |
| ANNUITIES. | Deeds. | Annuity. | OfficeYearlyPrem | | | | | |
| Immediate (Single Lives) | 84 | 2,529 | .. | .. | .. | .. | .. | 23,590 |
| Last Survivors | 1 | 20 | .. | .. | .. | .. | .. | 249 |
| Deferred | 2 | 26 | Prems. all paid up. | .. | .. | .. | .. | 364 |
| Contingent (payable) | 1 | 60 | .. | .. | .. | .. | .. | 242 |
| Arrears of Annuity Payments | .. | .. | .. | .. | .. | .. | .. | 788 |
| Decrease in value of Purchased Annuities | .. | .. | .. | .. | .. | .. | .. | 153 |
| | 88 | 2,635 | .. | .. | .. | .. | .. | 25,386 |
| Total of results | .. | .. | 137,893 | .. | .. | .. | .. | 1,513,486 |

## VALUATION BALANCE SHEET, AS AT 31ST DECEMBER 1870.

| DR. | | £ s. d. |
|---|---|---|
| To Net Liability under Life Assurance transactions, "New Series" (as per Summary Statement) | 1,282,969 0 0 | |
| To Surplus | 163,934 19 5 | |
| | | 1,446,903 19 5 |
| To Net Liability under Life Assurance transactions, "Old Series" (as per Summary Statement) | 205,131 6 2 | |
| To Surplus | 1,980 4 7 | |
| | | 207,111 10 9 |
| To Net Liability under Annuity transactions (as per Summary Statement) | 25,385 17 5 | |
| To Surplus | 1,429 17 7 | |
| | | 26,815 15 0 |
| | | £1,680,831 5 2 |

| CR. | £ s. d. |
|---|---|
| By Life Assurance and Annuity Funds, as per Consolidated Revenue Account— | |
| "New Series" | 1,446,903 19 5 |
| "Old Series" | 207,111 10 9 |
| Annuities | 26,815 15 0 |
| | £1,680,831 5 2 |

The average Rate of Interest per-cent was

| | New Series. £ s. d. | Old Series. £ s. d. |
|---|---|---|
| In 1866 | 4 10 0 | 5 3 10 |
| „ 1867 | 4 10 1 | 5 7 0 |
| „ 1868 | 4 10 0 | 5 3 7 |
| „ 1869 | 4 10 1 | 5 3 2 |
| „ 1870 | 4 10 1 | 5 3 5 |

The *minimum* value now allowed for the surrender of Assurances for the whole term of life, on which three or more Annual Premiums have been paid, is 60 per-cent of the full reserve value, ascertained by a special calculation, and 70 per-cent on Endowment Assurances. This percentage increases with the advance of age.

*Instances of Surrender Values of Participating Policies for £100, exclusive of the Value of Bonus.*

| Age at Entry. | 5 Years in force. | 10 Years in force. | 15 Years in force. | 20 Years in force. |
|---|---|---|---|---|
| | £ s. d. | £ s. d. | £ s. d. | £ s. d. |
| 20 | 2 15 6 | 5 15 3 | 8 14 6 | 12 1 3 |
| 30 | 3 6 9 | 7 0 5 | 11 6 5 | 16 3 10 |
| 40 | 4 15 8 | 10 4 0 | 15 13 0 | 21 6 6 |
| 50 | 6 7 8 | 13 0 9 | 21 19 2 | 32 7 1 |
| 60 | 9 12 8 | 21 2 1 | 34 6 6 | 49 10 3 |

The *extra* Premiums charged for Residence in other than European climates are taken off on the return home of the persons whose lives are assured, and being intended to cover the special temporary risk do not enter into the general calculation of the Policy values.

The *additional* Premiums charged on those lives which do not come up to the full standard of eligibility with regard to health, form part of the ordinary Premiums; and the lives being rated accordingly as so much older, in reference to the increased risk, the Policies are treated for all purposes as for the advanced ages, in conformity with the Premiums charged.

## PROVIDENT LIFE OFFICE.

*Established* 1806.

*This company publishes no Bonus Report: we can therefore only give extracts from the returns under the Life Assurance Companies Act.*

### SECOND SCHEDULE.

BALANCE SHEET *on* 31 *December* 1872.

LIABILITIES.

| | £ | s. | d. | £ | s. | d. |
|---|---|---|---|---|---|---|
| Shareholders' Capital paid up, as per Balance Sheet for 1871 . . | 49,376 | 15 | 0 | | | |
| Add excess of Annual Interest on Shareholders' Capital over the Annual Dividends for the past five years . | 286 | 16 | 0 | | | |
| Add amount of Dividends and Bonuses to Shareholders not claimed . . . | 72 | 15 | 0 | | | |
| Add non-divisible portion of Shareholders' Profit, 1868 | 36 | 17 | 8 | | | |
| | 49,773 | 3 | 8 | | | |
| Deduct sum paid for Surrender of Shares . . | 350 | 0 | 0 | | | |
| Shareholders' Capital paid up at 31st December 1872 . . . . | 49,423 | 3 | 8 | | | |
| Assurance Fund . . . . | 1,748,272 | 17 | 10 | | | |
| Annuity Fund . . . . | *nil.* | | | | | |
| Unclaimed Dividends . . . | 219 | 7 | 0 | | | |
| Amount reserved for Shareholders' Dividends, payable January 1873 | 2,376 | 5 | 0 | | | |
| Total Funds, as per First Schedule . . . . . | | | | 1,800,291 | 13 | 6 |
| Claims Admitted but not Paid . . . . | | | | 46,807 | 15 | 0 |
| Other Claims announced . . . . . . | | | | 10,544 | 15 | 4 |
| Premium reserved for a Reassurance Policy waiting (£5,000) . . . . . . . . . . | | | | 318 | 6 | 8 |
| | | | | £1,857,962 | 10 | 6 |

| ASSETS. | £ | s. | d. |
|---|---|---|---|
| Mortgages on Property within the United Kingdom | 1,098,041 | 12 | 2 |
| Mortgages on Property out of the United Kingdom | *nil.* | | |
| Loans on the Company's Policies | 73,597 | 0 | 0 |
| Investments— | | | |
| In British Government Securities | 210,762 | 10 | 0 |
| „ Indian and Colonial Securities | 57,335 | 0 | 0 |
| „ Foreign Government Securities | 65,373 | 16 | 0 |
| „ Indian Government 5 per-Cent Railway Guaranteed Stocks | 149,931 | 15 | 9 |
| „ House Property | 38,024 | 10 | 7 |
| Loans upon Personal Security with Provident Policies | 63,150 | 1 | 9 |
| Agents' Balances | 2,142 | 18 | 3 |
| Outstanding Premiums | 36,474 | 12 | 1 |
| Do. Interest | 20,985 | 6 | 3 |
| Accruing Interest | 17,804 | 10 | 1 |
| Accrued Dividends—Indian Railway Stocks and Indian 5 per-Cents | 3,755 | 15 | 0 |
| Cash in the Bankers' hands | 20,003 | 2 | 7 |
| Bills in the Bankers' hands undue | 580 | 0 | 0 |
| | £1,857,962 | 10 | 6 |

## FIFTH SCHEDULE.

The date up to which the valuation is made is the 31st day of December 1872.

The principle upon which the valuation is made is by ascertaining the present value of the sums assured under the various policies, deducting therefrom the present value of the future premiums: the difference being the present liability. Bonus additions have been valued as absolute reversions.

In all cases of extra premiums charged by the office on the ground that the lives were below the required standard of health, the ages have been raised in the valuation to correspond in each case with the actual premium charged.

In the case of extra premiums charged by the office for climate risks, reserve has been made for the unexpired portion of such premiums.

The principle upon which the valuation is made is, according to the original Deed of Constitution, determined by the Directors.

The amount of profit to be divided amongst the assured is allotted to each policy in proportion to the total amount of premium contributed from the commencement. This allotment, being in effect a cash bonus, is then converted into an equivalent reversionary sum according to the respective ages of the lives assured on the 31st December 1872.

The principle regulating the distribution of profit is determined by the Deed of Constitution which states that:—

> "One half of the surplus shall be left to accumulate and to be taken into account in the statement to be drawn out at the end of the next ensuing seven years [*a bye-law passed in* 1852 *reduced the period of division from seven years to five years*], and the other half shall be divided amongst the original Proprietors of the said Institution and their respective representatives and the persons then actually insured with the Society

in manner following (that is to say),—the sum of twenty-five thousand pounds advanced by the said original Proprietors and the amount of all the sums of money paid by the persons then actually insured as aforesaid shall be thrown into one aggregate sum, and such a proportion of the said moiety of profits as the sum of twenty-five thousand pounds shall bear to the amount of all the sums of money paid by the said insured shall belong to the said original Proprietors or their respective representatives, and shall be divided amongst them rateably according to their respective shares of the said twenty-five thousand pounds; and the residue of the said moiety of the profits shall be divided amongst the said insured rateably according to the amount of the sums of money respectively paid by them, and shall be either paid to them immediately or added to the money insured to them by their respective policies as shall be determined at the said general courts."—*Extract from the 54th clause of the Deed of Constitution.*

The following tables of mortality have been used in the valuation:—

Bonus policies issued previous to 1st of January 1859, and charged Northampton rate of premiums, have been valued by the Northampton table of mortality.

Bonus policies issued subsequently to 1st of January 1859, have been valued by a table of mortality agreeing with the premiums charged.

Without-bonus policies have been valued by the Carlisle Table of Mortality.

Three per-cent interest has been the rate assumed in all the calculations.

In respect to bonus policies no portion of the annual premium income is specifically assigned as a provision for future expenses and profits. The reserves (*i.e.* the present net liability under the policies) as brought out by the mode of valuation adopted, contain within themselves provision for future expenses and profits.

In respect to without-bonus policies, valued by the Carlisle Table of Mortality, the pure premium only has been brought into account,—the full loading has been reserved.

Inasmuch as a moiety only of the surplus is distributed at each periodical division of profits, in accordance with the before-mentioned clause of the Deed of Constitution, a further provision is thereby made to meet future expenses and possible contingencies.

### CONSOLIDATED REVENUE ACCOUNT *for Five Years, commencing January 1st 1868, and ending December 31st 1872.*

| | £ | *s.* | *d.* |
|---|---|---|---|
| Amount of Funds on January 1st 1868, the beginning of the Quinquennial period . . . . . . | 1,710,589 | 0 | 1 |
| Premiums (after deduction of Reassurance Premiums) | 779,270 | 16 | 5 |
| Consideration for Annuities granted . . . . | *nil.* | | |
| Interest and Dividends . . . . . . . . | 366,596 | 0 | 11 |
| Discount on Claims paid before due . . . . | 710 | 12 | 3 |
| Fined Renewals . . . . . . . . . . | 164 | 18 | 6 |
| Balance of Profit on Investments . . . . . | 2,390 | 5 | 7 |
| | £2,859,721 | 13 | 9 |

| | £ | s. | d. |
|---|---|---|---|
| Claims under policies (after deduction of Sums Reassured) | 874,188 | 17 | 5 |
| Surrenders | 54,227 | 7 | 4 |
| Annuities | 1,128 | 0 | 0 |
| Commission | 45,164 | 9 | 10 |
| Expenses of Management | 51,951 | 16 | 1 |
| Dividends and Bonuses to Shareholders | 24,492 | 16 | 7 |
| Income Tax | 7,589 | 18 | 3 |
| Loss on closing sundry Agents' Accounts | 686 | 14 | 9 |
| Amount of Funds on December 31st 1872, the end of the Quinquennial period | 1,800,291 | 13 | 6 |
| | £2,859,721 | 13 | 9 |

SUMMARY *and* VALUATION *of the* POLICIES (see p. 450).

### VALUATION BALANCE SHEET *as at December* 31*st* 1872.

| DR. | £ | s. | d. |
|---|---|---|---|
| To Net Liability under Assurance and Annuity Transactions (as per Summary Statement) | 1,392,729 | 9 | 2 |
| To Surplus | 355,543 | 8 | 8 |
| | £1,748,272 | 17 | 10 |

| CR. | £ | s. | d. |
|---|---|---|---|
| By Life Assurance and Annuity Funds (as per Balance Sheet) | 1,748,272 | 17 | 10 |
| | £1,748,272 | 17 | 10 |

After payment of one premium before the close of the quinquennial period, a policy is entitled to share in the division of profits, and the bonus *vests* immediately.

The results of the valuation show—

| | £ | s. | d. |
|---|---|---|---|
| (1). The total amount of profit made by the Company was | 355,543 | 8 | 8 |
| (2). The amount of profit divided among the policyholders was<br>(The number of policies which participated was 9,164; the amount insured thereby being £4,153,742. 19*s.* 5*d.*) | 170,661 | 14 | 4 |
| The amount allotted to the shareholders was | 7,110 | 0 | 0 |
| The moiety of profits or gross surplus, reserved in terms of the Deed of Constitution, was | 177,771 | 14 | 4 |
| Total amount as above | £355,543 | 8 | 8 |

## SUMMARY *and* VALUATION *of the Policies, as at December 31st* 1872.

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | | VALUATION. | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | VALUE BY THE TABLES REFERRED TO IN ANSWER TO QUESTION 3. INTEREST 3 PER-CENT. | | | |
| | Number of Policies. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Yearly Premiums, if ascertained. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Yearly Premiums, if computed. | Net Liability. |
| ASSURANCES. | | | | | | | | |
| I. WITH PARTICIPATION IN PROFITS. | | £ | £ | | | | | £ |
| For Whole Term of Life | 8,868 | 4,440,537 | 130,640 | Not ascertained. | The net liability for whole term of life assurances has been ascertained from prepared tables, which do not show separately the value of sums assured and value of office yearly premiums. | | Not computed. | 1,176,531 |
| Do. Limited Number of Premiums | 21 | 12,519 | 589 | | | | | 2,041 |
| Joint Lives | 61 | 20,157 | 922 | | | | | 5,602 |
| Survivor—Two Lives | 93 | 41,145 | 802 | | | | | 9,697 |
| Do. Three Lives | 58 | 38,773 | 450 | | | | | 7,405 |
| Contingent Survivorship | 8 | 2,750 | 43 | | | | | 26 |
| Endowment Assurances | 8 | 1,703 | 73 | | | | | 117 |
| Increasing Assurances | 47 | 29,777 | 708 | | | | | 19,641 |
| Extra Premiums payable | ... | ... | 592 | | | | | 325 |
| Total Assurances with Profits | 9,164 | 4,587,361 | 134,819 | | | | | 1,221,385 |
| II. WITHOUT PARTICIPATION IN PROFITS. | | | | | — | | | |
| For Whole Term of Life | 1,230 | 586,022 | 18,081 | | Each policy in the special classes of assurance has been valued separately; but the value of sums assured and bonuses, and the value of office yearly premiums have not in all cases been ascertained. | | | 166,071 |
| Do. Limited Number of Premiums | 1 | 700 | 51 | | | | | 133 |
| Joint Lives | 8 | 18,300 | 1,012 | | | | | 2,515 |
| Survivor—Two Lives | 9 | 2,900 | 68 | | | | | 701 |
| Do. Three Lives | 17 | 7,860 | 108 | | | | | 1,336 |
| Contingent Survivorship | 13 | 13,150 | 235 | | | | | 630 |
| Endowment Assurances | 131 | 36,600 | 1,593 | | | | | 3,130 |
| Short Term | 76 | 83,150 | 1,379 | | | | | 984 |
| Extra Premiums payable | ... | ... | 210 | | | | | 87 |
| Total Assurances Without Profits | 1,485 | 748,682 | 22,737 | | | | | 175,587 |
| Total Assurances | ... | 5,336,043 | 157,556 | | | | | 1,396,972 |
| Deduct Reassurances | ... | 60,536 | 1,726 | | | | | 5,170 |
| **Net Amount of Assurances** | **10,649** | **£5,275,507** | **£155,830** | | | | | **£1,391,802** |
| ANNUITIES. | | | | | | | | |
| Immediate . . . £175 per Annum | ... | ... | ... | | | | | 927 |
| **Total of the Results** | **10,649** | **£5,275,507** | **£155,830** | | | | | **£1,392,729** |

Specimens of reversionary bonuses allotted to policies of £100 each effected at the undermentioned ages.

| Age at Entry. | 5 Years' Duration. | 10 Years' Duration. | 15 Years' Duration. | 20 Years' Duration. | 25 Years' Duration. | 30 Years' Duration. | 35 Years' Duration. | 40 Years' Duration. | 45 Years' Duration. | 50 Years' Duration. |
|---|---|---|---|---|---|---|---|---|---|---|
| | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. | £ s. d. |
| 20 | 2 4 0 | 4 3 0 | 6 8 0 | 8 0 0 | 9 7 0 | 10 10 0 | 11 9 0 | 12 6 0 | 13 0 0 | 13 12 0 |
| 30 | 2 9 0 | 4 12 0 | 6 17 0 | 8 11 0 | 10 1 0 | 11 6 0 | 12 8 0 | 13 7 0 | 14 4 0 | 15 0 0 |
| 40 | 2 16 0 | 5 6 0 | 7 13 0 | 9 11 0 | 11 5 0 | 12 15 0 | 14 1 0 | 15 5 0 | 16 9 0 | 17 13 0 |
| 50 | 3 8 0 | 6 8 0 | 9 0 0 | 11 7 0 | 13 8 0 | 15 6 0 | 17 2 0 | 18 16 0 | ... | ... |

Policyholders have the option of surrendering at any time a reversionary bonus for a present cash payment, or for a permanent reduction in the annual premium.

The terms allowed are as follows:—

| Age at Date of Surrender. | Cash Payment per £100 Reversionary Bonus. | Permanent Reduction in Annual Premium per £100 Reversionary Bonus. | Age at Date of Surrender. | Cash Payment per £100 Reversionary Bonus. | Permanent Reduction in Annual Premium per £100 Reversionary Bonus. |
|---|---|---|---|---|---|
| | £ s. d. | £ s. d. | | £ s. d. | £ s. d. |
| 25 | 28 6 0 | 1 18 6 | 55 | 51 1 0 | 4 12 4 |
| 30 | 31 3 0 | 2 2 9 | 60 | 56 3 0 | 6 7 4 |
| 35 | 34 8 0 | 2 8 0 | 65 | 61 14 0 | 7 16 9 |
| 40 | 38 0 0 | 2 14 5 | 70 | 67 14 0 | 10 0 0 |
| 45 | 42 0 0 | 3 2 5 | 75 | 73 16 0 | 13 4 6 |
| 50 | 46 7 0 | 3 17 0 | 80 | 79 19 0 | 18 6 2 |

The average rate of interest at which the Assurance Fund was invested at the close of each year during the period since the last investigation was as follows:—

| | £ s. d. |
|---|---|
| December 31st 1868 | 4 4 2 |
| Do. 1869 | 4 6 0 |
| Do. 1870 | 4 6 0 |
| Do. 1871 | 4 7 1 |
| Do. 1872 | 4 6 10 |

The minimum values allowed for the surrender of policies are as follows:—

With bonus policies for the whole term of life, 33 per-cent of the premium paid.

Without bonus policies for the whole term of life, 25 per-cent of the premium paid.

Endowment policies, 40 per-cent of the premium paid.

These values are allowed any time after payment by the assured of one year's premium.

The manner in which policies granted upon unhealthy lives are dealt with for surrender is similar to that adopted in ascertaining the value for reserve.

The business of the company is confined to the United Kingdom. When any life assured goes beyond the limits allowed—as published in the prospectus of the company—an extra premium is charged to cover the extra risk incurred. In the event of the policy being subsequently surrendered, a portion of such extra premium is included in the surrender value.

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## SOVEREIGN LIFE ASSURANCE COMPANY.

*Established* 1845.

***This company has published no formal bonus report, and we can therefore only give the following extracts from the returns under the Fifth and Sixth Schedules.***

### BALANCE SHEET *on the* 31*st December* 1873.

LIABILITIES.

| | £ | *s.* | *d.* | £ | *s.* | *d.* |
|---|---|---|---|---|---|---|
| Shareholders' Capital paid up | 45,000 | 0 | 0 | | | |
| Bonus additions | 6,300 | 0 | 0 | | | |
| | 51,300 | 0 | 0 | | | |
| Life Assurance Fund | 344,994 | 18 | 8 | | | |
| Annuity Fund | 8,159 | 12 | 1 | | | |
| Total Funds | | | | 404,454 | 10 | 9 |
| Claims admitted but not paid | | | | 11,308 | 2 | 5 |
| Endowments admitted | | | | 713 | 0 | 0 |
| | | | | £416,475 | 13 | 2 |

ASSETS.

| | £ | *s.* | *d:* |
|---|---|---|---|
| Mortgages on Property within the United Kingdom | 170,905 | 14 | 3 |
| Loans on the Company's Policies | 21,430 | 12 | 2 |
| Ditto collaterally secured by real Property | 41,165 | 0 | 0 |
| INVESTMENTS:— | | | |
| Railway Debentures | 7,210 | 0 | 0 |
| House Property | 19,500 | 0 | 0 |
| General Annuity Reversionary Account | 32,379 | 7 | 7 |
| Reversion purchased | 762 | 0 | 0 |
| 6,212 Company's Shares | 17,704 | 4 | 0 |
| Loans upon Personal Security | 78,592 | 0 | 0 |
| Agents' Balances | 4,940 | 11 | 9 |
| Outstanding Premiums | 9,841 | 13 | 6 |
| Ditto Interest | 2,841 | 14 | 0 |
| Cash in hand and on Current Account | 8,586 | 19 | 1 |
| Other Assets:—House and Office Furniture | 574 | 15 | 9 |
| Stamps in hand | 41 | 1 | 1 |
| | £416,475 | 13 | 2 |

The valuation was made up to 31st December 1873.

Three-fourths of the profits of the assurance business are divided among the assured in proportion to the premiums paid by each since the last division of profits [in 1862]. These principles are determined by the Deed of Settlement.

The Institute of Actuaries table was employed for valuing the Assurances, and the Government Annuity Experience, 1860, for the Annuities. The General Annuity Fund has been valued by the Carlisle 4 per-cent tables.

The rate of interest used was 4 per-cent.

The proportion of premium reserved as a provision for future expenses and profits is on participating policies 24·15 per-cent, and on non-participating policies 15·55.

A participating policy is entitled to receive profits at each division, although only one premium may have been paid.

The results of the valuation show:—1st. A profit on the Life Assurance Fund of £36,503. 2nd. The amount divisible among the policyholders is £27,911. The number of policies was 6283, and the sum assured £1,454,875. There was a deficiency on the Sovereign Annuity Fund of £1,392, and a surplus on the General Annuity Fund of £31,317. 6*s*.

The following are specimens of reversionary bonuses allotted to policies of £100:—

| Age at Entry. | In force 5 Years. | In force 10 Years. | In force 15 Years. |
|---|---|---|---|
| | £ | £ | £ |
| 20 | 2·27 | 4·12 | 4·45 |
| 30 | 2·36 | 4·20 | 4·46 |
| 40 | 2·44 | 4·40 | 4·75 |
| 50 | 2·80 | 5·00 | 5·50 |

**The Office allows cash value of the reversionary bonus or an equivalent reduction of premium until the next valuation on policies of five years' standing.**

## CONSOLIDATED REVENUE ACCOUNT *for Six Years, commencing 1st January* 1868, *and ending* 31*st December* 1873.

| | £ | *s.* | *d.* |
|---|---|---|---|
| Amount of Funds on 1st January 1868 | 275,521 | 13 | 8 |
| Premiums (less reassurances) | 447,763 | 14 | 9 |
| Consideration for Annuities granted | 5,981 | 19 | 8 |
| Interest and Dividends | 86,369 | 16 | 1 |
| Other Receipts, namely, Fees, Fines, &c. | 423 | 0 | 6 |
| | £816,060 | 4 | 8 |

| | £ | *s.* | *d.* |
|---|---|---|---|
| Claims under Policies (after deduction of sums reassured) | 271,115 | 3 | 7 |
| Endowments | 8,620 | 6 | 6 |
| Surrenders | 27,083 | 1 | 4 |
| Annuities | 6,975 | 13 | 8 |
| Commission | 21,838 | 0 | 7 |
| Expenses of Management | 43,691 | 15 | 8 |
| Dividends to Shareholders | 15,364 | 0 | 6 |
| Profit and Loss Account, including Balance of purchase of other Companies' Account | 16,917 | 12 | 1 |
| Amount of Funds at end of the period | 404,454 | 10 | 9 |
| | £816,060 | 4 | 8 |

### VALUATION BALANCE SHEET *as at* 31*st December* 1873.

| DR. | £ | s. | d. |
|---|---|---|---|
| To Net Liability under Assurance transactions . . . | 307,210 | 4 | 0 |
| „ Half-year's Dividend on Share Capital to 31st December 1873 . . . . . . . . . . | 1,281 | 14 | 8 |
| „ Surplus on the Life Assurance Fund . . . . | 36,503 | 0 | 0 |
| | £344,994 | 18 | 8 |

| CR. | £ | s. | d. |
|---|---|---|---|
| By Life Assurance Fund . . . . . . . . | 344,994 | 18 | 8 |
| | £344,994 | 18 | 8 |

### SUMMARY *and* VALUATION *of the* POLICIES (see p. 455).

The average rate of interest at which the Life Assurance Fund was invested at the close of each year during the period since the last investigation was £4. 5*s*. 8*d*. per-cent, namely:—

| | £ | s. | d. |
|---|---|---|---|
| 31 December 1868 . . . . . . | 4 | 5 | 8 |
| „ „ 1869 . . . . . . | 4 | 5 | 7 |
| „ „ 1870 . . . . . . | 4 | 5 | 11 |
| „ „ 1871 . . . . . . | 4 | 5 | 8 |
| „ „ 1872 . . . . . . | 4 | 5 | 9 |
| „ „ 1873 . . . . . . | 4 | 5 | 8 |

The surrender value of policies effected for the whole of life and for endowment assurances, upon which at least four years' premium have been paid, ranges between 33 and 55 per-cent of the total ordinary premiums paid. The company has no minimum scale for other descriptions of policies, but makes a fair allowance according to the circumstances of each case.

The company has not transacted business at other than European rates. Extra charges for residence in climates deemed unhealthy are made on the principles usually adopted. Unhealthy lives are rated at advanced rates, the ages having been raised in the valuation so as to correspond with the premium paid.

## GENERAL ANNUITY FUND.

### CONSOLIDATED REVENUE ACCOUNT *of the* GENERAL ANNUITY TRUST FUND *for Six Years, commencing* 1*st January* 1868, *and ending* 31*st December* 1873.

| | £ | s. | d. |
|---|---|---|---|
| Amount of Funds on 1st January 1868 . . . . | 309,702 | 2 | 2 |
| Less Outstanding Premiums and Interest . . . | 6,188 | 3 | 1 |
| | 303,513 | 19 | 1 |
| Premiums . . . . . . . . . . . | 42,832 | 18 | 6 |
| Interest and Dividends . . . . . . . . | 83,554 | 4 | 1 |
| Profit on Sale of Estate . . . . . . . . | 3,906 | 13 | 2 |
| | £433,807 | 14 | 10 |

| | £ | s. | d. |
|---|---|---|---|
| Annuities . . . . . . . . . . . | 150,296 | 10 | 7 |
| Expenses of Management . . . . . . . | 1,345 | 18 | 3 |
| Amount of Funds at end of the period . . . . | 282,165 | 6 | 0 |
| | £433,807 | 14 | 10 |

### Summary *and* Valuation *of the* Policies, *as at* 31*st December* 1873.

| DESCRIPTION OF TRANSACTIONS. | PARTICULARS OF THE POLICIES FOR VALUATION. | | | | VALUATION. | | | |
|---|---|---|---|---|---|---|---|---|
| | | | | | Value by the Institute of Actuaries $H^M$ Table. Interest 4 per-cent. | | | |
| | Number of Policies. | Sums Assured and Bonuses. | Office Yearly Premiums | Net Yearly Premiums, as valued. | Sums Assured and Bonuses. | Office Yearly Premiums. | Net Yearly Premiums, as valued. | Net Liability. |
| **ASSURANCES.** | | | | | | | | |
| I.—With Participation in Profits. | | £ | £ | £ | £ | £ | £ | £ |
| For Whole Term of Life . . . . | 5,101 | 1,277,866 | 39,997 | 30,336 | 580,929 | 516,070 | 391,432 | 189,497 |
| Limited number of Premiums . . | 20 | 6,300 | 225 | 171 | 2,134 | 2,085 | 1,581 | 553 |
| Commuted Premiums . . . . . | 15 | 6,671 | .. | .. | 3,840 | .. | .. | 3,839 |
| Ascending Premiums . . . . . | 1 | 500 | 11 | 8 | 141 | 193 | 141 | .. |
| Endowment Assurances . . . . | 723 | 97,055 | 4,028 | 3,055 | 62,654 | 32,051 | 24,310 | 38,344 |
| Do. (Commuted Premium) . | 2 | 57 | .. | .. | 35 | .. | .. | 35 |
| Joint Lives . . . . . . . . | 404 | 60,729 | 2,852 | 2,163 | 33,272 | 30,850 | 23,399 | 9,873 |
| Last Survivor . . . . . . . | 13 | 3,525 | 105 | 80 | 1,556 | 1,289 | 978 | 578 |
| Contingent Survivorship Assurances . | 3 | 1,808 | 41 | 31 | 388 | 525 | 388 | .. |
| Do. (Commuted Premium) . . | 1 | 364 | .. | .. | 28 | .. | .. | 28 |
| Extra Premiums payable . . . . | .. | .. | 1,044 | .. | 522 | .. | .. | 522 |
| Total Assurances with Profits . | 6,288 | 1,454,875 | 48,303 | 35,844 | 685,499 | 583,063 | 442,229 | 243,269 |
| II.—Without Participation in Profits. | | | | | | | | |
| For Whole Term of Life . . . . | 1,084 | 521,010 | 16,625 | 14,041 | 223,401 | 218,964 | 184,919 | 38,483 |
| Limited number of Premiums . . | 1 | 300 | 9 | 7 | 124 | 89 | 75 | 49 |
| Commuted Premiums . . . . . | 17 | 1,761 | .. | .. | 912 | .. | .. | 912 |
| Ascending Premiums . . . . . | 27 | 19,400 | 467 | 394 | 7,752 | 9,166 | 7,741 | 10 |
| Endowment Assurances . . . . | 396 | 64,835 | 2,578 | 2,178 | 36,029 | 25,021 | 21,130 | 14,898 |
| Do. (Commuted Premium) . | 1 | 300 | .. | .. | 234 | .. | .. | 234 |
| Joint Lives . . . . . . . . | 58 | 12,753 | 631 | 533 | 7,008 | 6,884 | 5,814 | 1,195 |
| Last Survivor . . . . . . . | 3 | 500 | 8 | 7 | 171 | 118 | 100 | 72 |
| Do. (Commuted Premium) . | 2 | 65 | .. | .. | 27 | .. | .. | 27 |
| Temporary Assurances . . . . | 70 | 103,680 | 2,094 | .. | 1,047 | .. | .. | 1,047 |
| Endowments (with return) . . . . | 118 | 14,717 | 679 | .. | 6,278 | .. | .. | 6,278 |
| Do. (Commuted Premium) . | 10 | 1,394 | .. | .. | 1,047 | .. | .. | 1,047 |
| Endowments (without return) . . . | 4 | 400 | 13 | 10 | 300 | 55 | 46 | 253 |
| Do. (Commuted Premium) . | 3 | 650 | .. | .. | 455 | .. | .. | 455 |
| Endowments with two-thirds returnable | 7 | 1,000 | 40 | .. | 377 | .. | .. | 377 |
| Contingent Survivorship Assurances . | 15 | 12,600 | 223 | 188 | 1,616 | 1,660 | 1,402 | 214 |
| Do. (Commuted Premium) . | 1 | 60 | .. | .. | 4 | .. | .. | 4 |
| Special . . . . . . . . . | 1 | 3,000 | 68 | 57 | 1,027 | 821 | 694 | 334 |
| Additional Reserve for certain Policies . | .. | .. | .. | .. | 768 | .. | .. | 768 |
| Issue . . . . . . . . . . | 2 | 5,000 | .. | .. | 100 | .. | .. | 100 |
| Extra Premiums payable . . . . | .. | .. | 733 | .. | 366 | .. | .. | 366 |
| Total Assurances without Profits | 1,820 | 763,425 | 24,168 | 17,415 | 289,043 | 262,778 | 221,921 | 67,123 |
| Total Assurances . . . . . | 8,103 | 2,218,300 | 72,471 | 53,259 | 974,542 | 845,841 | 664,150 | 310,392 |
| Deduct Reassurances . . . . | .. | 56,303 | 1,353 | 375 | 7,719 | 5,587 | 4,537 | 3,182 |
| Net Amount of Assurances . . | 8,103 | 2,161,997 | 71,118 | 52,884 | 966,823 | 840,304 | 659,613 | 307,210 |
| **ANNUITIES.** | | Per annum. | | | | | | |
| Immediate . . . . . . . . | 38 | 1,372 | .. | .. | 8,894 | .. | .. | 8,894 |
| Reversionary . . . . . . . | 10 | 896 | 124 | 106 | 1,548 | 1,048 | 891 | 657 |
| | | | | | | | | 9,551 |

### SUMMARY *and* VALUATION *of the* ANNUITIES *of the* GENERAL ANNUITY TRUST FUND, *as at* 31*st December* 1873.

| | PARTICULARS OF THE ANNUITIES FOR VALUATION. | | | | | | | VALUATION. VALUE BY THE CARLISLE TABLE. INTEREST 4 PER-CENT. | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Number of Annuities. | Amount of Annuity. | | | Yearly Premium. | | | Annuities. | Yearly Premium. | Net Liability. |
| | | £ | *s.* | *d.* | £ | *s.* | *d.* | £ | £ | £ |
| Immediate ............ | 1,771 | 24,856 | 6 | 2 | ... | | | ... | ... | 201,826 |
| Survivorship ......... | 1,863 | 25,872 | 15 | 0 | 5,302 | 5 | 1 | 86,202 | 38,376 | 47,826 |
| Deferred.............. | 17 | 170 | 0 | 0 | 65 | 14 | 3 | ... | ... | 1,196 |
| Total of the results | 3,651 | 50,899 | 1 | 2 | 5,367 | 19 | 4 | 86,202 | 38,376 | 250,848 |

### VALUATION BALANCE SHEET *of the* GENERAL ANNUITY TRUST FUND, *as at* 31*st December* 1873.

| DR. | £ | *s.* | *d.* |
|---|---|---|---|
| To Net Liability under Annuity transactions . . | 250,848 | 0 | 0 |
| „ Surplus . . . . . . . . . . . | 31,317 | 6 | 0 |
| | £282,165 | 6 | 0 |

| CR. | £ | *s.* | *d.* |
|---|---|---|---|
| By Annuity Fund . . . . . . . . . | 282,165 | 6 | 0 |
| | £282,165 | 6 | 0 |

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